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DESIGN OF DIRECT-CURRENT MACHINES
DESIGN OF TRANSFORMERS
DESIGN OF ALTERNATING-CURRENT MACHINES

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed. At the end of the volume will be found a complete index, so that any subject treated can be quickly found.

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DESIGN OF ALTERNATING-CURRENT MACHINES—*Continued*

DESIGN OF DIRECT-CURRENT MACHINES

(PART 1)

FEATURES OF ARMATURE DESIGN

GENERAL REMARKS

1. When a direct-current generator or motor is to be designed, the requirements nearly always give the characteristics of the circuit with which the machine will be connected, the output, and some of the chief operating characteristics, as efficiency and heating limits; in case of engine-type generators or direct-connected motors, the speed is also given. The designer must usually determine the general form of the machine and must then aim to secure a skilful balancing of many factors of a widely diverging nature. Electrical designing is thus largely a matter of selecting values, within known limits, that will make a safe, reliable, and economical machine with the specified characteristics.

The greatest problem of the designing engineer is to produce good machines at low cost Good insulation of the windings makes for safety and reliability of operation; generous allowances for the cross-sectional areas of copper and iron reduce the machine losses and make for efficiency, but if these factors are increased unduly, the cost of the apparatus may be prohibitive.

The process of design as here treated is to assume *trial values*, taken from rules, tables, and curves, for some of the factors, carry through calculations until the probable results may be

determined, and then, if necessary, modify the original assumptions as indicated by the information thus secured.

When making trial calculations, only *approximate accuracy* should be sought, as the results are used simply to make an intelligent selection for the final value. Further calculation involving the final value just found in determining another *final step* in the design may be made with greater accuracy, although in almost every case, accuracy beyond the third significant figure is not essential. No definite rule can be made for the degree of accuracy for all design calculations.

A density of 70,000 lines per square inch may be economical for one particular shape of steel magnet yoke, but it may not be desirable to use this density for yokes of other shapes. The best density to use may be determined by assuming several densities, such as 65,000, 70,000, 75,000, and 80,000 lines per square inch and calculating the size and weight of the yokes and of the corresponding field coils. The density that is finally selected is the one that seems to give the best results at least cost.

This process of calculating with values either side of the first trial value involves a considerable amount of mathematical work, but the resulting design will probably be economically correct. An economically correct design does not become obsolete, because it is only when the same or better results can be obtained at a lower cost that designs are superseded. The mathematical work is usually done with the aid of a *slide rule*, which is a simple form of calculating device and which gives results close enough for practical purposes.

In order to assist reference to final values in the following design problems, the more important of these are printed in Italics.

DEVELOPMENT OF ARMATURE-DESIGN FORMULAS

2. Electromotive Force.—An electromotive force of 1 volt is generated in a conductor when it is cutting magnetic lines at the rate of one hundred million per second. From this fact the fundamental design formulas for direct-current generators and motors are deduced.

For any direct-current, closed-coil armature winding, let

p = number of poles on the field-magnet frame;

Φ = flux entering or leaving the portion of the armature core adjacent to each pole piece;

S = number of revolutions per minute of the armature;

f = number of face conductors on the armature;

m = number of paths, or circuits, through the winding.

Each face conductor will cut $p\,\Phi$ lines in a single revolution, or $\frac{p\,\Phi S}{60}$ lines in one second. The average voltage developed by each face conductor will be $\frac{p \cdot \Phi S}{60 \times 10^8}$.

In a closed-coil armature, the arrangement of the conductor is such as to form m paths, or routes, in parallel from one set of brushes through the winding to a set of brushes of opposite polarity. The value of m may be 2, 4, 6, or other multiple of 2. Each of the paths has substantially the same number of conductors in series along it; hence, the average number of face conductors in series per path will be $\frac{f}{m}$. The voltage E_i developed along each path, and, therefore, the *internal voltage* developed by the armature, will be $E_i = \frac{p\,\Phi S}{60 \times 10^8} \times \frac{f}{m}$; or,

$$E_i = \frac{p\,\Phi f S}{60 \times 10^8 \times m}$$

This equation is applicable to all types of closed-coil armature windings. For open-coil armature windings, the average number of face conductors in series may be substituted for $\frac{f}{m}$.

3. Power Developed Within the Armature.—The power developed within an armature, expressed in watts, is the product of its internal electromotive force and current. If I represents the total current either entering or leaving an armature having m paths in parallel, the current per conductor will be $i = \frac{I}{m}$, or the total current will be $I = m\,i$. The total electrical

power, in watts, developed within the armature winding will be

$$P_i = E_i \times I = \frac{p\,\Phi\,f\,S}{60 \times 10^8 \times m} \times m\,i;\text{ or}$$

$$P_i = \frac{p\,\Phi}{10^8} \times f\,i \times \frac{S}{60} \qquad (1)$$

The quantity $p\,\Phi$ is the product of the magnetic flux per pole and the number of poles around the armature and may be called the *total magnetic flux* of the machine. The quantity $f\,i$ is the product of the number of face conductors and the current in each and may be called the *total armature ampere-conductors*. The speed of rotation of the armature, expressed in revolutions per second, will be $\frac{S}{60}$. Equation **1** may, therefore, be written

$$P_i = \frac{\text{Total flux} \times \text{total amp.-cond.} \times \text{rev. per sec.}}{10^8} \qquad (2)$$

4. It is convenient, for purpose of analysis and design, to measure the total magnetic flux and the total ampere-conductors of an armature in units of the dimensions of the armature core. Thus,

Let d = diameter of armature core, in inches;
l = length of armature core, in inches;
$\%$ = per cent. of armature core surface covered by main pole faces;
B = average magnetic density in air gap.

Then, the total area of the cylindrical surface of the armature core will be $\pi\,d\,l$; the surface of the armature core covered by the main poles will be $\%\,\pi\,d\,l$; and the total flux will be

$$p\,\Phi = \%\,\pi\,d\,l\,B \qquad (1)$$

The total ampere-conductors $f\,i$ may be divided by the circumference of the armature core, in inches, and the quotient will be the ampere-conductors per inch of circumference, which may be represented by K and called an *electric unit of activity*. Then, $f\,i = \pi\,d\,K$; or

$$K = \frac{f\,i}{\pi\,d} \qquad (2)$$

By substituting these values for $p\,\Phi$ and $f\,i$ in formula **1**, Art. **3**,

$$P_i = \frac{\% \, \pi^2 \, d^2 \, l \, B \, K}{10^8} \times \frac{S}{60} \qquad (3)$$

The electrical power developed within the armature is now expressed in terms of the spread of the poles, of the dimensions of the core, of a unit of magnetic density, of an electrical unit of activity, and of the speed. Since the electromotive force, current, number of poles, number of face conductors, and number of paths do not directly enter the equation, the output of an armature, measured in watts, may be calculated independently of the variations of these quantities.

5. Conversion of Mechanical Into Electrical Energy. The internal voltage E_i and the internal power P_i of the armature have been considered in the preceding formulas. The output of a generator, measured at its terminals, is less than the internal electrical power because of internal electrical losses; likewise, the output of a motor, measured at its pulley, is less than the internal mechanical power, because of the internal mechanical losses.

A generator is usually self-excited; therefore, part of the electrical energy generated is required to energize the field coils and maintain the magnetic flux. If the field windings are connected in series with the armature, such as commutating-pole windings and series-field windings, the terminal electromotive force will be less than the internal electromotive force by the sum of the drops of potential in the armature winding, the brushes, the commutator, and in the series-connected field windings. If there is also a shunt-field winding, the external load current will be less than the armature current by the value of the current diverted through the shunt-field circuit.

The electrical energy supplied to a motor must not only cause it to deliver its rated horsepower at the pulley or shaft, but must also maintain the armature in rotation against the friction of the bearings, the windage friction, the brush friction on the commutator, and against the iron losses in the armature core.

The power required for friction, windage, and core losses in a generator is supplied by the source of mechanical energy before the conversion of mechanical into electrical energy. In the case of a motor, the current required for exciting the field magnets and the electromotive force for maintaining the main currents through the commutator and windings are supplied by the source of electrical energy before the conversion of electrical into mechanical energy. Thus, there are losses in both generators and motors not only before but after conversion. The conversion of mechanical into electrical energy or electrical into mechanical energy does not in itself involve any loss.

In a generator, the mechanical losses occur before the conversion and the electrical losses afterwards; therefore, the internal power converted is the sum of the output and the electrical losses.

In a motor, the electrical losses occur before conversion and the mechanical losses afterwards; hence, the internal power of a motor is the sum of the output and the mechanical losses.

6. Efficiency.—The efficiency of a generator or motor is usually expressed in percentage, and is the ratio of the power output to the power input. The power input is the sum of the power output and the losses. If U represents the total efficiency; P, the power output; P_e, the power lost electrically; and P_m, the power lost mechanically, the last three values being expressed in watts, then,

$$U = \frac{P}{P+P_e+P_m} \qquad (1)$$

Since there are both electrical and mechanical losses, the efficiency may be divided and expressed as the mechanical efficiency, and the electrical efficiency. In a *generator*, the power converted from mechanical into electrical power is the output plus the electrical losses, and this may be considered the mechanical output; that is, $P+P_e$. The *mechanical efficiency* of a generator will then be

$$U_m = \frac{P+P_e}{P+P_e+P_m} \qquad (2)$$

The *electrical efficiency* of a generator will be

$$U_e = \frac{P}{P+P_e} \qquad (3)$$

The *total efficiency*, also called the *commercial efficiency*, or, simply, the *efficiency*, will be $U = U_m \times U_e = \frac{P+P_e}{P+P_e+P_m} \times \frac{P}{P+P_e} = \frac{P}{P+P_e+P_m}$.

7. If there are no data to the contrary, the electrical and mechanical efficiencies may be considered as equal and each may be taken as equal to the square root of the total efficiency U, expressed as a decimal. The power converted from mechanical to electrical power in a generator may be taken as $P_i = P+P_e = \frac{P}{U_e}$; or

$$P_i = \frac{P}{\sqrt{U}}, \text{ approximately}$$

In the case of a motor, the mechanical efficiency will be $U_m = \frac{P}{P+P_m}$, and the electrical efficiency $U_e = \frac{P+P_m}{P+P_m+P_e}$. The electrical power converted into mechanical power in a motor will be $P_i = P+P_m = \frac{P}{U_m}$, or $P_i = \frac{P}{\sqrt{U}}$, approximately. This value of P_i should be used in formula **3**, Art. **4**, for generator- or motor-design problems.

8. The efficiencies of generators and motors vary with the output. In machines of the same output, usually those of higher speed will be most efficient. Machines developing higher electromotive forces will have relatively smaller currents to commutate, hence the commutator losses will be smaller and the efficiency will be slightly higher. Low-speed machines will usually have low mechanical losses and relatively higher electrical losses, whereas in high-speed machines the mechanical losses may exceed the electrical. A tabulation of the efficiencies of generators and motors would probably be more misleading than helpful, but for the purpose of obtaining the

value of P_i, Table I may be considered as applying to moderate-speed belted machines operating in connection with circuits having voltages ranging from 230 to 250. The ratings of the motors in horsepower or of the generators in kilowatts are indicated in the first column.

TABLE I

MACHINE EFFICIENCIES

Horsepower or Kilowatts	Efficiency Per Cent.	Horsepower or Kilowatts	Efficiency Per Cent.
$\frac{1}{4}$	50 to 60	25	88 to 90.
1	76 to 82	50	89 to 91.
3	81 to 85	100	90 to 91.5
10	86 to 88	300 and up	91 to 93.

SELECTION OF ARMATURE DESIGN VALUES

9. Percentage of Armature Surface Covered by Poles.—The following instruction relates to the range of values for the terms used in the design formulas.

The percentage of the armature surface covered by main poles, designated by the symbol %, should be as large as is practicable, because the larger the area of the pole faces, the greater the flux for a given density, or the less the density for a given flux. To provide for commutation a space must be left between adjacent poles. This space also serves to limit to a reasonable value the flux leaking from pole tip to pole tip either between main poles or between main poles and commutating poles.

The value of % usually ranges from 50 per cent. to 80 per cent., depending chiefly on the pole pitch, measured at the armature circumference. The values in Table II may be considered reasonable.

10. Density in the Air Gap.—The density B in the air gap between the pole faces and the armature core ranges from

25,000 to 75,000 lines per square inch, depending on the size of the machine, the material in the pole faces, and the resulting density in the armature teeth. The density B can be higher in large machines than in small ones. It cannot economically be over 35,000 lines per square inch with cast-iron pole faces, while with cast steel or laminated iron pole faces so much of the metal in the armature is cut away by the slots that the tooth densities will usually be as high as is practicable when B is from 50,000 to 70,000 lines per square inch.

Consideration of the air-gap density is useful in the first approximate calculations, but a more important consideration

TABLE II

PORTION OF ARMATURE SURFACE COVERED BY MAIN POLES

Pole Pitch, in Inches, Center to Center Distance Between Adjacent Main Poles, Measured at the Armature Circumference	Polar Span	
	With Commutating Poles Per Cent.	Without Commutating Poles Per Cent.
3		50 to 60
5	50 to 60	55 to 65
7	55 to 65	60 to 70
9	60 to 68	65 to 75
12	65 to 72	68 to 78
15 and above	68 to 75	70 to 80

in the further development of a design problem is the density in the teeth. A very considerable expenditure of ampere-turns is necessary when the density in the teeth exceeds a value that may be from 120,000 to 140,000 lines, depending on the steel used. It is necessary to so proportion the different cross-sections of the magnetic circuit that the expenditure of ampere-turns to maintain the flux is reasonable for the size of the machine. High densities are allowable and economical on larger machines, whereas lower densities are necessary on small ones.

11. Ampere-Conductors and I^2R Losses.—The value of the ampere-conductors per inch of circumference of the armature core is restricted by the room available for the armature slots. If the room is restricted, the number of amperes will depend on the allowable heating of the conductors by the current.

The losses in an armature are the core losses and the I^2R losses in the conductors. In machines of low speed, the core losses are low and the I^2R losses constitute the principal heating factor. In higher-speed machines, the increased cooling effect of the armature by the faster rotation is sufficient not only to dissipate heat occasioned by the increased core losses, but also to dissipate additional I^2R losses, thus allowing the armature windings to conduct safely a larger current. The armature heating may, therefore, be estimated as based on the I^2R loss only, provided data on this heating is observed and tabulated without including the core losses. The conductors are distributed around the armature circumference, and it is convenient to reduce the I^2R loss to the watts per square inch of cylindrical surface.

12. Ordinary generators and motors are guaranteed by their makers to operate continuously without exceeding 35° C., 40° C., or 50° C. rise above a room temperature of 25° C. In operation, the copper conductors in a machine will be at a temperature of from 60° C. to 75° C.

The resistance of a copper wire of uniform cross-section at a temperature of 25° C. may be computed from the formula $R = \frac{10.8 \times \text{length in feet}}{\text{area in circular mils}}$, in which 10.8 ohms is taken as the resistance of a mil-foot of copper wire at 25° C. If the wire has a temperature of 60° C., the value 12 ohms should be used as the resistance of a mil-foot of copper wire, and the formula becomes $R = \frac{12 \times \text{length, in feet}}{\text{area in circular mils}}$, or

$$R = \frac{\text{length, in inches}}{\text{area in circular mils}}$$

13. In Fig. 1 is shown a portion of an armature core indicating four slots and the conductors therein. The conductors per square inch of the surface of the armature core are to be considered. Let the number of ampere-conductors per inch of circumference be represented by K. Consider that the average number of conductors under one inch of circumference are placed in one stranded cable, each strand conductor being of the same size and carrying the same current as a slot conductor. The cable will, therefore, carry a total current of K amperes, that is, one large conductor $\times K$ amperes gives the same number of ampere-conductors as the product of any number of conductors and of any value of current that gives as a result the numerical value of K. If the current in each strand is i amperes, the number of strands in the cable will be $\frac{K}{i}$. If the sectional area of each strand is a circular mils, the circular mils per ampere will be $\frac{a}{i}$, and the total area of $\frac{K}{i}$ strands will be $\frac{K}{i}a$, or $K\frac{a}{i}$. According to the formula of Art. **12,** the warm resistance of 1 inch of length of this cable will be $R=\frac{1}{K\frac{a}{i}}$. As this length of cable covers 1 inch each way on the armature surface, the heating loss per square inch of cylindrical surface of the armature core with K amperes will be $K^2\times\frac{1}{K\frac{a}{i}}=\frac{K}{\frac{a}{i}}$, or

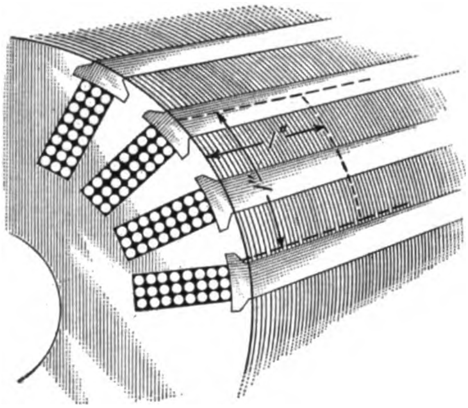


FIG. 1

$$I^2R \text{ loss per square inch} = \frac{\text{ampere-conductors per inch}}{\text{circular mils per amp.}}$$

14. In Fig. 2 are shown curves indicating the rise in temperature of an armature core at various peripheral speeds in which the I^2R loss in the armature conductors is 1 watt for every square inch of cylindrical surface. The rise in temperature for other values of loss per square inch is proportional to these losses.

By well-ventilated armatures is meant those having air spaces, or vents, $\frac{3}{8}$ or $\frac{1}{2}$ inch wide and spaced not over 3 inches

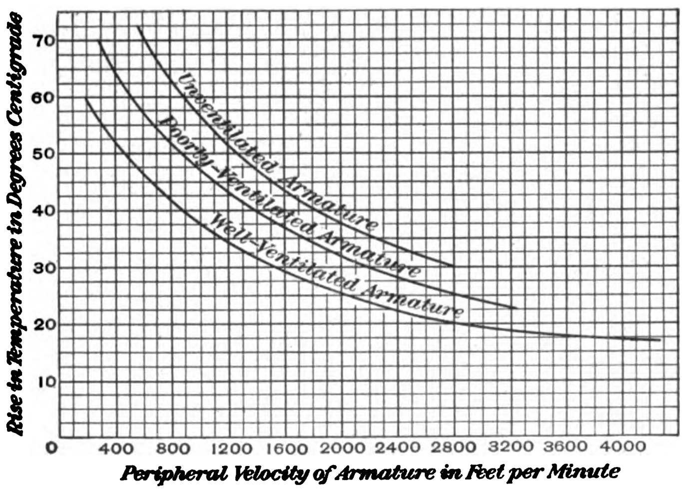


FIG. 2

apart on the axial length of the core. Poorly ventilated and unventilated armatures have fewer ventilating spaces or perhaps none at all.

The rise in temperature represents the difference in readings between a thermometer placed in the room a few feet from the machine and one placed on the surface of the armature core at the finish of a 6- to a 10-hour run under full-load conditions.

EXAMPLE.—A well-ventilated armature core is $11\frac{1}{2}$ inches in diameter and is rotated at the rate of 400 revolutions per minute. What is the least number of circular mils per ampere that should be provided for the armature conductors if there are 450 ampere-conductors per inch and the temperature rise is not to exceed 40° C.?

SOLUTION.—The peripheral speed is $\frac{11.5 \times 3.1416 \times 400}{12} = 1{,}200$ ft. per min., approx. The lower curve, Fig. 2, indicates that for this speed and 1 watt per sq. in., the rise in temperature is 34° C. above that of the room. For 40° C. rise, the number of watts per square inch can be $\frac{40}{34}$. Then, by the formula of Art. **13**, in which $K = 450$, $\frac{40}{34} = \frac{450}{\frac{a}{i}}$, and $\frac{a}{i} = 450 \times \frac{34}{40}$ $= 383$, cir. mils per amp., at least. Ans.

15. In machines not equipped with commutating poles, the ampere-conductors per inch of armature circumference are restricted not only by the allowable heating, but by commutation. If the value of the ampere-conductors per inch is too high, fluxes will be established in the neutral spaces that will cause appreciable electromotive forces in the coils undergoing commutation.

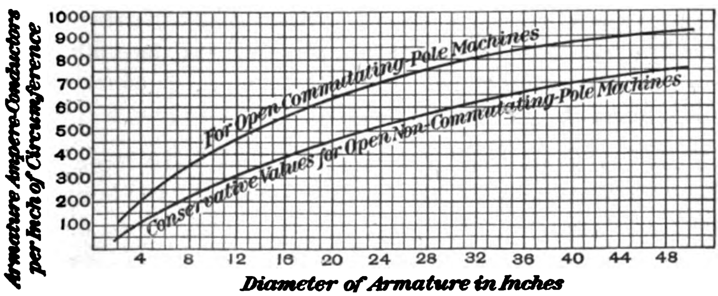


FIG. 3

The curves in Fig. 3 show the relation between the diameter of the armature and the ampere-conductors per inch of circumference and relate to open machines under continuous operation. These values may be slightly exceeded if the ventilation is very good or when the service is of an intermittent character.

16. Peripheral Speeds.—The designer can use some discretion in determining the speed of belted generators and motors. But belt speeds exceeding about 5,000 feet per minute on pulleys 15 inches or more in diameter are not good practice, nor should the speed be much below 4,000 feet per minute. Moreover, the peripheral velocity of the armature

cannot be made much over 5,000 feet per minute, because of high centrifugal stresses, nor can the commutator surface be run at much over 4,500 feet per minute, because of the difficulty of keeping the brushes in good contact. The latter difficulty is more serious in the case of commutators of small diameter than in the case of those of large diameter and also more difficult in the case of long commutators than in the case of short ones.

Generators for direct connection to steam turbines sometimes operate at armature peripheral speeds of 8,000 to 10,000 feet and at commutator speeds up to 6,000 or 7,000 feet per minute. Machines of such high speeds are specially constructed mechanically to withstand the stresses, and the brush rigging is usually most carefully designed. Notwithstanding all this careful construction, machines of these extreme speeds are difficult to operate successfully and, therefore, may be considered as exceptional.

17. Dimensions of Armature Core.—In formula **3** of Art. **4**, the values of all the quantities except d and l are known or can be assumed. The formula can be transformed thus:

$$d^2 l = \frac{60 \times 10^8 P_i}{\% \, \pi^2 B K S}$$

The expression $d^2 l$ is often called the *cylindrical inches of the armature core*. When its value is known, the values of d and l can be determined so that their relation will be within the approximate limits indicated in Table III.

TABLE III

RELATIVE DIMENSIONS OF ARMATURE CORE

Number of Poles	$\frac{\text{Armature Length}}{\text{Armature Diameter}} = \frac{l}{d}$
2	.50 to 1.00
4	.40 to .70
6	.30 to .55
8	.25 to .40
10	.20 to .30

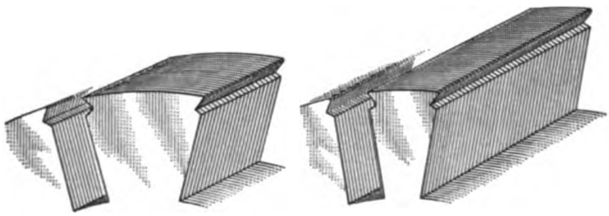
The armature core is rarely made longer than 20 inches in any case, because it becomes more difficult to secure satisfactory ventilation and also because the voltage developed by a single turn consisting of two face conductors may be excessive.

18. Total Flux and Total Ampere-Conductors.—The quantities K and B in formula **3**, Art. **4**, cannot be selected without regard to their relative values, because of the limiting conditions relating to slot room and density at the roots of the armature teeth that affect both values. If the total flux $p\,\Phi$, formula **1**, Art. **4**, be divided by the ampere-conductor $f\,i$, the quotient should come within the limits 500 and 2,500. In other words, the magnetic flux for each ampere-conductor, or, as it may be expressed, per *circumferential ampere*, should be from 500 to 2,500 magnetic lines. Machines having 500 lines per circumferential ampere would be considered *low-flux*, or *high armature-reaction*, machines; while those having 2,500 lines per circumferential ampere would be considered *high-flux*, or *low armature-reaction*, machines.

19. The cost of the material entering into the make-up of a motor or generator is influenced by the magnetic flux per circumferential ampere. A high-flux design, other things being equal, should have a larger part of its material cost expended for the magnetic circuit than would be required for a low-flux design. A high-flux design really implies greater cross-sections for the magnetic circuit, and if these circuits are equally long in the two designs, the high-flux design would require greater cost for magnetic material. It sometimes happens, however, that a short magnetic circuit may be arranged for a high-flux design, thus keeping down the cost of magnetic material. Conversely, with a low flux and high armature reaction there is implied an increased cost for copper in the electric circuits of the windings.

It is the duty of the designer to so proportion the flux and the armature reaction to the particular magnetic and electric circuits selected as to obtain the lowest possible total cost, for the materials and labor required to produce a motor or generator having a given output. A high-flux machine in combination with a long magnetic circuit should be avoided, and a high armature-reaction machine should not be so proportioned as to require a long length of turn for the coils on its armature or on its field coils.

20. The cross-section of the armature teeth for a high-flux machine should be considerably larger than that for a low-flux design. To obtain a larger cross-section, either the diameter or the length of the core may be increased, the cylindrical inches, $d^2 l$, remaining constant. Fig. 4 (*a*) shows a short core and (*b*) a long core of smaller diameter. The tooth in (*a*) is larger than the slot, while the root of the tooth in (*b*) is about the same size as the slot. The arrangement shown in (*b*) permits the use of more ampere-conductors per inch than that



(*a*) (*b*)

FIG. 4

indicated in (*a*), but does not permit of as high an average air-gap density with the same saturation at the roots of the teeth.

In design problems it is necessary that the various quantities be *relatively* satisfactory. Any quantities assumed or selected from tables or curves in the following examples are considered as trial assumptions by means of which calculations may proceed and the relative fitness of the assumed quantities are later determined. As the designs progress and more information is developed, the original assumptions may be found to need correction.

ARMATURE DESIGN PROBLEMS

ARMATURE FOR A 5-HORSEPOWER MOTOR

21. The following design problem relates to an armature for a *5-horse-power, shunt-wound motor* to be directly connected to a machine operating at *200* revolutions per minute. Electrical energy is supplied from a 115-volt circuit. After carrying full load until the temperatures are constant, the maximum temperature rise in the armature must not exceed 40° C.

22. Referring to Table I, an efficiency of 85 per cent. at full load may be selected. The electrical power converted into mechanical power P_i, formula of Art. **7**, will be $\frac{5\times746}{\sqrt{.85}}=4{,}050$ watts, approximately.

The speed of the motor is low and the short-circuited currents during commutation should not be serious, hence commutating poles will not be required. A reasonable value for the percentage of armature surface covered by the main poles is 70.

A trial value of 10 inches for the diameter of the armature is taken, and the ampere-conductors per inch, as indicated by Fig. 3, is 300, approximately. For trial, the average air-gap density may be taken as about 40,000 lines per square inch.

23. Substituting values in the formula of Art. **17** and solving the equation, $d^2\,l=\frac{60\times10^8\times4{,}050}{.7\times9.87\times40{,}000\times300\times200}=1{,}465.$

If d were 10 inches, l would have to be $\frac{1{,}465}{10^2}=14.65$ inches, which would be much too long for an armature having so small a diameter. If d were 13 inches, l would be 8.7 inches; if 14 inches, l would be 7.5 inches; and if 15 inches, l would be 6.5 inches. This machine is too large to have only two poles;

either four or six will be better. The diameter of the armature should be rather greater for a six-pole machine than for one having four poles. From $12\frac{1}{2}$ inches to 13 inches would be a good diameter for a four-pole design; whereas, if six poles are used, 14 or 15 inches would be preferable. Let it be assumed that *six poles* and a *diameter of 14 inches* are decided upon.

The curve, Fig. 3, indicates a value of 350 ampere-conductors per inch for a 14-inch armature. The pole pitch, or the distance from center to center of poles, measured on the armature circumference, is $\frac{\pi \times 14}{6} = 7.33$ inches. From Table II, a value of 70 per cent. of the pole pitch for the polar span, or spread, is shown to be reasonable.

24. The watts converted into mechanical power are about 4,050, and the line voltage is 115. The internal voltage developed by the armature conductors is less than 115 volts by the amount of the $I R$ drop necessary to maintain the current through the armature and brush resistances. The internal voltage may be taken roughly as the line voltage times the square root of the assumed efficiency, or $E_i = 115 \times \sqrt{.85} = 115 \times .922 = 106$ volts. This value will be used as a trial value, to be modified later if necessary.

The trial value of the armature current will then be $\frac{4,050}{106} = 38.2$ amperes, approximately. When the current per path does not exceed 200 or 300 amperes, a series-, or two-path, armature winding is usually employed. In this design problem, a series winding is used and the trial value of the current per path will be about 19.1 amperes.

25. The peripheral velocity of the armature is $\frac{14 \times \pi \times 200}{12} = 733$ feet per minute.

With this velocity and a radiation allowance of 1 watt per square inch of armature surface, the lower curve of Fig. 2 indicates a rise in temperature of 43° C. For 40° C. rise, the loss per square inch should be $\frac{40}{43}$ watt. By the formula of

Art. **13**, $\frac{40}{43}=\frac{350}{\frac{a}{i}}$, or the circular mils per ampere $\frac{a}{i}=350\times\frac{43}{40}$ $=376$. For 19.1 amperes per conductor at 376 circular mils per ampere, the conductor should have a cross-sectional area of 7,180 circular mils, approximately. The next larger standard wire is *No. 11 B. & S.*, having an area in circular mils of 8,234; and this wire will be selected.

26. The product of the number of ampere-conductors per inch, 350, and the circumference in inches, 14 π, gives the total number of ampere-conductors, and this number divided by the amperes per conductor, 19.1, gives the number of conductors, or $\frac{350\times 14\,\pi}{19.1}=806$, which is equivalent to 403 turns.

The number of slots in the neutral space between poles should not be less than three and preferably should be four or five. Three slots will be selected for trial calculations.

The poles cover 70 per cent. of the armature core, and the neutral spaces 30 per cent. With three slots in a neutral space,

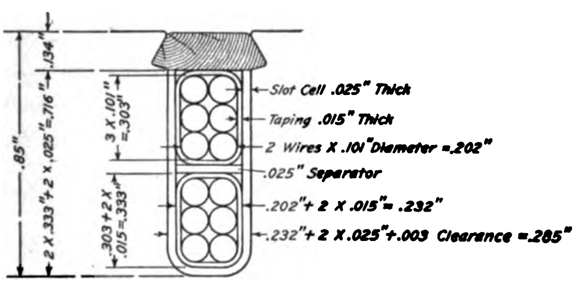


FIG. 5

there would be $\frac{100}{30}\times 3=10$ slots in the space between the centers of adjacent poles. Since there are six poles, the armature should have not less than $10\times 6=60$ slots. If 806 conductors and 60 slots are used there would not be a whole number of conductors per slot, as $\frac{806}{60}=13.4$. The trial arrangement will be twelve conductors per slot and sixty-seven slots, making a total of 804 conductors or 402 turns. The winding

will be in two layers having six conductors in the top and six in the bottom of each slot.

27. The dimensions of a slot and of the sides of the two coils placed within it are indicated in Fig. 5. The outside diameter of a No. 11 B. & S. double-cotton covered conductor is .101 inch. There are six conductors in each side of each coil and they are bound together by half-lapped tape wound to a thickness of .015 inch on each side of the coil. The slot cell has a wall thickness of .025 inch, and a separator .025 inch thick is placed between the two coils. A wooden wedge .134 inch thick is used to keep the coils in place. The main dimensions of the slot are *.285 inch wide* by *.85 inch deep*.

28. The formula of Art. 2 may be modified to read: $p\,\Phi = \frac{60 \times 10^8\, E_i\, m}{S f}$. Substituting known values in the second member of the equation, $p\,\Phi = \frac{60 \times 10^8 \times 106 \times 2}{200 \times 804} = 7{,}910{,}000$ lines, trial value of total flux.

This is a small machine and it would be inadvisable to use excessive tooth densities because high field excitation would then be required. A reasonable value of tooth root density is 125,000 lines per square inch. Since only 70 per cent. of the armature will be covered by poles, 70 per cent. of the tooth roots must carry a flux of 7,910,000 lines, hence the trial value of the area of all tooth roots will be $\frac{7{,}910{,}000}{125{,}000 \times .70} = 90.4$ square inches.

The armature slots are .85 inch deep and the armature diameter is 14 inches, hence the diameter at the roots of the teeth will be $14 - 2 \times .85 = 12.3$ inches and the circumference at the roots, $12.3 \times 3.1416 = 38.642$ inches.

The sixty-seven slots occupy $67 \times .285 = 19.095$ inches of space, leaving $38.642 - 19.095 = 19.547$ inches of space for the sum of all the tooth roots. The net length of steel in the armature will be $\frac{90.4}{19.547} = 4.62$ inches.

29. Ordinary armature punchings are made of sheet steel .014 to .025 inch thick; the thinner punchings are preferred for magnetic frequencies of forty cycles per second or over. Each punching has on its surface a small amount of scale and some japan or enamel to insulate it from its neighbors. For this reason the net length of steel in the armature core is only from 85 per cent. to 90 per cent. of the length of stacked punchings.

The magnetic frequency, in cycles per second, is equal to the product of the number of pairs of poles (one north and one south) and the speed in revolutions per second. In the case of a six-pole motor running 200 revolutions per minute, the magnetic frequency is $\frac{6}{2} \times \frac{200}{60} = 10$ cycles. As this is much below forty cycles, steel .025 inch thick will be used and 90 per cent. of the punchings will be considered as net steel.

If the calculated length of the steel is 4.62 inches, the total length of punchings is $\frac{4.62}{.90} = 5.13$ inches. As this armature was assumed to be well ventilated, one air-vent plate $\frac{3}{8}$ inch thick should be included in the middle of the core. This makes the total length of the core $5.13 + .375 = 5.505$, or $5\frac{1}{2}$ inches.

30. The armature coils should have a spread, or pitch, as nearly that of the poles as possible. If six poles cover sixty-seven slots, one pole will have a pitch of $11\frac{1}{6}$ slots, and the coils should, therefore, spread over eleven slots. The mean diameter of the armature windings, that is, the average diameter between the top and bottom of the coils, will be about $\frac{14 + 12.3}{2} = 13$ inches, approximately. The spread of the coils, measured along the cylindrical surface of the winding, will be $13\,\pi \times \frac{11 \text{ slots}}{67 \text{ slots}} = 6.71$ inches.

Since the end connections are diagonal to the spread, the length of either the front or the rear end connections may be taken approximately 25 per cent. greater than the cylindrical spread of 6.71 inches, or $6.71 \times 1.25 = 8.4$ inches for one end

connection and 16.8 inches for both. The armature core is $5\frac{1}{2}$ inches long, but to insulate the coils properly they must extend straight out beyond the core from $\frac{1}{2}$ inch to 1 inch at each end of the coil. In this case an allowance of $\frac{5}{8}$ inch will be made. The straight side of each coil will be $5\frac{1}{2}+2\times\frac{5}{8}=6\frac{3}{4}$ inches, and for both sides of the coil $6\frac{3}{4}\times 2=13\frac{1}{2}$ inches. The mean length of an armature turn will be $16.8+13.5=30.3$ inches.

31. The resistance of the windings on the armature may be calculated by a modification of the formula of Art. **12.** The resistance of one turn having a mean length of l inches and formed of a conductor having a cross-sectional area of a circular mils will be equal to $\frac{l}{a}$. There are m paths in parallel in the armature circuit; therefore, the equivalent cross-sectional area of the conductor from brush to brush will be $a\,m$. There are n turns on the armature core, but as there are m parallel paths for the current, the number of turns in series in each path will be $\frac{n}{m}$, and the length in inches of the conductor in one path will be $\frac{l\,n}{m}$. At 1 ohm per circular-mil inch, the hot resistance of the armature windings between brushes will be

$$R=\frac{\frac{l\,n}{m}}{a\,m}=\frac{l\,n}{a\,m^2}$$

Substituting values and solving the equation,

$$R=\frac{30.3\times 402}{8{,}234\times 2\times 2}=.37 \text{ ohm}$$

32. The value of the armature current can now be estimated more closely. This motor will have shunt-wound field coils only, therefore the armature resistance and brush-contact resistance are those affecting the current in the armature circuit. A brush drop of $2\frac{1}{2}$ volts may be taken as an average value for carbon brushes. The voltage impressed on the armature windings may now be taken as $115-2\frac{1}{2}=112\frac{1}{2}$ volts.

This impressed voltage will establish a current I through the winding against the generated internal voltage E_i and the resistance R. There will be 4,050 watts for conversion into mechanical power; therefore, the following equations may be formed:

$$E_i + R\,I = 112\tfrac{1}{2}$$

and

$$E_i\,I = 4{,}050, \text{ or } E_i = \frac{4{,}050}{I}$$

In this case, $R = .37$ ohm, therefore, by substituting values for E_i and R, the equation $\dfrac{4{,}050}{I} + .37\,I = 112\tfrac{1}{2}$ is formed. Then, multiplying both members by I, $4{,}050 + .37\,I \times I = 112.5\,I$, or $(112.5 - .37\,I)\,I = 4{,}050$, or $I = \dfrac{4{,}050}{112.5 - .37\,I}$. This, expressed as a formula, is

$$I = \frac{P_i}{E_c - R\,I} \qquad (1)$$

where E_c = voltage impressed on the armature windings, which will equal line voltage minus brush drop;
R = resistance of armature;
P_i = power converted in the motor (Art. 7).

The value of I may be determined by the trial method. The value $.37\,I$, which is the drop in voltage in the windings, is small compared to 112.5; therefore, I is equal, approximately, to $\dfrac{4{,}050}{112.5}$, or 36. Substituting this trial value of $I = 36$ and other known values in the right-hand member of formula 1 and solving the equation, $I = \dfrac{4{,}050}{112.5 - .37 \times 36} = 40.83$. When the correct value of I is found, its substitution in the right-hand member of the formula should result in the calculated value of I being the same as the trial value employed. In this case it does not. Therefore, 40.83 is used as a new trial value of I in the right-hand member, or $\dfrac{4{,}050}{112.5 - .37 \times 40.83} = 41.58$.

Using 41.58 as a trial value, the result is 41.7; using 41.7, the result is 41.72; using 41.72, the result is 41.727; using 41.727, the result is 41.727. The new value of I will be taken as 41.73 amperes.

The following formula may also be used for determining the value of I:

$$I = \frac{E_c}{2R} - \sqrt{\left(\frac{E_c}{2R}\right)^2 - \frac{P_i}{R}} \qquad (2)$$

Substituting values, $I = \frac{112.5}{2 \times .37} - \sqrt{\left(\frac{112.5}{2 \times .37}\right)^2 - \frac{4{,}050}{.37}} = 41.73$ amperes, which is the same as determined by the trial method.

The current per path in the armature winding will be $41.73 \div 2 = 20.9$ amperes, and the generated internal voltage will be $E_i = 4{,}050 \div 41.73 = 97$ volts.

33. It was assumed in Arts. **23** and **24** that there would be 350 ampere-conductors per inch with a current of 19.1 amperes per path. The new value of 20.9 amperes per path is 9 per cent. higher than the former value. If the value of 350 ampere-conductors per inch is to be retained and the new trial value is 20.9 amperes per path, the number of conductors should be reduced by 9 per cent. This can be done, with approximate accuracy, by reducing the number of slots from 67 to *62*, keeping the size of the armature conductor, the size of the slot, and the number of conductors per slot the same. The number of conductors will now be $12 \times 62 = 744$ conductors, the turns being *372* and the new trial value of ampere-conductors per inch being $\frac{744 \times 20.9}{14\,\pi} = 353$, which differs but little from the value 350.

34. The change in design reduces the armature resistance 9 per cent., or from .37 ohm to $.37 - (.37 \times .09) = .34$ *ohm*. The new value of the internal voltage may be determined by trial. Assume an internal voltage of 98.5 volts. Dividing 4,050 watts by 98.5 volts gives 41.12 amperes, and 41.12 amperes multiplied by .34 ohm gives a drop of 14 volts, which, added to 98.5 volts, gives 112½ volts impressed on the windings. This

value of 98.5 checks, but if it had not, other trial values could be taken.

35. The new value of the total flux may be determined by substituting known values in a modification of the formula of Art **2,** and solving the equation. Then $p\,\Phi = \frac{60 \times 10^8 E_i\, m}{S\, f}$ $= \frac{60 \times 10^8 \times 98.5 \times 2}{200 \times 744} = 7{,}940{,}000$ lines, approximately.

The flux per pole will be $\frac{7{,}940{,}000}{6} = 1{,}323{,}000$ lines. This flux enters the armature core near a north pole, passes through the teeth in two paths, as indicated in Fig. 6, and leaves the core near the adjacent south poles. There will be $1{,}323{,}000 \div 2 = 662{,}000$ lines, approximately, in each path.

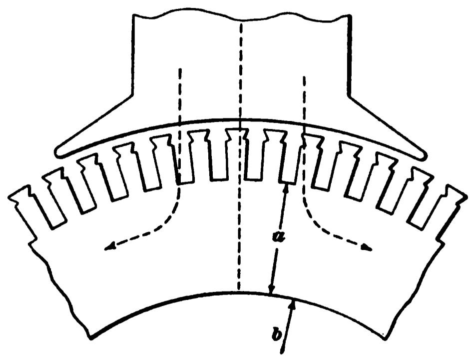


Fig. 6

A reasonable value for the density in the inner armature core is 75,000 lines per square inch. At this density, the cross-sectional area of steel in the flux path in the inner core must be $\frac{662{,}000}{75{,}000} = 8.83$ square inches. The net length of steel parallel to the shaft was calculated as 4.62 inches; therefore, the dimension a, Fig. 6, will be $\frac{8.83}{4.62} = 1.9$ inches. The diameter of the armature at the roots was calculated to be 12.3 inches; deducting twice the dimension a, Fig. 6, gives $12.3 - (2 \times 1.9) = 8\frac{1}{2}$ inches, as the inner diameter b, Fig. 6, of the punchings.

36. The core losses may be estimated if the volume of steel in the inner armature core, that in the teeth, and the

densities in these parts are known. To obtain the densities, it is best to estimate the areas of the sections through which the total flux $p\ \Phi$ passes. Dimension a, Fig. 6, is 1.9 inches, and the net length of steel parallel to the shaft is 4.62 inches. Half of the flux from one pole passes through this cross-sectional area of 1.9×4.62; therefore, since there are six poles, the area for the total flux will be $1.9 \times 4.62 \times 12 =$ *105 square inches.*

The magnetic densities in the teeth vary from moderate values at the tops to fairly high values at the roots. It is best to estimate the areas at both tops and roots and use the average value of density for estimating core losses. The circumference at the top of the teeth will be $14\ \pi = 43.98$ inches, and at the roots $12.3\ \pi = 38.64$ inches. Deducting the space occupied by sixty-two slots each .285 inch wide, or 17.67 inches, from each of these circumferences leaves 26.31 inches and 20.97 inches for the tops and roots, respectively. At the middle of the teeth the space occupied by steel is $\frac{26.31 + 20.97}{2} = 23.64$ inches. The net length of steel parallel to the shaft is 4.62, and only 70 per cent. of the circumference is covered by the poles; therefore, $26.31 \times 4.62 \times .7 =$ *85 square inches* is the effective area at the tops, and $20.97 \times 4.62 \times .7 =$ *67.8 square inches* that at the roots.

The volume of steel in the teeth will be equal to 23.64 (circumferential space occupied by steel at the middle point of teeth) $\times 4.62 \times .85$ (depth of teeth) $= 92.8$ cubic inches.

The volume of steel in the armature core, exclusive of the teeth, will be equal to the product of the net length of steel parallel to the shaft and an area equal to the difference in areas of two circles, one with the armature diameter at the roots of the teeth and the other with the diameter at the inner surface of the core. The volume, in this case, will be $[(12.3^2 \times .7854) - (8.5^2 \times .7854)] \times 4.62 = 287$ cubic inches.

37. The approximate densities for the parts of the armature core may now be calculated, the total flux being taken as 7,940,000 lines. The data shows whether or not the approximate values are satisfactory and may be collected as follows:

Part	Effective Area	Magnetic Density
Armature core (below slots) ...	105	75,600
Armature teeth at tops........	85	93,400
Armature teeth at roots.......	67.8	117,100
Armature teeth, average.......		105,250

The final value for the total flux will be somewhat greater than 7,940,000 lines, and the densities in the parts of the magnetic circuit based on the larger flux will be calculated in *Design of Direct-Current Machines*, Part 3.

38. As calculated in Art. **29**, the magnetic frequency is ten cycles per second. Fig. 7 shows a curve that may be

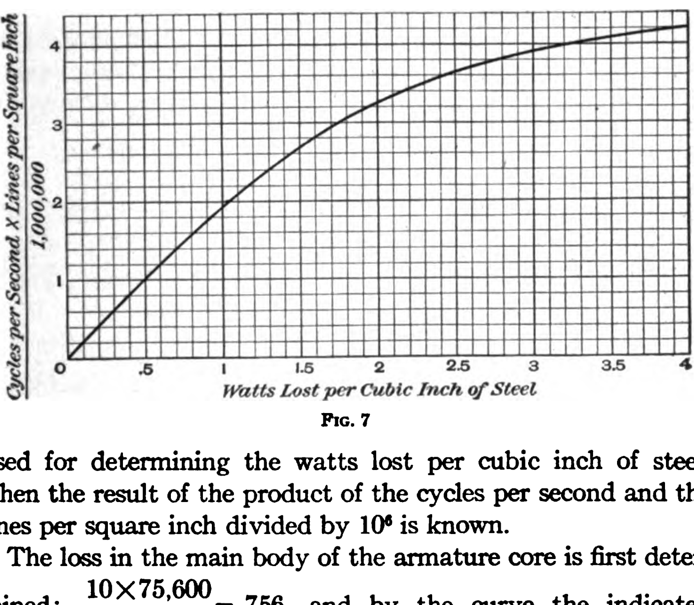


Fig. 7

used for determining the watts lost per cubic inch of steel, when the result of the product of the cycles per second and the lines per square inch divided by 10^6 is known.

The loss in the main body of the armature core is first determined: $\frac{10 \times 75,600}{1,000,000} = .756$, and by the curve the indicated watts loss per cubic inch of steel is .35. For 287 cubic inches, the loss will be $287 \times .35 = 100$ watts.

The loss in the teeth is then determined: $\frac{10 \times 105,250}{1,000,000}$ $= 1.053$, and by the curve the indicated watts loss per cubic

inch of steel is .53. For 92.8 cubic inches, the loss will be $92.8 \times .53 = 49$ watts. The total steel loss in the whole armature core will be $100 + 49 = 149$ watts.

39. Besides the core losses the other mechanical losses are brush friction, bearing friction, and windage. Bearing friction and windage are usually very small, totaling only from $\frac{1}{2}$ to 2 per cent. of the output of the machine; hence, an intelligent guess as to their value is often sufficiently accurate. In this case the speed is low and the loss will be low; 1 per cent. of the output will be ample. The output is 5 horsepower, or 3,730 watts, hence the bearing friction and windage may be taken as 40 watts.

The brush friction may be roughly estimated by multiplying together the commutator diameter, expressed in inches, the revolutions per minute, and the full-load amperes, and dividing this product by 4,000. The full-load current at a motor efficiency of 85 per cent. is $\frac{5 \times 746}{115 \times .85} = 38.2$ amperes. For purposes of estimation, the diameter of the commutator may be assumed to be 70 per cent. of that of the armature, or $14 \times .7 = 10$ inches, approximately. The loss in brush friction will be $\frac{10 \times 200 \times 38.2}{4{,}000} = 19.1$ watts.

40. The total armature losses equal 149 (teeth and core) +40 (bearing friction and windage)+19 (brush friction) =208 watts, or *210 watts*, approximately. The output of the 5-horsepower motor is 3,730 watts, and adding 210 watts for the armature losses gives a total of 3,940 watts for P_i, the watts converted from electrical into mechanical power. The first rough estimate of P_i of 4,050 watts is now corrected to *3,940 watts*.

41. The amperes in the armature may now be finally calculated. The electromotive force impressed on the windings, which is the line electromotive force less the drop in the brushes and commutator, will be $112\frac{1}{2}$ volts; the armature resistance will be .34 ohm; the power converted will be 3,940

watts. Use either of the formulas of Art. **32.** Trial values give *99* for internal voltage and *39.8 amperes* as the armature current. Substituting the values for P_i, E_c, R, and I in the right-hand member of formula **1** of Art. **32,** $I = \frac{3{,}940}{112.5 - .34 \times 39.8}$ $= 39.8$ amperes.

The current 39.8 amperes × the resistance .34 ohm = 13.5 volts $I\,R$ drop in the armature winding, and $112\frac{1}{2} - 13\frac{1}{2}$ = 99 volts for E_i the internal volts.

The new value of ampere-conductors per inch will be $\frac{39.8 \times 744}{2 \times 14\,\pi} = \mathit{337}$; and of circular mils per ampere will be $\frac{8{,}234}{19.9}$ $= \mathit{414}$.

The total flux, taking the value for the generated internal voltage as 99 and using a modification of the formula of Art. **2,** is $p\,\Phi = \frac{60 \times 10^8\,E_i\,m}{S\,f} = \frac{60 \times 10^8 \times 99 \times 2}{200 \times 744} = \mathit{7{,}980{,}000}$ *lines.*

42. The new values of flux current and generated electromotive force do not differ sufficiently from the trial values to necessitate changes in the dimensions of the armature core or its windings.

The armature calculations now being complete, the data obtained may be collected as follows:

Output, horsepower	5
Revolutions per minute	200
Outside diameter, inches	14
Inside diameter, inches	8.5
Length over laminations, inches	5.5
Air vent, one, inch	.375
Net steel in laminations, per cent	90
Slots	62
Width of slot, inch	.285
Depth of slot, inch	.85
Size of conductor	No. 11 B. & S., d. c. c.
Conductors per slot	12
Resistance when hot, ohm	.34
Total current for the two paths, amperes	39.8

Current per path, amperes.............	19.9
Counter electromotive force, volts......	99
Effective area of core for total flux, square inches......................	105
Effective area of teeth at tops for total flux, square inches..................	85
Effective area of teeth at roots for total flux, square inches..................	67.8
Mechanical losses, estimated, watts.....	210
Ampere-conductors per inch...........	337
Circular mils per ampere..............	414
Total flux, lines of force...............	7,980,000

ARMATURES FOR A 150-KILOWATT GENERATOR

43. The design problem to be taken up here relates to three armatures for a *150-kilowatt generator*, the armatures to develop outputs at *125*, *250*, and *550* volts, with a speed of *575* revolutions per minute. Such an armature might be belt-driven or be connected to an alternating-current motor having a synchronous speed of 600 revolutions per minute. In either case, the no-load speed will be about 600 and the full-load speed 575 revolutions per minute. On account of speed and output, the generator is to be provided with commutating poles.

44. An efficiency U of 91 per cent., Table I, will be selected, and from the formula of Art. **7**, $P_i = \frac{150{,}000}{\sqrt{.91}} = 157{,}000$ watts.

A reasonable value for the percentage of armature surface covered by the main poles for this commutating-pole machine is 70.

As the generator is of considerable size, densities of from 130,000 to 140,000 lines per square inch at the teeth roots and an average air-gap density of 57,500 lines may be taken as trial values.

An armature of this output and speed will be at least 24 inches in diameter. For this diameter Fig. 3 indicates a value of 700 ampere-conductors per inch, and this value will be selected for trial.

45. Substituting values in the formula of Art. **17**,

$$d^2 l = \frac{60 \times 10^8 \times 157{,}000}{.7 \times 9.87 \times 57{,}500 \times 700 \times 575} = 5{,}891$$

If d were 24 inches, l would be 10.2 inches; if d were 26 inches, l would be 8.7 inches; if d were 28 inches, l would be 7.5 inches. Any of these diameters would be reasonable. If, for instance, an armature punching die of 27 inches is available, that fact would probably determine the diameter of the core; *27 inches* will therefore be selected. The required length l of core with this diameter will be determined later.

46. When the core is wound for 550 volts a large number of commutator segments will be required; therefore, if many poles are provided, there will be too little space between brushes for the segments. If the commutator diameter is 70 per cent. of that of the armature core, it will be $27 \times .7 = 19$ inches, nearly; and the circumference will be 60 inches, nearly.

The distance from center to center of brushes, if eight poles are employed, will be $\frac{60}{8} = 7\frac{1}{2}$ inches. This is a rather small allowance for 550 volts, and, therefore, more than eight poles would be impracticable. The value, however, will be taken for trial, to be modified later.

47. According to Fig. 3, the ampere-conductors per inch for an armature diameter of 27 inches may be taken as about *750*. For a 150-kilowatt generator operating at 550 volts, the current output will be $150{,}000 \div 550 =$ *273 amperes*. When the generator has commutating poles, a current of 400 or even 500 amperes for each path of the armature windings may be successfully commutated. Therefore, a *series winding* for this armature will be chosen.

48. The current for each conductor will be $273 \div 2 =$ *136.5 amperes*. The total number of ampere-conductors for the whole core will be $3.1416 \times 27 \times 750 = 64{,}000$, approximately. The number of conductors when the current in each is 136.5 amperes, will be $64{,}000 \div 136.5 = 468$, approximately; the

number of turns and, with one turn per coil, the number of commutator segments, 234; the number of segments between centers of brushes for eight poles, $234 \div 8 = 29.3$; the average voltage between segments for a 550-volt machine, $550 \div 29.3 = 18.8$. This is a high value, but not prohibitive. The average voltage per segment could be reduced by increasing the armature diameter and putting in more conductors or by using six poles instead of eight.

If more of these generators are to be wound for 550 volts than for the lower voltages, the 550-volt design should be favored. Let it be decided to use a lesser average voltage than 18.8 between segments, and let this be accomplished by employing *six poles*.

49. As explained in *Direct-Current Generators*, the number of coils for a series winding must be such as to fit the formula

$$\text{number of coils} = \frac{p}{2} \times y \pm 1$$

in which p represents the number of poles; and y, the commutating pitch, which must be a whole number.

The number of poles is 6 and $\frac{p}{2} = 3$. For a 550-volt armature, about 240 segments or 240 coils will be required; substituting values, $240 = 3 \times y \pm 1$. The value of y must be about 80, and 240 coils substituted in the formula does not fulfil the condition that y be a whole number. If the value of y is taken as 80 and the equation solved for the number of one-turn coils, $3 \times 80 \pm 1 = 241$ or 239 coils.

Both 241 and 239 are prime numbers, and as many slots as active coils would have to be used. If a smaller number of slots were selected it would be necessary to use some dummy, or unconnected, coils, in order to make the total number of coils divisible evenly by the number of slots. In this case 240 coils might be used, 239 active and 1 dummy, with either 60 slots with 4 coils per slot or 80 slots with 3 coils per slot. It would be better, in a series winding for six poles, to avoid using 3 coils per slot. Four coils per slot with either 59 or 61 slots may be used. Let it be decided to use *59 slots* and 59×4

$= 236$ *coils*, all active. Then, $236 = \frac{p}{2} \times y \pm 1$, and $y = (236+1) \times \frac{2}{p} = 79$. The commutator pitch will be *79*; hence, if one end of a coil connects to segment No. 1, the other end of this coil will connect to segment No. 80, which is y segments removed from the starting point.

50. For the 250-volt armature, the current output will be $150{,}000 \div 250 = $ *600 amperes*. The current for each path of a *series armature winding* will be 300 amperes. Since the machine will have commutating poles, this current allowance for each path will be satisfactory. The current for each conductor, or path, will be a little greater than 300 amperes, because of the current required for the shunt field-coil circuit.

The total ampere-conductors have been estimated as 64,000. The number of conductors, when the current in each is 300 amperes, will be $\frac{64{,}000}{300} = 213$. This value may be increased to 220 conductors, which will form 110 coils; then fifty-five slots, with two coils per slot, could be used, but the number of slots per pole would be rather low and it would be better to adopt *110 slots* with *1 coil per slot*.

51. For the 125-volt armature, the current output will be $150{,}000 \div 125 = $ *1,200 amperes*. If a series winding is used, the current in each path will be 600 amperes. This is excessive, and a parallel winding with as many paths as poles will be selected. With six paths, the current per path will be $1{,}200 \div 6 = 200$ amperes.

The trial number of conductors will be $64{,}000 \div 200 = 320$, and the number of turns 160. When equalizer connections are installed, the number of coils must be divisible by the number of pairs of poles. In this case the number of pairs of poles is three, and 160 is not divisible by 3. It would be possible to use 165 turns with 55 slots and 3 coils per slot, but this is rather too few slots. Two coils per slot will be used, and the number of coils must, therefore, be an even number. The number

of coils may be 168 or 162 and the number of slots 84 or 81. It is advisable, in the case of parallel-wound armatures to be used with commutating poles, to have the number of slots divisible by the number of poles; therefore, *84 slots* and *168 coils* and segments will be selected.

52. The peripheral velocity of the armature will be $\frac{3.1416 \times 27 \times 575}{12} = 4{,}060$ *feet per minute*, approximately. From the lower curve, Fig. 2, the rise in temperature indicated for a loss of 1 watt per square inch is 17° C. For 40° C. rise in temperature, the armature I^2R loss may be as high as $40 \div 17 = 2.35$ watts per square inch.

From a modification of the formula of Art. **13,** the circular mils per ampere is equal to the ampere-conductors per inch of circumference divided by the permissible radiation allowance, expressed in watts per square inch. Substituting the selected data for $K = 750$, and loss allowance $= 2.35$ watts, the least value for $\frac{a}{i}$, or circular mils per ampere, $= K \div$ watts per square inch $= 750 \div 2.35 = 320$, approximately. With this value, there will be little margin for overloads and the efficiency will be lower than with larger armature conductors. The value, therefore, will be raised to about *400 circular mils per ampere.* This value will be used as the approximate minimum in the three armatures.

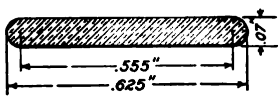


FIG. 8

53. For the 550-volt armature, the current in each conductor is 136.5 amperes, hence, at 400 circular mils per ampere, the cross-sectional area of the conductor should be $136.5 \times 400 = 54{,}600$ circular mils or 42,900 square mils, approximately. A conductor, Fig. 8, will be adopted *.07 inch thick,* $\frac{5}{8}$ *= .625 inch wide, and with rounded edges.* The cross-sectional area of the conductor, in circular mils, will be the sum of the areas of the rectangular portion $\frac{70 \times 555}{.7854} = 49{,}500$, approximately, and of the two rounded end sections 70×70, or $49{,}500 + 4{,}900 =$ *54,400.*

There are to be 4 coils per slot, as indicated in Figs. 9 and 10, and each coil is to have a single turn of conductor .07 in.$\times\frac{5}{8}$ in. Each conductor is to be wrapped with linen or cotton tape half lapped so that the thickness of tape is double on each side. The wrapped conductor is to be larger than the bare conductor in each dimension by about two times the thickness of the wrappings or four times the thickness of the tape.

Tape .005 inch thick is usually employed, but a half-lapped wrapping will usually add more than .020 inch to the conductor dimensions, due to several reasons. First, it cannot always be put on absolutely tight; and, second, the coils, after being taped, are usually coated with, or dipped into, insulating varnish or compound. The tape absorbs some of the insulating material and consequently increases in thickness. After the insulating treatment, the coils may be heated and pressed to reduce the thickness of the insulation. The final total thickness will depend somewhat on the treatment that the coils receive. In this case it is assumed that the tape will add .028 inch to the conductor dimensions. Each taped conductor will then measure *.098 in.$\times$.653 in.*

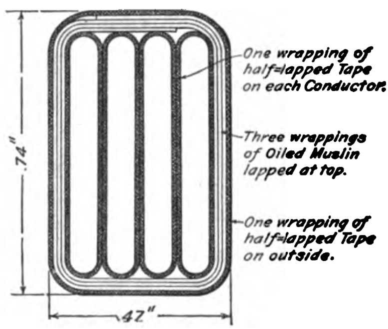


FIG. 9

54. The bundle of four coils is next wrapped along the slot portion with oiled muslin—that is, with muslin cloth coated with a linseed-oil varnish. This material serves as the more important insulation between the armature conductors and the armature core. The insulating cloth is prepared in a number of thicknesses, the thickness depending on the quality of the cloth used, which varies from the thinnest muslin to a heavy canvas. A thickness of .008-inch oiled muslin will be used for the coils; three wrappings will be used for the 550-volt coils and two wrappings for the 250- and 125-volt coils.

Each taped conductor measures .098 by .653 inch, and the bundle of four will measure .392 in. × .653 in. Three wrappings of oiled muslin, each .008 inch thick, will add .024 inch to each side, or .048 inch to the width, but since the ends of the oiled muslin are lapped over the top, the wrappings will add .056 inch to the depth. The wrapped bundle will measure (.392 + .048 = .440 inch) × (.653 + .056 = .709 inch). The bundle of coils is next taped over the entire length with .005-inch tape half lapped, which adds .028 inch to each dimension. The completely insulated bundle will measure through the slot portion .468 in. × .737 in., or practically *.47 in.* × *.74 in.*, as shown in Fig. 9.

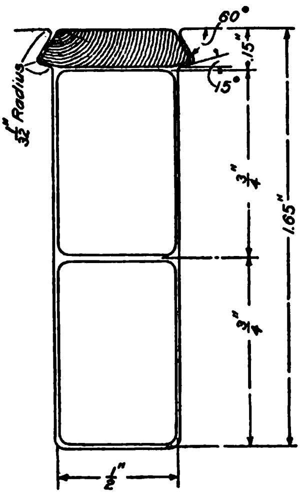


Fig. 10

When assembling the punchings on a shaft or on an armature spider, it is difficult to line them up exactly true, consequently the finished size of a slot is always a little smaller than it is punched. The size of slot in this case is selected as *.5 in.* × *1.65 in.*, as indicated in Fig. 10. The allowance of .5 − .47 = .03 inch in width is partly for additional clearance and partly because the finished slot will be smaller than the slot punched in a single lamination. Two groups of conductors and a wooden wedge are placed in each slot.

55. For the 250-volt armature, 110 slots are to be provided, and a side of each of two coils is to be placed in each slot. It will be well to punch out as much iron for the 250-volt armature as for the 550-volt armature; therefore, the punched size of the slots will be $\frac{59 \text{ slots} \times .5}{110} = .268$ *inch wide*. Allowing, in this case, about .018 inch for irregularity in lining up the punchings leaves *.25 inch* for the maximum thickness of the insulated conductor.

The copper conductor will be insulated first with two wrappings of .008-inch oiled muslin, lapped at the top, which adds .032 inch to the width; and then for the outside protection, one layer of .005-inch half-lapped tape will add .028 inch more to the width. The total width of insulation will be $.032+.028=.06$ inch.

If the finished slot has a width of .25 inch, the maximum width of the bare copper will be $.25-.06=.19$ inch, approximately. The depth of the conductor will be taken as .625 inch, as in the 550-volt design. For a trial, two conductors in parallel each .09 in. $\times$.625 in. will be considered; the corners of each conductor are to be rounded to a $\frac{1}{32}$-inch radius. The total cross-sectional area of the two conductors, Fig. 11, may be calculated by adding to the areas of the large rectangular portions of the two conductors, the areas of the circles composed of quarter sections at the corners and the areas of the small rectangles formed at the ends of the conductors. The dimensions in the following equation are in mils. The total cross-sectional area

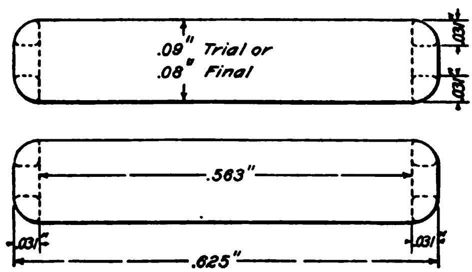


FIG. 11

$$=2\left[\frac{[625-(2\times 31)]\times 90}{.7854}+(2\times 31)^2+\frac{2[90-(2\times 31)]\times 31}{.7854}\right]$$

$=141{,}140$ circular mils. This conductor must carry a current of 300 amperes, approximately; thus, the current density will be $\frac{141{,}140}{300}=470$ circular mils per ampere.

This is a rather larger value than is necessary; a satisfactory conductor will be composed of *two strands, each of .08 in.* $\times$.625 *in. copper strip, which, with edges rounded to a radius of* $\frac{1}{32}$ *inch*, will have a total cross-sectional area of *125,200 circular mils*. The current density will be $\frac{125{,}200}{300}=417$ circular mils

per ampere. The width of the insulated conductor will be .08 +.08+.06=.22 inch.

56. For the 125-volt armature, 84 slots and 168 coils are to be provided. There will be $168 \div 84 = 2$ coils per slot, or four conductors per slot. Two insulated conductors will be grouped together and there will be two insulated groups in each slot. The amount of iron punched out for the slots will be assumed to be the same as for the 550-volt armature, which has fifty-nine $\frac{1}{2}$-inch slots. The width of the punched slot will be $\frac{59 \times .5}{84} = .351$ *inch;* allowing, in this case, about .015 inch for irregularities in assembling the punchings leaves *.336 inch* for the finished size of the slot and the size of the coil. Deducting .06 inch for tape and two wrappings of oiled muslin around each group of conductors leaves .276 inch for the width of two insulated conductors, or .138 inch for each insulated conductor. Deducting .028 inch for half-lapped tape on each conductor leaves .11 inch for the thickness of the bare conductor. A conductor $.625 \times .11$ inch with corners rounded to $\frac{1}{32}$-inch radius will have an area of $\frac{[625-(2\times31)]\times110}{.7854}+(2\times31)^2$ $+\frac{2[110-(2\times31)]\times31}{.7854} = 86,400$ *circular mils*, approximately. The current in each conductor will be $\frac{150,000}{125 \times 6} = 200$ amperes, and the current density, $\frac{86,400}{200} = 432$ circular mils per ampere.

57. To find the length of the armature core, it is necessary first to determine the total flux through the teeth, then to assume a suitable tooth density and compute the length of core that will give this density. The internal electromotive force developed in an armature winding of a generator is greater than the terminal electromotive force by the voltage required to maintain the current through the internal resistance. In the absence of more accurate data, the internal voltage may be taken as equal to the terminal voltage divided by the square

root of the efficiency. Thus, for the 550-volt armature, $E_i = \frac{E}{\sqrt{U}} = \frac{550}{\sqrt{.91}} = \frac{550}{.954} = 577$ volts.

The trial value for the total flux, calculated by a modification of the formula of Art. 2, will be $p\,\Phi = \frac{60 \times 10^8\, E_i\, m}{S f} = \frac{60 \times 10^8 \times 577 \times 2}{575 \times 472} = 25.5$ megalines.

58. In ordinary machines, the ampere-turns required to maintain the flux through the armature teeth should be between 10 per cent. and 20 per cent. of the armature reaction, in ampere-turns per pole. The computation for the excitation ampere-turns for the teeth is explained later. If less than 10 per cent. of the armature ampere-turns per pole are expended in maintaining the flux in the teeth, the machine may be too expensive, owing to the use of too large an armature core, but if more than 20 per cent. is expended, the cost of the field coils may be excessive.

For this machine a tooth density of 140,000 lines per square inch may be assumed. For 25.5 megalines, the effective area of the roots of the teeth should be $\frac{25,500,000}{140,000} = 180$ square inches, approximately. Since but 70 per cent. of the armature is covered by poles, the total tooth-root area will be $180 \div .7 = 257$ square inches. The armature diameter is 27 inches and the slots are 1.65 inches deep; therefore, the diameter at the roots of the teeth will be $27 - (2 \times 1.65) =$ *23.7 inches*, and the circumference at that point $23.7 \times 3.1416 = 74.46$ inches. Out of this circumference fifty-nine slots each ½ inch wide, or 29½ inches, are punched, leaving *44.96 inches* as the sum of the width of all teeth at the roots.

For an area of 257 square inches, the net length of steel should be $\frac{257}{44.96} = 5.72$ inches. Assuming armature punchings .025 inch thick, 90 per cent. of the armature length will be net steel. The punching should total $\frac{5.72}{.9} = 6.36$ inches. The

length of the punchings will be taken as *6½ inches*, and a *½-inch air vent* provided in the middle of the core, will make the core *7 inches* in length. The net length of steel will be $6.5 \times .9 = 5.85$ *inches*.

59. The inner diameter of the armature core may be determined by assuming a suitable magnetic density for the central part of the core. This density may be taken anywhere between 60,000 and 95,000 lines per square inch, the former being preferable for frequencies of 60 and up and the latter for frequencies of 20 and under. A density of *70,000 lines per square inch* is a good value for this machine, the frequency of which is $\frac{575}{60} \times \frac{6}{2} = 28.75$, *or nearly 30 cycles*. If $p\,\Phi$ is 25.5 megalines, the flux per pole will be about $25.5 \div 6 = 4.25$ megalines. In the armature core this flux divides as indicated in Fig. 6, and the flux each way will be 2.13 megalines. With a density of 70,000 lines per square inch, an area of $\frac{2{,}130{,}000}{70{,}000} = 30$ square inches will be required. The net length of steel is 5.85 inches, as previously determined, therefore the radial depth of steel below the bottom of the slots, or dimension *a*, Fig. 6, will be $30 \div 5.85 = 5.13$ inches. The diameter at the roots of the teeth is 23.7 inches; therefore, if the inner diameter of the core is made 13.5 inches, the radial depth of steel under the slots will be $\frac{23.7 - 13.5}{2} = 5.1$ *inches*. The inner diameter will be taken as *13.5 inches*. The cross-sectional area for the path of one-half of the flux to or from a pole will be $5.85 \times 5.1 = 29.84$ *square inches*. The area for six poles will be $2 \times 6 \times 29.84 = 358$ *square inches*.

60. The diameter of the armature core is 27 inches, and its outer circumference will be 84.82 inches. Out of this is punched fifty-nine slots each ½ inch wide, leaving $84.82 - 29.5 = 55.32$ inches as the net circumference of all of the teeth. There are but 70 per cent. of these teeth effective in conducting the flux at any one time. The net length of the armature core

is 5.85 inches, and the effective area of all the teeth at their tops for carrying the total flux will be $55.32 \times .7 \times 5.85 = 227$ *square inches.*

The net circumference at the tooth roots, after deducting the slots, is 44.96 inches; therefore, the effective area of the teeth at the roots for conducting the total flux is $44.96 \times .7 \times 5.85 = 184$ *square inches.*

The total area of the teeth at the tops will be $55.32 \times 5.85 = 323.6$ square inches; at the roots the area will be $44.96 \times 5.85 = 263$ square inches; and the average area will be *293.3 square inches.* The teeth are 1.65 inches long, and the volume of the steel will be *484 cubic inches.*

The volume of steel below the slots will be $[(23.7^2 \times .7854) - (13.5^2 \times .7854)] \times 5.85 = 1{,}740$ *cubic inches,* approximately.

61. To approximate the mean length of an armature turn, let it be assumed that the coils will extend straight out from the slots 1 inch at each end of the armature core. As the armature core is 7 inches over all, this will make the straight conductors 9 inches long. The armature coils lie in cylindrical surfaces whose mean is the surface that passes between the two groups of coils in a slot, Fig. 10. The diameter at the bottom of the slots is 23.7 inches, and the distance from the bottom to the middle of the space between the two groups of coils is $\frac{3}{4}$ inch; therefore, the diameter of the mean cylindrical surface of the coils is $23.7 + .75 + .75 = 25.2$ *inches.* The mean pitch of the coils for a six-pole machine, measured along the arc, must average $\frac{25.2 \times 3.1416}{6} = 13.2$ *inches.* The end connections of the conductors in the slots will be about 25 per cent. longer than the calculated pitch of 13.2 inches of arc, or $13.2 \times 1.25 = 16.5$ *inches.* Each armature turn includes two face conductors and two sets of end connections; therefore, the calculated mean length of an armature turn will be $2(9 + 16.5) = 51$ inches.

62. If the above rough approximation of the mean length of an armature turn is not considered sufficiently accurate, the coil may be drawn to scale and the length of the turn determined. Fig. 12 shows a coil for the 125-volt winding developed

into a flat surface. There are to be eighty-four slots and six poles, therefore each coil will span fourteen slots. One side of the coil is in the bottom of slot *1* and the other side in the top of slot *15*, the span being fourteen slots. The center lines of slots *1* and *15* are placed to scale 13.2 inches apart; this distance is the length of the arc of the mean circumference that is spanned by the coil.

Within the slots the coils are .336 inch wide, but beyond the wrappings of oiled muslin the coils measure only $.336-.032=.304$ inch thick. A space of .04 inch is allowed between the

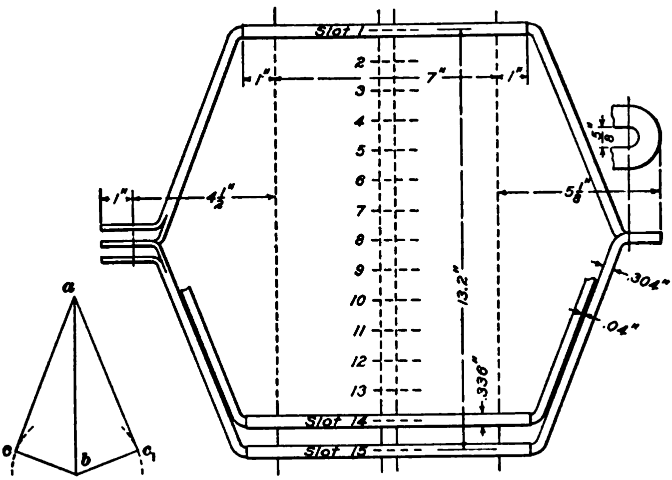


FIG. 12

end connections of the coils. The distance between centers of adjacent slots, measured on the arc of the mean circumference, will be $13.2 \div 14 = .943$ inch. Lay off to any convenient scale the distance $a\,b$ equal to .943 inch, and from b draw an arc of a circle having a radius $b\,c$ equal to $.304+.04=.344$ inch. Draw from a a line tangent to the arc of the circle, making the angle $a\,c\,b$ a right angle. The end portions of the coil should be drawn parallel to either the line $a\,c$ or $a\,c_1$ in order to secure the spacing of .04 inch between the ends of the coils.

At the rear ends the coils are bent on edge around a pin with a diameter of $\frac{5}{8}$ inch; at the front ends the conductors are

straight for an inch where they enter the commutator necks. By completing the drawing of the coil, as indicated in Fig. 12, the length of the conductor per turn may be accurately determined by scaling. The mean length of the turn, as determined from the original drawing, is *52¾ inches*.

63. The resistance of the 125-volt armature is, from the formula of Art. **31,** $R = \frac{52.75 \times 168}{86{,}400 \times 6^2} = .00285$ ohm. With a current of 1,200 amperes, the drop in voltage in the armature windings will be $1{,}200 \times .00285 = 3.42$ volts; adding 2.5 volts as an allowance for the average drop for carbon brushes, and 40 per cent. of the armature drop, or $3.42 \times .4 = 1.37$ volts, for the drop in the series and commutating-pole windings, makes the total drop $3.42 + 2.5 + 1.37 = 7.3$, or practically *7.5 volts*. The internal electromotive force of the 125-volt machine will be $125 + 7.5 = 132.5$ *volts*. The total flux will be $p\,\Phi = \frac{60 \times 10^8\, E_i\, m}{S f}$ $= \frac{60 \times 10^8 \times 132.5 \times 6}{575 \times 336} = 24.7$ *megalines*.

64. The resistance of the 250-volt armature will be $R = \frac{52.75 \times 110}{125{,}200 \times 2^2} = .0116$ ohm. With a current of 600 amperes, the drop in voltage in the armature windings will be 6.96 volts; adding 2.5 volts as an allowance for the average drop for carbon brushes and 40 per cent. of the armature drop, or $6.96 \times .4 = 2.78$ volts, for the drop in the series and commutating-pole windings, makes the total drop $6.96 + 2.5 + 2.78 = 12.24$, or practically 12.25 volts. The internal electromotive force of the 250-volt machine will be $250 + 12.25 = 262.25$ *volts*. The total flux will be $p\,\Phi = \frac{60 \times 10^8 \times 262.25 \times 2}{575 \times 220} = 25$ *megalines*, approximately.

65. The resistance of the 550-volt armature will be $R = \frac{52.75 \times 236}{54{,}400 \times 2^2} = .0572$ *ohm*. With a current of 273 amperes, the

drop in volts in the armature windings will be *15.6 volts;* adding 2.5 volts as an allowance for the average drop for carbon brushes and 40 per cent. of the armature drop, or $15.6 \times .4 = 6.24$ volts, for the drop in the series and commutating-pole windings, makes the total drop $15.6+2.5+6.24=24.34$, or practically 24.35 volts. The internal electromotive force of the 550-volt machine will be $550+24.35=$ *574.35, or practically 575 volts.* The total flux will be $p\,\Phi = \frac{60 \times 10^8 \times 575 \times 2}{575 \times 472}$ $=$ *25.4 megalines.*

66. It should be noted that the windings selected for the three voltages require practically the same total flux, and since the steel sections in the teeth and armature core are the same for all three armatures, a single calculation of the core loss is sufficient in this case for all three armatures. The core loss depends very largely on the care taken when annealing the steel, and as this is a difficult and delicate process somewhat wide variations in results may be expected. On this account very accurate core-loss calculations are not essential.

With a total flux of *25 megalines* passing through a total main armature core section, Art. **59,** of 358 square inches, the magnetic density will be $\frac{25,000,000}{358} =$ *70,000 lines per square inch*, approximately.

For an effective tooth area, Art. **60,** at the roots of 184 square inches and at the tops of 227 square inches, the densities will be 136,000 and 110,100 lines per square inch, respectively; the average density in the teeth is *123,000 lines per square inch.*

67. The frequency of the magnetic periods in a six-pole machine operating at 575 revolutions per minute will be $\frac{6}{2} \times \frac{575}{60}$ $=28.75$ cycles.

The product of the density 70,000 lines per square inch in the main core and the frequency, 28.75, divided by 1,000,000 is 2.01. Referring to Fig. 7, the indicated loss per cubic inch of steel in the main core will be 1.05 watts. In a similar

manner, for the average density of 123,000 lines per square inch in the teeth, the loss per cubic inch of steel in the teeth will be (3.54 and 2.35 on curve) 2.35 watts.

The steel loss in the main part of the armature core, which contains 1,740 cubic inches, at 1.05 watts per cubic inch, will be $1{,}740 \times 1.05 =$ *1,830 watts;* and in the teeth, which contain 484 cubic inches, at 2.35 watts per cubic inch, will be $484 \times 2.35 =$ *1,140 watts*. The total core loss will be *2,970 watts*.

68. The armature data obtained may be collected as follows:

Output, kilowatts	150
Revolutions per minute, full-load	575
Outside diameter, inches	27
Inside diameter, inches	13.5
Length over laminations, including an air vent ½ inch wide, inches	7
Net steel in lamination, inches	5.85

	125 Volts	250 Volts	550 Volts
Slots	84	110	59
Width of slot, inch	.351	.268	.5
Depth of slot, inches	1.65	1.65	1.65
Coils	168	110	236
Dimensions of conductor, inch	$.11 \times .625$	Two strands $.08 \times .625$	$.07 \times .625$
Area of conductor, circular mils	86,400	125,200	54,400
Armature resistance, ohm	.00285	.0116	.0572
Effective area of main armature core, square inches	358		
Effective area of teeth at roots, square inches	184		
Effective area of teeth at tops, square inches	227		
Estimated core loss, watts	2,970		

COMMUTATOR DESIGN

COMMUTATOR AND BRUSH DATA

69. Function of Commutator.—The purpose of a commutator of a direct-current motor or generator is to provide means of connecting the moving conductors on the armature to stationary brushes bearing on the commutator and connected to the external circuit.

In a motor, the current enters the armature windings through the positive brushes and passes out through the negative brushes; in a generator, the current leaves the armature through the positive brushes and returns through the negative brushes. In either case, the current must pass through the commutator and brush contact surfaces twice, half of the brushes carrying the current to the commutator and the other half taking it away from the commutator.

The commutator must be so designed that the armature current may be carried by the brushes and commutator without undue heating, without excessive wear or roughening of the rubbing surfaces of either, and without sparking or burning of the brushes or commutator segments.

70. Commutator and Brush Materials.—Commutators are almost invariably built up of hard-drawn or drop-forged copper segments insulated from each other by amber, mica, or micanite. Machines of 25 volts and over practically always use brushes of carbon or graphite or a combination of carbon and graphite. The materials copper and mica for the commutator and carbon and graphite for the brushes have been selected because they do not wear excessively when in rubbing contact.

The internal resistance of the carbon or graphite brush and the contact resistance between the brush and the commutator

are comparatively high, but a high resistance in the brush and in the contact surface is desirable and essential in order that the current in the coil during commutation shall not be excessive.

71. Current Density in Brush Contact.—The current density in the brush contact surface should not exceed about 30 amperes per square inch for pure carbon, or about 50 amperes per square inch for pure graphite. For combination carbon and graphite brushes, the current density may be between 30 and 50 amperes per square inch, depending on the proportions of the two materials used in making the brushes. For the purpose of designing, a current density in brushes of carbon or graphite may be taken as 30 to 35 amperes per square inch of contact surface.

72. At no load, the electromotive force between two adjacent neutral points on the commutator is practically the full voltage developed in the armature. This electromotive force is generated chiefly in the coils whose conductors are moving across the pole faces, yet the coils whose conductors are in the so-called neutral space are not entirely inactive except for a very short space. If the brushes are so thick that they span too great an angle on the commutator surface, excessive currents will be established in the coils under commutation and in the brushes even though the latter are of high resistance.

73. Thickness of Brushes.—The voltage generated in the conductors within a given angle for any relative position of conductors and pole piece will be greater for high- than for low-voltage armatures; therefore, brushes should be thinner on machines of higher voltage, assuming that the specific resistance of the brush material is about the same for the different voltages.

A rule that may be followed is to have the brush cover not more than three commutator segments in any case; for 125-volt machines the thickness should not exceed 10 per cent. of the angular space from one neutral point to the next, measured on the commutator surface; for 250-volt machines, 7.5 per cent.; and for 500-volt machines, 5 per cent.

74. Brush Resistance Loss.—The brush contact resistance depends on the material of the brush, the current density in the contact surface, and the pressure between the brush and the commutator. On ordinary stationary motors and generators, the brush-holder springs should be adjusted to give a pressure of about 2 pounds for every square inch of contact surface between the brush and the commutator.

Fig. 13 shows the total drop in volts for both the positive and negative brushes. The data is based on a brush pressure of 2 pounds per square inch of contact surface, and on brushes of average quality, containing approximately 15 per cent. of

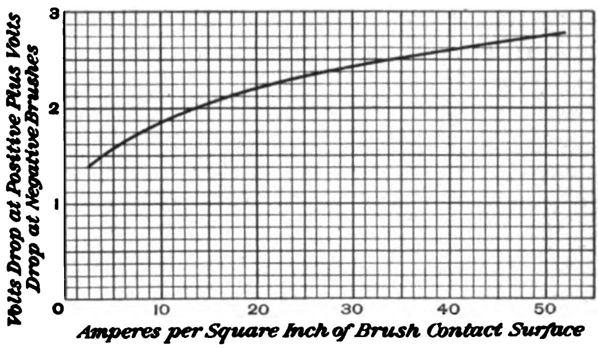


FIG. 13

graphite. Had the brush pressure been less, the voltage drop for a particular current density would have been greater; whereas, with increased brush pressure, the voltage drop would be less.

The total watts resistance loss in both of the sets of brushes and contacts is equal to the product of the total drop in volts for a positive and a negative set of brushes and the current in each set.

75. Brush Friction Loss.—Besides the loss due to brush internal resistance and contact resistance there is also the loss due to friction between the brush and moving commutator. The friction loss may be calculated, if the brush pressure per square inch, the total area of the brush contact surface, the commutator surface speed, and the coefficient of friction are

known. For brushes, composed of various mixtures of carbon and graphite, rubbing on copper, the coefficient of friction varies from .1 to .3. As much depends on the condition of the rubbing surfaces, it is well to allow the value .3 for calculations in order not to underestimate the friction loss.

If the brush pressure is 2 pounds per square inch and if the coefficient of friction is .3, then the tangential force due to the rubbing of the brush on the commutator surface will be $2\times.3=.6$ pounds for each square inch of brush contact surface. If this tangential force is multiplied by the velocity of rubbing, in feet per minute, the foot-pounds of work expended per square inch of contact surface per minute will be obtained.

The velocity v of rubbing, in feet per minute, will be $\frac{d}{12}\pi S$, in which d is the diameter of the commutator, expressed in inches; and S, the number of revolutions of the commutator per minute.

With the value .6 pound per square inch of brush-contact surface, the work per square inch of brush-contact surface, expressed in foot-pounds, will be $\frac{.6\,d\,\pi S}{12}$.

One horsepower is equivalent to 33,000 foot-pounds per minute, or 746 watts. The brush friction loss per square inch of brush contact surface, expressed in watts, will be $\frac{746\times.6\,d\,\pi S}{33{,}000\times12}$ $=.00355\,d\,S=\frac{d\,S}{282}$.

If I is the current output of the armature and i the current density in amperes per square inch of brush-contact surface, the total number of square inches of brush-contact surface, considering that current I both enters and leaves the commutator surface, is $\frac{2\,I}{i}$. The total watts P lost in brush friction will be $\frac{2\,I}{i}\times\frac{d\,S}{282}$, or

$$P=\frac{I}{i}\times\frac{d\,S}{141}$$

76. In Art. **39** it was stated that the commutator brush-friction loss may be roughly estimated by multiplying together the commutator diameter, expressed in inches, the speed, and the full-load armature amperes, and dividing the product by 4,000. Comparing this statement with the equation for P, the product of i and 141 in this case should equal 4,000, and, if so, the current density is $\frac{4,000}{141} = 28.4$ amperes per square inch.

77. Results Due to Excessive Brush Losses.—The resistance and friction losses must not be so large as to cause the commutator to heat excessively, since roughening of the current-collecting surfaces may result. With some kinds of brushes, the coefficient of friction increases with the temperature to such an extent that the brushes chatter, that is, vibrate in the holders. Any roughness or chattering will impair the brush contact surface and greatly increase its contact resistance.

78. Temperature Rise of Commutator.—It is best to keep the temperature rise of the commutator under 55° C.; temperature rises of 40° C. and 50° C. are commonly permitted for full-load continuous operation.

To estimate the rise in temperature of a commutator it is necessary to determine the number of watts it must radiate and compare this with the area of the radiating surface.

In addition to the brush losses due to their internal and contact resistances and the friction losses, there are others due to the extra currents established in the bars connected to coils undergoing commutation. These losses are liable to be higher in machines without commutating poles than in those that are so provided, unless the strength of these poles is improperly adjusted. When the currents in the short-circuited coils are excessive, visible sparking will exist, but they often cause commutator heating without visible sparking. There does not seem to be any simple method of estimating the commutator loss due to these currents. The best that can be done is to base the losses on those due to normal resistance and friction and base the heating on the total estimated loss.

79. The ability of a commutator to dissipate heat depends very much on its construction. Commutators 6 inches and larger in diameter are usually ventilated by air passages either

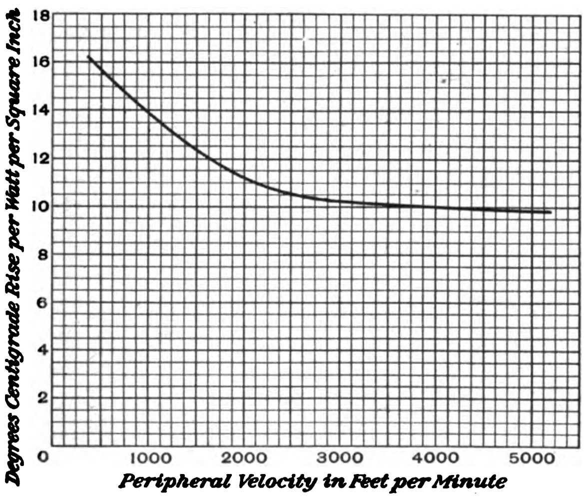


FIG. 14

between the segments and the shaft, or between the shell in in which the segments are clamped and the shaft.

Although the heat-dissipating surface of a ventilated commutator is practically all of the exposed area of the segments and shell, it is convenient, for the purpose of making calculations, to consider only the outside surface of the segments themselves.

80. Relation Between Temperature Rise and Speed of Commutator.—In Fig. 14 is shown the relation between the peripheral velocity of ventilated commutators and the degrees centigrade rise per watt per square inch of surface. In preparing the curve, certain assumptions were made in estimating the radiating area, because the results were more consistent when thus calculated.

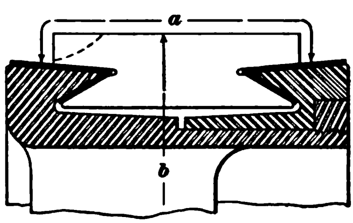


FIG. 15

Commutators in which the armature conductors were soldered directly into the segemnts, such as that shown in Fig. 15, were considered as having a length of face a equal to the length of the actual face plus twice the radial depth of the end of a segment exposed for radiation. The area was estimated as though the ·total estimated face a had the maximum diameter b.

81. The radiating-surface area of commutators in which the armature conductors are soldered into necks, which, in turn, are soldered to the segments, as shown in Fig. 16, was calculated by adding to the assumed face a the dimension c, which is the height of the commutator neck. Wherever c exceeded 4 inches, the dimension 4 inches was used. It is found that the use of these necks aids considerably in dissipating the heat of the commutator, for they not only add considerable radiating surface in themselves, but they set up by their fanning action a draft of cool air across the face of the commutator. The maximum diameter b of the main portion of commutator was used in the calculation for radiating surface area.

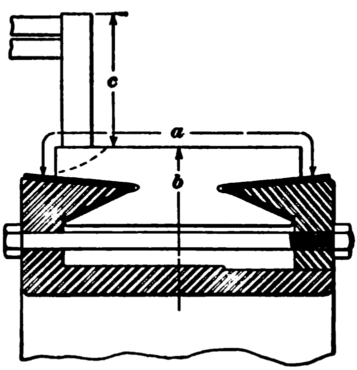


FIG. 16

COMMUTATOR DESIGN PROBLEM FOR A 5-HORSEPOWER MOTOR

82. This design problem relates to a commutator for the 5-horsepower motor armature previously designed. The number of core slots is 62 and the number of conductors per slot is 12; therefore, there are $\frac{62 \times 12}{2} = 372$ turns. The number of turns per slot is 6. The number of commutator segments cannot be more than one per armature turn, or 372, nor less than one per slot, or $372 \div 6 = 62$. The number of segments should be such

that the average volts per segment around the commutator will not be excessive, and that commutating conditions will be satisfactory.

83. In motors that are not equipped with commutating poles the following formula may be used to predetermine roughly the commutating condition:

$$X = c \times i \times t^2 \times l \times S$$

In which X = commutating condition, the value of which should not exceed 75,000,000 and preferably not exceed 50,000,000;
c = number of segments in the commutator;
i = current, in amperes per face conductor;
t = number of turns per armature coil;
l = over-all length of armature core, in inches;
S = revolutions per minute.

The length l of the armature core is 5.5 inches; the current i in each conductor, or per path, is $39.8 \div 2 = 19.9$ amperes; and the revolutions per minute S are 200. The product of c and t for any values of c and t that may be determined $= 372$ turns. The right-hand member of the equation may be written $(c \times t) \times t \times i \times l \times S$, and, by substituting values, $372 \times t \times 19.9 \times 5.5 \times 200$, or $8{,}140{,}000\,t$. If t is taken as 6, its greatest value for this armature, $X = 8{,}140{,}000 \times 6 = 48{,}840{,}000$. This is just within the limit of 50,000,000. It will be better to make $t = 3$, which will insure good commutation as the product of 8,140,000 and 3 is only 24,420,000.

84. If the number of turns per coil is 3, the number of segments will be $372 \div 3 = 124$. The machine has six poles, and a voltage of 115 exists between one commutator brush point and the next brush point $360° \div 6 = 60°$ distant. The sum of the voltages around the commutator will be $115 \times 6 = 690$ volts; and the voltage between adjacent segments will be $690 \div 124 = 5.56$, which is a low value; *124 segments* are, therefore, satisfactory.

85. The commutator diameter is usually between 60 per cent. and 75 per cent. of the armature diameter, which, in this

case, is 14 inches. The diameter of the commutator will be taken as 71.5 per cent. of 14 inches, or *10 inches*.

The mica segments between the copper segments are usually from .02 to .04 inch thick, .03 inch being an average value. To insure a good wearing surface, the total thickness of mica should not exceed 15 per cent. of the commutator circumference. Assuming a thickness per mica segment of *.03 inch*, the total thickness of mica will be $124 \times .03 = 3.72$ inches, which is $\frac{3.72}{10 \times 3.1416} \times 100 = 11.8$ per cent. of the commutator circumference, which may be considered satisfactory.

The armature conductor is No. 11 B. & S. double-cotton covered round copper wire. This can be readily bent down into a slot sawn into the end of the copper segments, a commutator neck, or lug, being unnecessary. The commutator will be similar to that shown in Fig. 15.

86. There will be three positive and three negative brush points, and since the total armature current is 39.8 amperes, the current at each brush point will be $39.8 \div 3 = 13.3$ amperes. Assuming a current density of 30 amperes per square inch of brush contact surface, the contact area per brush point should be approximately $13.3 \div 30 = .44$ square inch.

It is customary, in motors of more than one horsepower, to use two carbons per brush point so that, in the event of the commutator roughening, the lifting of a single brush will not break the circuit to that brush point. In this case, however, *one carbon per brush point* will be sufficient, since the speed is very low and the current per brush point small.

87. The percentage of the armature covered by the poles is 70 per cent. The space between the edges of adjacent poles is 30 per cent. of the pole pitch. The brush must not span over about a third of the space between the edges of adjacent poles, or $30 \times \frac{1}{3} = 10$ per cent. of the pole pitch. If all the armature coils are not alike even 10 per cent. is too great.

The commutator is 10 inches in diameter and 31.4 inches in circumference. There are six poles, the pole pitch is 31.4 $\div 6 = 5.23$ inches, and 10 per cent. of this is .52 inch. The

thickness of the brushes should be rather less than this value. Either $\frac{3}{8}$ or $\frac{7}{16}$ inch would do, but brushes $\frac{3}{8}$ *inch thick* will be selected.

If the brush area is to be approximately .44 square inch, the width will be $.44 \times \frac{8}{3} = 1.17$ inches. Either brushes $1\frac{1}{8}$ or $1\frac{1}{4}$ inches wide would do, but brushes *$1\frac{1}{4}$ inches wide* will be selected. The current density in the contact surface of the brush will be $\frac{13.3}{\frac{3}{8} \times 1\frac{1}{4}} = 28.4$ *amperes per square inch.*

From the curve, Fig. 13, the drop in voltage for both positive and negative brush contact surfaces with a current density of 28.4 amperes per square inch will be 2.4 volts; hence, the *brush resistance loss* for the full-load current of 39.8 amperes will be $2.4 \times 39.8 = 95.5$ *watts.*

The *brush friction loss* as calculated by the formula of Art. 75 will be

$$P = \frac{39.8}{28.4} \times \frac{10 \times 200}{141} = 19.9 \text{ watts}$$

The total commutator loss will be $95.5 + 19.9 = 115.4$ *watts.*

88. Fig. 17 indicates the general dimensions of the upper portion of a commutator bar. The radiating surfaces of the end portions of the bar will have a depth of $\frac{1}{2}$ inch; an allowance of $\frac{1}{2}$ inch will be provided for the slot that holds the armature leads; and a clearance for armature movement will be allowed of $\frac{1}{4}$ inch on each side of the brush, which will be $1\frac{1}{4}$ inches wide.

The total length of the segment will be $2\frac{1}{4}$ inches.

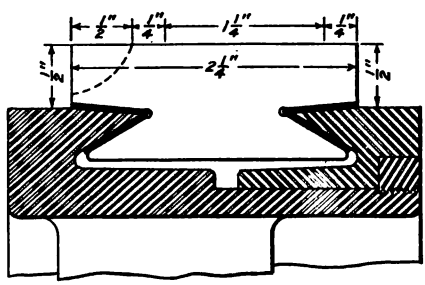


FIG. 17

The total face of the radiating surface will have an effective length of $\frac{1}{2} + 2\frac{1}{4} + \frac{1}{2} = 3\frac{1}{4}$ inches. The circumference of the commutator is 31.4 inches, and the effective radiating surface is $31.4 \times 3.25 = 102$ square inches. This surface must dissipate a total of 115.4 watts, or $\frac{115.4}{102} = 1.13$ *watts per square inch.*

89. The peripheral speed of the commutator will be $\frac{10 \times 3.14 \times 200}{12} = 523$ feet per minute. From the curve, Fig. 14, the rise in temperature per watt per square inch will be 15.6° C.; therefore, the probable rise in temperature for 1.13 watts per square inch will be $1.13 \times 15.6 = 17.6°$ *C.* As 40° or even 50° C. rise may be allowed for ordinary motors, this commutator will be considered satisfactory.

The size of this commutator is determined more by the rubbing area required by the brushes than by the radiating area necessary to prevent it from overheating. This is not an unusual condition with small outputs and low speeds.

COMMUTATOR DESIGN PROBLEMS FOR A 150-KILOWATT GENERATOR

90. The armatures for the three voltages are all 27 inches in diameter, and the commutator diameters can be from 16 to 20 inches. With one segment per coil, the number of segments each for the 125-, 250-, and 550-volt commutators will be *168*, *110*, and *236*, respectively. The total voltage around the 550-volt commutator will be the product of the number of poles, 6, and 550, or 3,300 volts. The average number of volts per segment will be $3{,}300 \div 236 = 14$. It is best to have the thickness of a mica segment at least equal to the product of .002 and the average number of volts per segment. In this case a mica segment should not be less than $.002 \times 14 = .028$ inch; mica segments of a thickness of *.03 inch* will be selected. It is well to have the thickness of the mica segment not more than 15 per cent. of the combined thickness of one mica and one copper segment; then the least combined thickness will be $.03 \div .15 = .2$ inch. For 236 combined segments, the least diameter for a 550-volt commutator will be $\frac{236 \times .2}{3.1416} = 15$ inches. Other considerations will lead to the selection of a commutator of larger diameter.

91. The current output for the **125-volt armature** is 1,200 amperes, or $1{,}200 \div 3 = 400$ amperes at each brush point.

At a current density of 30 amperes per square inch, the contact area of one set of brushes must be about $400 \div 30 =$ *13.3 square inches*.

If the commutating poles of the machine are properly designed, an angle of 10 per cent. of the pole pitch may be spanned by the brushes. The pole pitch will be $360 \div 6 = 60°$; 10 per cent. of this is 6°, or $\frac{1}{60}$ of a circumference. If the commutator had a diameter of 15 inches, a brush to span one-sixtieth of the circumference would be $\frac{15 \times 3.1416}{60} = .7854$, or about $\frac{3}{4}$ inch thick. In the same proportion, a brush $\frac{7}{8}$ inch thick will require a commutator $\frac{.875 \times 60}{3.1416} = 17$ inches in diameter. A brush 1 inch thick will require a commutator $\frac{1 \times 60}{3.1416}$ $= 19$ inches in diameter.

92. For a $\frac{3}{4}$-inch brush and an area of 13.3 square inches, the brush contact surface will be 17.7 inches long; for a $\frac{7}{8}$-inch brush, 15.2 inches long; and for a 1-inch brush, 13.3 inches long. If each of these lengths is multiplied by the respective commutator circumferences, the approximate areas of the commutator surface that comes in contact with one set of brushes during one revolution will be obtained. The cost of the commutators will be nearly proportional to these areas. There is no great difference between the values, and there is no definite advantage of one diameter over another.

Since the diameter of the armature core is 27 inches and its length is 7 inches, it would not be in keeping to use a commutator of small diameter and great length. A commutator *19 inches in diameter* will be selected for the three armatures, and on the 110-volt commutator brushes *1 inch thick* will be used. The thickness of the individual commutator segments and the mica segments for the three commutators depends on the number of segments required and on the voltage between segments. The combined maximum thickness of copper and the uniform thickness of the mica segments should equal the circumference of a circle 19 inches in diameter for the three commutators.

93. If each brush is 1 inch thick and $1\frac{1}{4}$ inches wide, eleven brushes will have a contact area of *$13\frac{3}{4}$ square inches.* The current density in the brushes will be $400 \div 13.75$ $= 29.1$ *amperes per square inch.* The drop in voltage in the brushes, as indicated in Fig. 13, is *2.41 volts*, approximately; therefore, the resistance loss in the carbons and commutator will be $2.41 \times 1{,}200 = 2{,}890$ *watts.*

94. The peripheral velocity of a commutator 19 inches in diameter at a speed of 575 revolutions per minute will be

$$\frac{19 \times 3.1416 \times 575}{12} = 2{,}860 \text{ feet per minute.}$$

The brush friction loss as calculated by the formula of Art. **75** will be $P = \frac{1{,}200}{29.1} \times \frac{19 \times 575}{141} = 3{,}200$ *watts*, approximately.

The total commutator loss is $2{,}890 + 3{,}200 = 6{,}090$ *watts.*

95. The area of the commutator radiating surface may be estimated from the data indicated in Fig. 18. The length of the brush contact surface per brush holder, provided that the holder is so arranged that the brushes abut each other on their sides, will be $13\frac{3}{4}$ inches. If the pocket type of holder is pre-

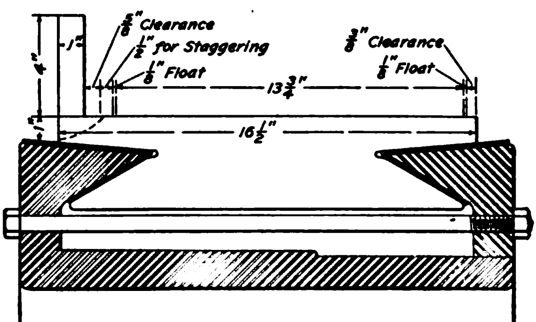


FIG. 18

ferred, ten brushes each $1\frac{1}{8}$ inches thick by $1\frac{1}{4}$ inches wide may be used. If an allowance of $\frac{1}{8}$ inch per brush is made for the metal spacing strips of the pockets and clearances, the total length between the extremities of the brush group will be $(10 \times 1\frac{1}{4}) + (10 \times \frac{1}{8}) = 13\frac{3}{4}$ inches, which is the same as with the

eleven-brush group. The brush contact surface of the ten-brush group will be $1\frac{1}{8} \times 1\frac{1}{4} \times 10 = 14$ square inches, which is practically that of the eleven-brush group.

The allowance for armature float, or end movement, Fig. 18, will be $\frac{1}{8}$ inch either way; for clearances near the ends of the commutator surface, $\frac{3}{8}$ inch and $\frac{5}{8}$ inch; and for staggering the brushes, $\frac{1}{2}$ inch. By staggering the brushes on alternate studs, the brushes on one stud tend to wear down the ridges left by the spaces between the brushes on the next stud.

Adding these allowances, the length of the net wearing surface, and the 1-inch space for the commutator neck, the length of the segment, Fig. 18, will be *$16\frac{1}{2}$ inches*. To determine the length of the effective radiating area, 4 inches may be used for one side of the commutator neck and 1 inch at either end of the segment, making a total segment length of $16\frac{1}{2}+4+1+1 = 22\frac{1}{2}$ *inches*. The diameter of the commutator is 19 inches, and the area of the effective radiating surface will be $22.5 \times 19 \times 3.1416 = 1{,}340$ square inches, approximately. This surface radiates 6,090 watts, or $6{,}090 \div 1{,}340 =$ *4.5 watts per square inch*.

From the curve, Fig. 14, for a peripheral velocity of 2,860 feet per minute, the temperature rise indicated is 10.3° C. per watt per square inch. The estimated rise in temperature will be *46.5° C.*, approximately.

If desirable to design for a lower temperature rise, the commutator may be lengthened and more carbon brushes used. In order to maintain the same area of contact and not increase the friction loss with the larger brush group, the carbons should be less in thickness than those for the shorter group.

96. The current output for the **250-volt armature** is 600 amperes. Although this armature has a series winding, it is better to use as many brush groups as poles. Also, it will be better to use brushes somewhat thinner than for the 125-volt armature, although when wide commutating poles are employed brushes covering 10 per cent. of the pole pitch may be used on 250-volt machines. Brushes *$\frac{3}{4}$ inch thick* will be selected for this machine.

For a current density of 35 amperes per square inch of brush-contact surface and for $(600\times2)\div6=200$ amperes per brush point the total contact area per brush point should be 5.7 square inches. With brushes $\frac{3}{4}$ inch thick, the length of the contact surface should be $5.7\div.75=7.6$ inches. *Six brushes $\frac{3}{4}$ inch by $1\frac{1}{4}$ inch* and having a contact area of $6\times.75\times1.25=5.625$ square inches will be used, and the current density then will be $200\div5.625=$ *35.5 amperes per square inch.* The length of the contact surface will be $6\times1.25=$ *7.5 inches.*

97. From Fig. 13, the indicated brush contact drop for a current density of 35.5 amperes per square inch is *2.52 volts;* therefore, the resistance loss for 600 amperes will be *1,510 watts.*

The brush friction loss as calculated by the formula of Art. **75** will be $P=\frac{600}{35.5}\times\frac{19\times575}{141}=$ *1,310 watts.* The total commutator losses will be *2,820 watts.*

98. The area of the radiating surface of the commutator may be estimated from Fig. 18 by substituting $7\frac{1}{2}$ inches for the $13\frac{3}{4}$ inches there shown. It is assumed that a type of brush holder will be used in which the brushes abut each other. The reduction in the length of the segment and of the effective radiating surface will be $13\frac{3}{4}-7\frac{1}{2}=6\frac{1}{4}$ inches. The total length of the segment will be $16\frac{1}{2}-6\frac{1}{4}=10\frac{1}{4}$ inches, and the length of the effective radiating surface will be $10\frac{1}{4}+4+1+1=16\frac{1}{4}$ inches. The area of the effective radiating surface will be $16.25\times19\times3.1416=$ *970 square inches.* The loss that must be radiated will be $2,820\div970=$ *2.9 watts per square inch.*

From the curve, Fig. 14, for a peripheral velocity of 2,860 feet per minute, the temperature rise indicated is 10.3° C.; therefore, for 2.9 watts per square inch, the estimated temperature rise is $10.3\times2.9=$ *30° C.*

99. The current output for the **550-volt armature** is 273 amperes, or 91 amperes per brush point. Four brushes *$\frac{1}{2}$ inch thick by $1\frac{1}{4}$ inches wide* will be used per group. The contact area will be $4\times\frac{1}{2}\times1\frac{1}{4}=2\frac{1}{2}$ square inches, and the current density will be $91\div2.5=$ *36.4 amperes per square inch.*

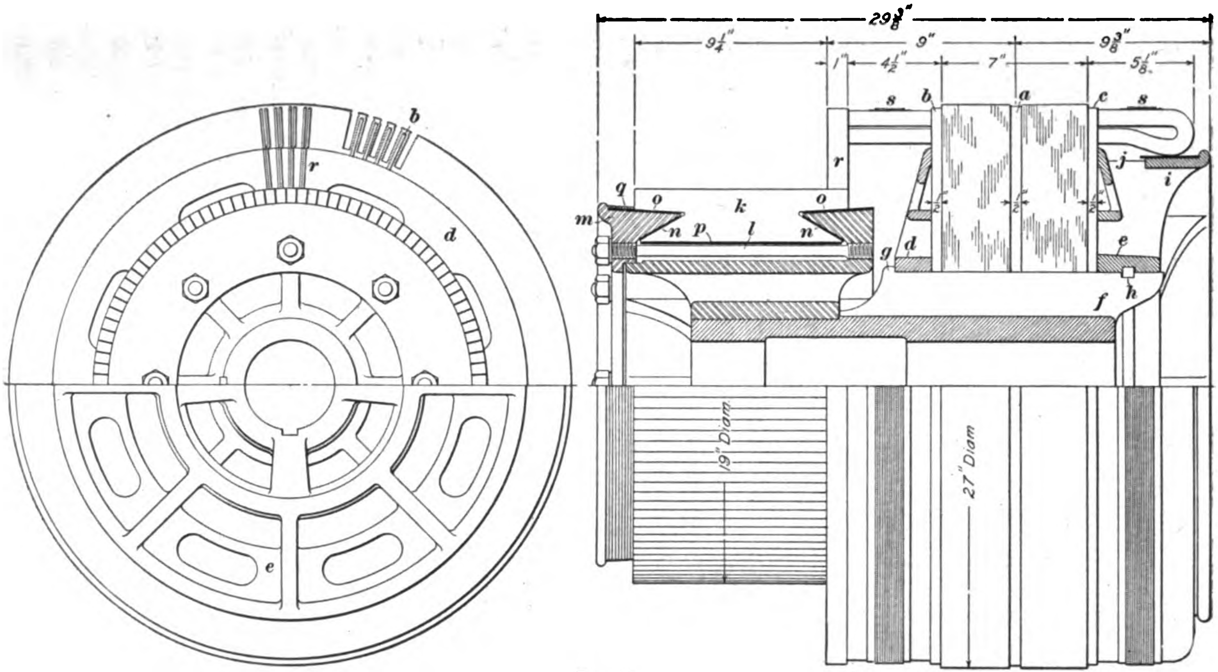


FIG. 19

From Fig. 13, the indicated brush contact drop for a current density of 36.4 amperes per square inch is 2.55 volts; therefore, the resistance loss for 273 amperes will be 696 watts.

The total length of the four $1\frac{1}{4}$-inch brushes will be 5 inches; therefore, the length of the commutator segment will be $7\frac{1}{2}-5=2\frac{1}{2}$ inches less than for the 250-volt commutator, or $10\frac{1}{4}-2\frac{1}{2}=7\frac{3}{4}$ *inches.*

When designing a set of commutators for 125-, 250-, and 550-volt machines, if the designs of the commutators for the two lower voltages result in satisfactory heating conditions, the heating of the higher-voltage commutator will be satisfactory when moderate current density in the brushes is employed.

100. Some of the more important features of construction for the complete **250-volt armature** are indicated in Fig. 19. The compressed armature punchings, including a $\frac{1}{2}$-inch air vent *a*, have a length of 7 inches. At *b* and *c* finger plates are installed similar to the vent plate at *a*. The fingers support the teeth at their ends, as indicated at *b* in the view at the left. The laminations and spacing plates are clamped between the end plates *d* and *e* mounted on the armature spider *f*. Plate *d* rests against a shoulder *g* on the spider, and plate *e* is pressed on until some small keys *h*, which lock the plate in position, may be inserted in the plate and spider arms. The ring *i* supports the rear end windings and is in turn supported by the arms of plate *e*. These arms act as vanes and fan the air through the rear end windings at *j*. The commutator shell is made of steel and is mounted on an extension of the hub of the armature spider. The copper segments *k* are clamped by means of bolts *l* between the commutator shell and the wedge ring *m*. Micanite insulating cones *n* and *o* and a mica sleeve *p* insulate the segments from the shell and ring. The exposed part of the outer micanite cone *o* should be protected from oil and dust by a covering *q* of twine or tape. The commutator necks shown at *r* serve to support the front armature end windings. Bands *s* serve to retain the end windings in position. End plate *e* at the rear of the armature is indicated in the lower portion of the left-hand view.

DESIGN OF DIRECT-CURRENT MACHINES

(PART 2)

FEATURES OF FIELD-MAGNET DESIGN

THE MAGNETIC CIRCUIT

1. Function of the Poles and Yoke.—The magnet frame, or yoke, and the poles, together with the air gaps and armature, form one or more complete magnetic circuits. It is the primary function of the poles and yoke to conduct the magnetic flux from the armature surface near one pole through the field coils and return it to the armature surface near the next pole.

The poles must also support the field coils, and the magnet yoke must be able to withstand the magnetic pull between the armature and the poles. In small machines, the stress does not usually prove troublesome, but in large ones, especially those having ten or more poles, it is often necessary to so design the section of the yoke as to obtain the greatest mechanical stiffness from the magnetic material required.

2. Function of the Field Coils.—It is the function of the main field coils to supply the magnetomotive forces necessary to establish the fluxes through the magnetic circuits. Space must be provided not only for the coils but for the circulation of air around them in order that they may not overheat. The design of the poles and yoke, therefore, requires a knowledge of the field coils, and the space they will need.

3. Comparative Magnetomotive Forces Required for Parts of the Magnetic Circuit.—The greatest expenditure of magnetomotive force in the magnetic circuit of an ordinary generator or motor is that required to establish the flux across the air gaps between the armature teeth and the pole faces. The length of an air gap is usually quite small and therefore only slightly affects the size of the machine, but is usually so selected that the magnetomotive force expended in the gap will bear a certain ratio to the armature reaction per pole.

A considerable portion of the total magnetomotive force must be expended in the armature teeth, the field-magnet cores, and the yoke. It is also necessary to supply sufficient magnetomotive force to compensate for and to overcome the armature magnetomotive forces which interfere with and oppose those of the field windings.

The total magnetomotive force that is required for all purposes usually bears a certain relation to the armature ampere-turns and it may be assumed that in a motor or generator of the ordinary radial-pole type, the field excitation in ampere-turns per pole will be somewhere from one and one-fourth to two times the armature ampere-turns per pole.

4. Materials Used for the Magnetic Circuit.—Magnet cores and yokes are made from one or more of the following metals, the selections being made usually from considerations of economy: Cast iron, malleable iron, low-carbon cast steel, wrought iron, steel forgings, or sheet-iron or sheet-steel punchings.

5. Saturation Curves.—The values indicated in the saturation curves shown in Fig. 1 are not to be considered absolutely correct, but rather as average values. It is not uncommon to find materials whose properties differ by 10 per cent. or more from those shown. The chemical composition and the treatment of the irons have much to do with their magnetic qualities. In general, the purer the steel or wrought iron and the softer it is, the higher will be its magnetic permeability. Chilling lowers the permeability, while annealing raises it. Any impurities, such as carbon, which tend to harden

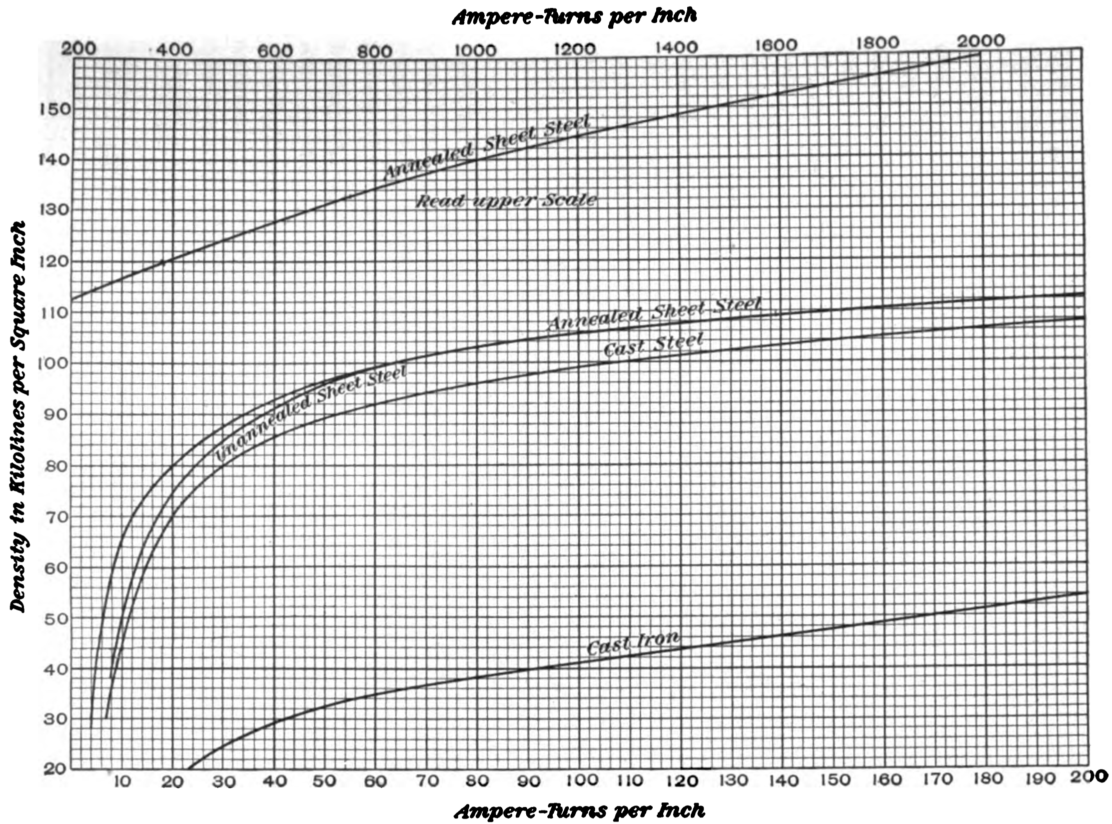

Ampere-Turns per Inch
200
400
600
800
1000
1200
1400
1600
1800
2000
Density in Kilolines per Square Inch
150
140
130
120
110
100
90
80
70
60
50
40
30
20
Annealed Sheet Steel
Read upper Scale
Annealed Sheet Steel
Cast Steel
Unannealed Sheet Steel
Cast Iron
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
Ampere-Turns per Inch

FIG. 1

the metal usually lower its permeability, and those elements that tend to soften the metal, when present in small quantities, either cause no appreciable change or tend to raise its permeability.

The saturation curves, Fig. 1, indicate the practice of one designing office, and relate to the magnetic properties of soft gray-iron castings, low-carbon steel castings, and unannealed and annealed steel punchings. Wrought iron is not ordinarily used because it is not so economical as cast steel or sheet steel, whose magnetic properties it resembles. Malleable-iron castings are used only for very small machines.

The curves show the number of ampere-turns which must be supplied for each inch of length of the material magnetized at various densities, measured in lines per square inch of cross-section of the magnetic circuit. It is not uncommon for the density at the tooth roots to be 120,000, 130,000, or even 150,000 lines per square inch. The saturation curve for annealed sheet steel is continued on the upper part of Fig. 1 from 200 to 2,000 ampere-turns excitation per inch.

6. Comparison of Properties of Cast Steel and Cast Iron.—The magnetic properties of cast steel are much better than those of cast iron, but as cast iron is more readily molded into intricate shapes and is more easily available, cast iron is sometimes employed. The cost of steel castings per pound is not usually as much as double the cost of iron castings; and as steel is somewhat more than twice as good magnetically as iron, steel is the more economical. It is, therefore, the more extensively used, even though, because of its high melting temperature, it is difficult to mold and is subject to greater shrinkage than iron.

Where lightness is desired, as in the frames of railway motors, cast steel, pressed sheet-steel, or punchings, are necessary. Sheet-steel punchings may be readily assembled and riveted together into parts having rectangular sections with comparative economy, because the parts so made do not require machining. The use of punchings for the construction of pole pieces is very common. The magnetic flux does not change

rapidly through the poles or yoke and when these parts are made of sheet-steel punchings, the laminations may be riveted together without objectionable eddy currents being set up.

7. Form of Magnetic Circuit.—The usual form of the frame for a direct-current generator or motor consists of a circular yoke having inwardly projecting poles on which the field coils are placed. In Fig. 2 is shown a bipolar magnet frame of this construction.

The magnetic lines, in passing through the magnetic circuit, traverse all possible paths. For purposes of estimating the magnetomotive forces necessary to maintain flux in a magnetic circuit, it is customary to approximate the position of the mean path; that is, the average between the shortest and the longest paths. The dotted lines in Fig. 2 indicate approximately the positions of the mean path. It will be noticed that the flux leaves the armature, enters the south pole piece and divides at the yoke, half returning to the north pole piece through one side of the yoke and the other half returning through the other side. This is very much the same as the division of the flux in the armature core, and, like the armature core, the yoke section on one side need only be sufficient to accommodate one-half of the flux in one of the pole pieces.

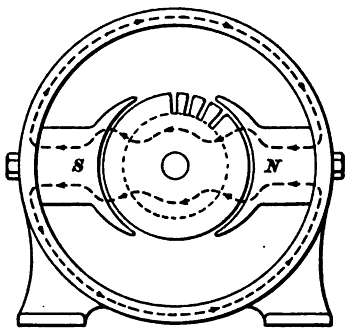


FIG. 2

8. Proportioning the Yokes and Poles.—In proportioning the magnet yoke and poles, there is no fixed rule to follow to determine the amount of space to be allowed for the field coils. If the coil space is excessive, the length of the poles makes the diameter of the machine too great, and the weight and cost of the poles and yoke are increased. On the other hand, too small a coil space causes the field copper to be crowded and its radiating area to be decreased. This usually requires that the losses within the coil must be decreased, which, with

the same excitation, can only be done by increasing the quantity of copper in the coil. A very short coil becomes very deep, and requires a greater average length of wire to encircle the pole so that the weight of the copper is in this way still further increased.

9. If the field excitation, in ampere-turns per pole, is assumed to be from one and one-fourth to two times the armature reaction in ampere-turns per pole, then the coil space, measured radially along the pole, should bear some relation to the pole pitch, measured along the armature surface.

In the general type of machine shown in Fig. 3, the coil space *a* should be within the broad limits of from 40 per cent. to 75 per cent. of the pole pitch *b*. In a small bipolar machine of such proportions as that shown in Fig. 2, it is best to make the coil space rather less than 40 per cent. of the pole pitch; the figure shows it about 25 per cent.

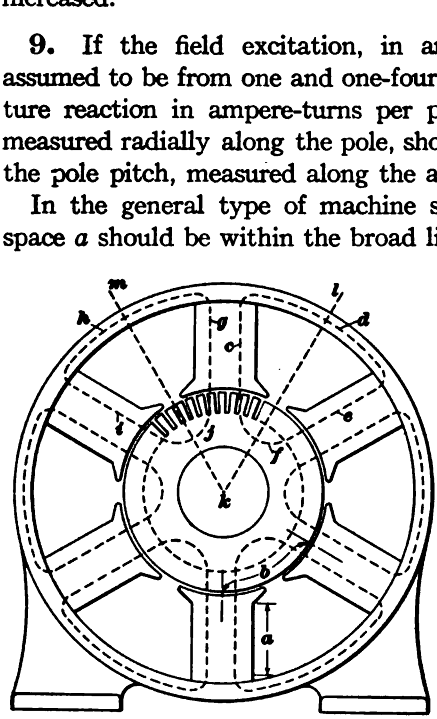


FIG. 3

Magnet frames that are to be equipped with commutating poles should have a somewhat greater proportionate field-coil space than frames without commutating poles. With frames of six poles and over, it should be satisfactory to make an allowance for the coil space of from 50 to 60 per cent. of the pole pitch for machines without commutating poles, and from 60 to 75 per cent. for those with commutating poles.

10. Distribution of Magnetomotive Forces for the Magnetic Circuits.—The mean paths of six magnetic circuits are indicated in Fig. 3 by dotted lines. Each path is through two magnet cores and hence is encircled by two exciting

coils, each of which must supply half of the excitation required by the complete magnetic circuit. Thus, the coil on the upper vertical magnet core must supply the magnetomotive force expended in forcing the fluxes of the two magnetic circuits c, d, e, f, and g, h, i, j from the lines $k\,l$ and $k\,m$ through the yoke to the upper part of the magnet core, down through the magnet core, across the air gap, through the teeth, and through the armature core from the bottom of the teeth to the lines $k\,l$ and $k\,m$.

In like manner each of the coils on the six poles may be considered as supplying magnetomotive forces for the corresponding portions of the adjacent magnetic circuits.

LEAKAGE FLUX

11. Coefficient of Magnetic Leakage.—The field coils not only establish the fluxes along the paths, or routes, through the steel, but they also establish fluxes along all other possible routes between any and all points of different magnetic potential. These stray fluxes add very little to the useful ones that enter the armature and develop the electromotive force, but they add materially to the fluxes through the pole pieces, magnet cores, and yoke.

The excess of flux in the magnet cores over the flux passing through the armature is termed the *magnetic leakage*, and the ratio of flux in the magnet cores to that in the armature is called the *coefficient of magnetic leakage*. Usually the coefficient of magnetic leakage is nearly the same for a certain type of magnet yoke regardless of the size. In practical designs it varies from 1.05 to 1.20, which means that the leakage flux varies from 5 per cent. to 20 per cent. of the useful armature flux.

12. Allowance for Magnetic Leakage.—It is impossible to prevent magnetic leakage, but to make allowance for it, the sectional areas of the poles and yoke must be increased to make them of suitable size to accommodate the correct total flux passing through them.

13. Assumptions Relating to Magnetic Leakage. Fig. 4 shows a part of the frame shown in Fig. 3, indicating the paths of the leakage flux by dotted lines drawn between the pole pieces. The value of the leakage flux is proportional to the magnetomotive force that produces it, and inversely proportional to the length of the path or route. Thus, the flux density along the portion *a b* of the magnetic circuit *a, b, c, d* depends on the magnetomotive force acting between the points *a* and *b*. Along the complete magnetic circuit *a, b, c, d*, there are displayed

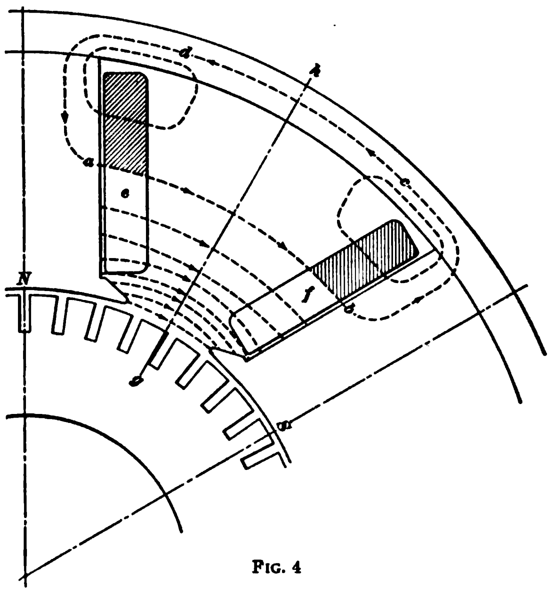


FIG. 4

as many ampere-turns as there are amperes in all of the conductors in the shaded part of the coils *e* and *f*. If from the number of these ampere-turns that are required for the whole circuit *a, b, c, d* are deducted those required to maintain the flux in the iron from *b, c, d* to *a*, the remaining ampere-turns will be the magnetomotive force that establishes the flux from *a* to *b* through the leakage path.

The density of the leakage flux will be greatest in the space between the pole tips of adjacent poles because the air path

is shortest and the path of the leakage flux is linked with all of the ampere-turns of coils *e* and *f*. It is customary, for purposes of calculation, to assume that all of the leakage flux in the air path is established between the tips of the poles.

14. Relation Between Field Flux and Armature Flux. In calculating the densities in the magnetic circuit, the *armature flux*, as it is sometimes called, is computed from the formula of Art. 2, *Design of Direct-Current Machines*, Part 1. The flux so obtained is assumed to pass through the armature core, teeth, and air gap. The *field flux* is obtained by multiplying the armature flux by the leakage coefficient and this flux is assumed to pass through the poles and yoke.

15. Leakage Constant, or Factor, Without Commutating Poles.—Tests and calculations relating to leakage flux show that it may be assumed that each ampere-turn on a pole will establish from 4 to 5 leakage-lines on each side of the pole for each inch of the length of pole piece parallel to the shaft plus a distance equal to that between the pole tips of one pair of poles. This extra distance is added to the length of the pole pieces to allow for the spreading out of the lines at the ends of the poles. It is also assumed that the excitation of each coil establishes the leakage fluxes between the side of the pole and a plane midway between the adjacent pole pieces, the position of one of which is indicated by the line *g h*, Fig. 4.

In Fig. 4, suppose that each pole piece is 10 inches long and that the pole tips are 4 inches apart; the effective length of the pole piece, parallel to the shaft, may be taken as 14 inches. Assume that $4\frac{1}{2}$ leakage lines on each side of the pole per inch of this effective length are set up by each establishing ampere-turn on the coil, then the leakage flux per establishing ampere-turn per pole for each side of the coil will be $4\frac{1}{2} \times 14 = 63$ lines. For both sides of the pole, the leakage flux per ampere-turn per coil will be 126 lines and for 6 poles will be $126 \times 6 = 756$ lines.

The factor 756 lines may be considered as a constant for the frame, and may be termed its *leakage constant*, or *factor*. To obtain the total leakage flux, it is necessary to multiply the leakage constant by the ampere-turns per pole active in setting

up the leakage flux. The calculation of these ampere-turns will be explained later.

16. Leakage Flux With Commutating Poles.—When commutating poles are used, the length of the magnetic leakage path in air is shortened, as indicated in Fig. 5. The magnetomotive force required to establish the leakage flux across the main or commutating poles may be entirely neglected because these are of iron of high permeability.

The excitation of the coils on the main poles causes the leakage flux from one side of a main pole to pass from the surface i of the north pole through the air to the surface j of the commutating pole, through the pole, out of surface k, through the air to the surface l of the adjacent south main pole. These surfaces include the whole sides of the poles.

The excitation of the coils on the commutating poles causes their leakage flux to pass out of the surfaces j and k, through the air paths and into the surfaces i and l of the main poles, through the main poles and across the air paths to the commutating poles beyond these main poles. In each case the *air paths* for these leakage fluxes are practically the same.

17. Suppose that the coil m on the commutating pole, Fig. 5, has A ampere-turns available for establishing leakage flux and that coils e and f each have B ampere-turns establishing leakage flux. The ampere-turns establishing leakage flux from the commutating pole N to the main pole N are equal to the difference between A and B, while the ampere-turns establishing the flux between the commutating pole N and the main pole S are the sum of A and B. If K_l is the leakage constant for one commutating pole, the leakage flux between the surfaces j and i will be $K_l\ (A-B)$. Likewise, the leakage flux between the surfaces k and l will be $K_l\ (A+B)$. The total leakage flux for both sides will be $K_l\ (A-B)+K_l\ (A+B)=2\,K_l A$.

The leakage flux $2K_l A$ is the same as would have occurred had B been zero, and hence it follows that the total leakage flux due to the excitation of the coils on the commutating poles is not influenced by the excitation on the main poles. This is

so because the assistance rendered on one side of the commutating pole by the excitation of the coil on the adjacent main pole is equal to the opposing influence of the excitation of the coil on the adjacent main pole on the other side of the commutating pole.

18. The leakage constant for both main and commutating coils may be estimated by allowing from 6 to 7½ magnetic lines

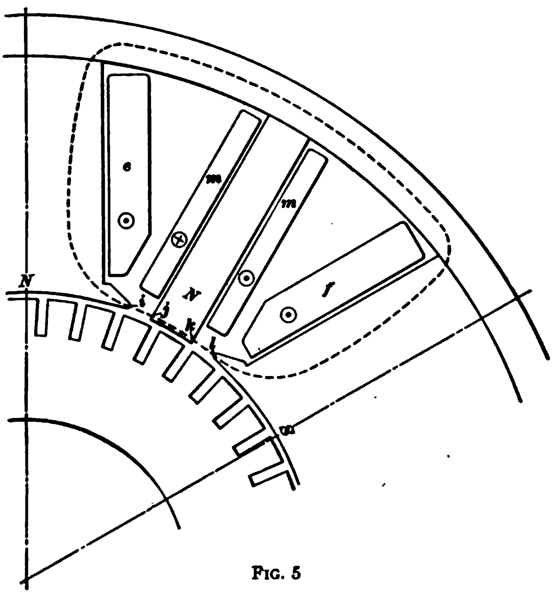


FIG. 5

for each side per ampere-turn and per inch of effective length of pole piece parallel to the shaft, making allowance for fringe of lines at the ends.

For example, if the main pole pieces, Fig. 5, are 10 inches long parallel to the shaft and the distances between the tips of adjacent main pole pieces is 4 inches, the effective length of the main poles will be taken as 14 inches. The commutating poles are 1½ inches thick and 7 inches long, and if 2 inches is allowed for spreading of the lines at the ends of the poles, the effective length of the commutating poles will be taken as 9 inches. The main-pole leakage flux, for a frame without commutating poles,

is assumed to be $4\frac{1}{2}$ lines per ampere-turn per inch of effective length of main pole, as in the case indicated in Fig. 4. The leakage constant for both main and commutating poles, Fig. 5, is assumed to be 7 lines per ampere-turn per inch of effective length of commutating pole.

The effective length of the commutating poles, 9 inches, multiplied by 7 lines equals 63 lines per ampere-turn and per side, or 126 lines per ampere-turn per commutating pole. For the main poles, 9 inches of the 14 inches effective length have 7 leakage lines per ampere-turn and per inch, and the other 5 inches have $4\frac{1}{2}$ lines per ampere-turn and per inch, or $(9\times7)+(5\times4\frac{1}{2})=85.5$ lines per ampere-turn per main pole per side, and for both sides 171 lines per ampere-turn per pole. For six poles the leakage factor for the frame with commutating poles will be $171\times6=$ *1,026 lines*. The leakage factor for a similar frame, but without commutating poles, was found to be 756, so that the leakage factor has apparently increased $\frac{1026-756}{756}\times100=36$ per cent., nearly.

19. With commutating poles, however, the excitation of the main poles is usually much less than without them, hence the total magnetic leakage flux, which is the product of the leakage factor and the ampere-turns of excitation that establish the leakage flux, may be very little changed by adding the commutating poles.

20. The flux in the main poles affects the armature as a whole and it is convenient to consider the total flux in all main poles. On the other hand, the fluxes of all the commutating poles are not added together for any purpose; hence, the flux for one commutating pole is more convenient to consider. Likewise, the leakage factors have been computed as applying to all main poles for the main flux, and to one commutating pole for the commutating flux.

ARMATURE REACTION

21. Grouping of Conductors.—By **armature reaction** is meant the magnetic effect that the current in the armature winding has on the magnetic field in which the armature rotates.

The conductors of an armature that carry current from brush to brush may be considered as being divided into as many groups as there are poles on the frame, the direction of the currents in all conductors of a group being the same.

Fig. 6 shows, between the radial lines *a b* and *c d*, the armature slots in which the conductors of a group are located. The

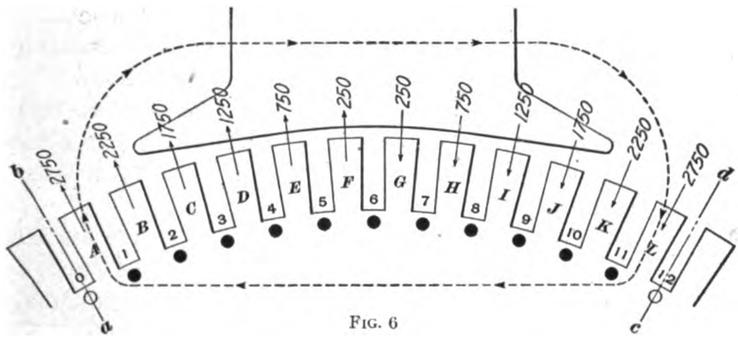


FIG. 6

radial lines are drawn midway between the pole piece shown and adjacent pole pieces. There are 12 slots between the radial lines, making 12 slots per pole.

In Fig. 6 and those following, the directions of the currents in the conductors of each slot are indicated by the small circles below the slots. A full-black circle indicates a current away from the observer, or downwards, and a dotted circle, a current toward the observer. Slots *0* and *12* are directly on the center lines between poles and are represented in Fig. 6 as having no current in the conductors located in them, but the direction of the current in all of the conductors in the other slots of this group is downwards.

22. Magnetomotive Force of Conductor Groups. For purposes of general calculation, let it be considered that

the conductors in all of the 12 slots per pole are active, and that the sum of the currents in the conductors of each slot is 500 amperes; this is equivalent to the product of the current in each conductor and the number of conductors per slot, or, as it is stated, 500 ampere-conductors per slot. For the group of 12 slots, there will be $12 \times 500 = 6{,}000$ ampere-conductors.

The magnetomotive forces of this group of conductors set up magnetic swirls, and if there is no interference by other magnetomotive forces these swirls will be symmetrical around the group. The path of the largest swirl, shown in Fig. 6, will be up the tooth A, through the air, across the pole, through the air, down the tooth L, and through the armature core back to tooth A. This swirl will be linked with the conductors in 11 slots; therefore, the magnetomotive force that establishes the flux of the swirls will be $11 \times 500 = 5{,}500$ ampere-turns.

It will be convenient to consider these 5,500 ampere-turns as divided into two parts; 2,750 ampere-turns acting to establish a flux passing outwards in tooth A and 2,750 ampere-turns acting to produce a flux inwards in tooth L.

In a similar manner the swirl that passes up through tooth B, across the pole, and into the symmetrically located tooth K will be linked with the ampere-turns of 9 slots, and the magnetomotive force establishing the flux will be $9 \times 500 = 4{,}500$ ampere-turns, or 2,250 ampere-turns acting outwards through tooth B and inwards through tooth K. In a similar manner the magnetomotive forces expressed in ampere-turns have been calculated for the positions of the other teeth and the values are indicated in Fig. 6 for positions A to L.

23. Symmetrical Arrangement of Armature Fluxes. If armature currents alone are active, a symmetrical arrangement of the lines of force will be produced approximately similar to that indicated in Fig. 7. It will be noticed that the left side of the pole is south and the right side north, as indicated by S and N, while the teeth A to F are north poles and those from G to L are south poles. The magnetic effect of the group of conductors from A to L *on the armature* is to establish a north pole about the line $a\,b$ and a south pole about the line $c\,d$.

The magnetomotive force of these poles at the teeth A and L, Fig. 6, is 2,750 ampere-turns at the instant shown. Were all of the group of 12 slots considered active, the armature ampere-turns *per pole* would have been $\frac{12 \times 500}{2} = 3{,}000$. It is customary in calculating the armature reaction per pole to consider all conductors as active. Thus, because the group of conductors supply the magnetomotive force of two poles, numerically the armature reaction per pole is always a half of the ampere-conductors per group.

Fluxes between slots are also indicated in Fig. 7; the magnetomotive forces establishing these vary from zero, at the lower side of the bottom conductors, to 500 ampere-turns

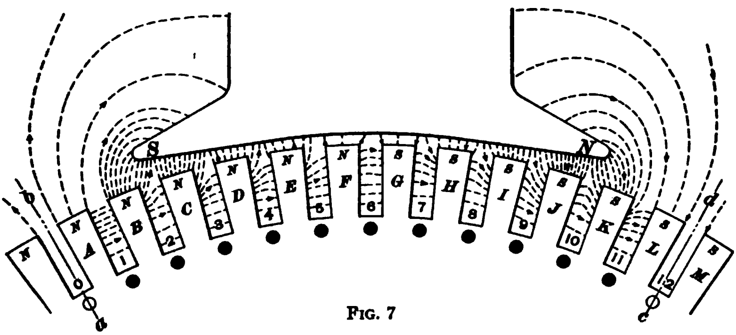


FIG. 7

at the upper side of the upper conductors and above them. The slot flux is the same for all slots, as each has the same establishing magnetomotive forces; therefore, considered as a whole, the path of the slot flux may be taken as passing up through the tooth A and crossing successively the slots *1* to *11*, returning downwards through tooth L and the armature core.

The flux established between the teeth F and G and the pole is low, because the magnetomotive forces at these positions are low, as indicated by the values shown in Fig. 6. The flux is greater between the pole and the teeth E and H, Figs. 6 and 7, and still greater between the pole and the teeth D and I, because of the increased magnetomotive forces at these positions. Between the pole and the successive teeth C, B, A, and J, K, L

the air path increases in length; and, even though the magnetomotive forces are also increased, the fluxes established between the teeth and the pole may not necessarily be increased at the successive positions *C* to *A* or *J* to *L*.

24. Resultant of Field and Armature Fluxes.—In a generator under load, the magnetomotive forces of the field coils and of the armature windings are both active, and the approximate positions of the paths of the resultant fluxes are indicated by the dotted lines in Fig. 8.

The reason for the distortion of the flux emanating from the pole face may be understood better by a consideration of the corresponding values of the field magnetomotive force and

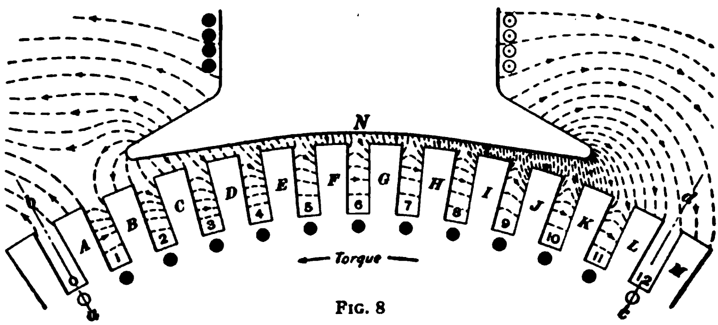


FIG. 8

the armature magnetomotive force for each of the positions of the various teeth between lines *a b* and *c d*, Fig. 9.

Suppose that the magnetomotive force of the exciting coil on the north main pole is assumed to impress 3,000 ampere-turns between the pole piece and the armature teeth. Between the pole and tooth *A*, Fig. 9, there will then be a field magnetomotive force of 3,000 ampere-turns acting downwards, and an armature magnetomotive force, shown in Fig. 6, of 2,750 ampere-turns acting upwards, leaving a net magnetomotive force, acting downwards, of 250 ampere-turns, as indicated in Fig. 9; the effect, however, of the south pole on the left (not shown) is to cause some lines to pass from tooth *A* toward the left to the south pole. In a similar manner the other resultant magnetomotive forces are calculated.

In the case of teeth G to L, the field and the armature magnetomotive forces agree in direction, and the values are added to determine the resultant magnetomotive force active in each tooth; the results of the calculations are as indicated in Fig. 9.

The spacing of the lines of force in Fig. 8 shows that a flux of low density is produced at the left end of the north-pole piece where the ampere-turns impressed between the pole face and the teeth are small, and that it gradually increases to a flux of much higher density at the right end because of the increasing impressed magnetomotive forces.

25. Development of Torque.—Another important effect indicated in Fig. 8 is that the lines of force at any position in

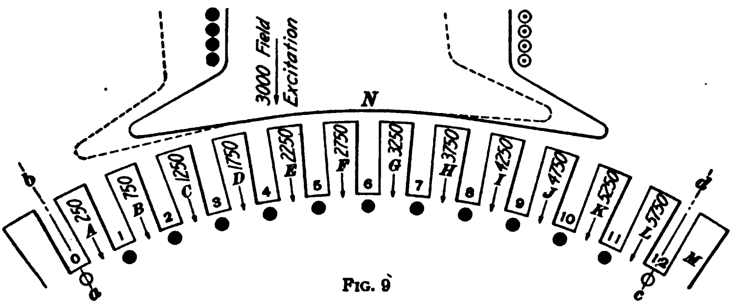


Fig. 9

passing from the pole face to the teeth cross the air gap at an inclination to a radial line through the position.

At tooth D, Fig. 9, the resultant magnetomotive force is 1,750 ampere-turns, while at tooth E it is 2,250. A magnetic line emanating from the pole face, Fig. 8, at a point opposite the middle of slot 4 will pass down through tooth E because of the greater magnetomotive force active between the pole face and tooth E. Other lines emanating from the pole face well over on the D side of midpoint position will also pass down through tooth E and thus cause the lines of force over the slot to slant. The same inequality of magnetomotive forces exists between any two adjacent teeth; therefore, the inclination of the lines will occur over every slot under the pole face. The lines of force in spreading to fill all of the region of the air gap will

slightly incline over the teeth as well as over the slots. Magnetic lines of force tend to shorten themselves, thus acting like stretched elastic bands; hence, these inclined lines exert a torque, or twisting force, upon the armature in the direction indicated by the large arrow in Fig. 8.

26. Relation Between Total Magnetomotive Force and Total Flux.—The armature magnetomotive forces, indicated in Fig. 6, are symmetrically placed about slot *6* midway between the limit lines *a b* and *c d*. The teeth to the left of slot *6* have the same values of magnetomotive forces acting upwards that the corresponding teeth to the right of slot *6* have acting downwards.

If the pole piece is so located that its center line coincides with the center line of slot *6*, as is the case in Fig. 6, there will be on the armature exactly as much armature magnetomotive force assisting the pole flux on the right of slot *6* as there is opposing it on the left.

The armature currents in this position of the pole piece do not affect the *total magnetomotive force* of the field coil, and were the flux in every tooth proportional to its magnetomotive force, the *total flux* leaving a north-pole face or entering a south-pole face would not be changed. Flux in air is always strictly proportional to the magnetomotive force, but in iron, because of magnetic saturation, this is not true. Tooth densities as high as 140,000 to 150,000 lines per square inch may be used, and the curves in Fig. 1 indicate that such values represent high saturation.

27. If magnetic saturation of the teeth is encountered in the case indicated in Fig. 9, then the flux in every tooth will not be directly proportional to its magnetomotive force. *Without* the interference of any armature currents, the field magnetomotive force of 3,000 ampere-turns would be impressed at every tooth. *With* armature currents, the teeth having magnetomotive forces greater than 3,000 ampere-turns *will not have proportionately greater fluxes* established in them, while the teeth with magnetomotive forces correspondingly smaller than 3,000 ampere-turns *will have proportionately lower fluxes*. The total

flux, therefore, with the same field excitation, will be less when there are armature currents than when there are none.

The effect of the armature currents is to shift the flux as shown in Fig. 8, and this action by causing excessive tooth densities in some of the teeth tends to increase the magnetic reluctance of the armature teeth, considered as a whole, to the passage of the main-field flux, and this results in a lessened flux.

In a machine with fixed excitation, such as a shunt-wound motor, less flux is established at full load than at no load. To maintain the field flux in such a machine, the excitation must be greater at full load than at no load. It is not a simple matter to calculate the magnetomotive force that must be added to the field excitation to compensate for armature reaction, but ordinarily it is less than 10 per cent. of the armature ampere-turns per pole.

28. Magnetic Effect of Changing the Length of the Air Gap.—The interference of armature currents with the main-field flux may be prevented by means of compensating pole-face windings as stated in *Direct-Current Generators*, or it may be controlled by using an air gap of suitable size between the main pole and the armature.

Suppose that in Fig. 9 the length of the air gap be changed in three machines so that the magnetomotive forces, considered as stated in Art. 24, necessary to establish the fluxes across the gaps are 3,000 ampere-turns in the first; 2,000 ampere-turns in the second; and 1,000 ampere-turns in the third case. Assume that the armature ampere-turns have the values indicated in Fig. 6. Consider tooth *J*, Figs. 6 and 9, to be under the strong pole tip in which there are 1,750 armature ampere-turns assisting the field excitation. Then, under no-load conditions, the field excitation in this tooth may be considered as equivalent to 3,000, 2,000, and 1,000 ampere-turns, respectively, for the three cases. Under load, 1,750 ampere-turns will be added, making the excitation at this tooth 4,750, 3,750, and 2,750 ampere-turns, respectively.

The increase in excitation in the first case from no load to full load is from 3,000 ampere-turns to 4,750, or 58.3 per cent.; in

the second case it is from 2,000 to 3,750, or 87.5 per cent.; and in the third case, from 1,000 to 2,750, or 175 per cent.

The shifting of the main flux is greatest when the magnetomotive force expended in the air gap is least. If the teeth are reasonably saturated at no load with 3,000, 2,000, and 1,000 field ampere-turns, respectively, then the saturation will be extreme when these quantities are increased 58.3 per cent., 87.5 per cent. and 175 per cent.

29. Field Excitation to Compensate for Flux Shift. The number of ampere-turns that must be added to the field excitation to maintain the same field flux at full load as at no load in the three cases under consideration will be approximately proportional to 58.3 per cent., 87.5 per cent., and 175 per cent.; and these quantities bear the ratio of 1, $1\frac{1}{2}$, and 3. In the first case, the added ampere-turns will be from 3 to 8 per cent. of the armature ampere-turns per pole, depending upon the densities in the teeth and their length; in the second case, they will be from 5 to 12 per cent.; and, in the third case, from 9 to 24 per cent.

The armature magnetomotive force totals 3,000 ampere-turns per pole, and the ratio of the armature ampere-turns per pole to the air-gap ampere-turns per pole is 1, $1\frac{1}{2}$, and 3 in the three cases.

In order to obtain a general expression, it may be stated that the ampere-turns to be added to the field excitation to compensate for armature interference is equal to from 3 to 8 per cent. of the square of the value of the armature ampere-turns per pole divided by the gap ampere-turns per pole, or, expressed as a formula,

$$\text{added ampere-turns} = .03 \text{ to } .08 \times \frac{(\text{armature ampere-turns per pole})^2}{\text{gap ampere-turns per pole}}$$

The value of the right-hand member and of the equation may be increased by using values of gap ampere-turns per pole obtained by shortening the air gap, and may be decreased by using values obtained by lengthening the air gap, since the value of the gap ampere-turns is in the denominator of the fraction, and this value is lessened if the air gap is decreased, and increased if the air gap is lengthened. By selecting a suitable length of

air gap, the ampere-turns necessary to be added to compensate for armature interference may be kept below 10 per cent. of the armature ampere-turns per pole.

30. Magnetic Effect of Change in Brush Position. It was assumed in Fig. 9 that the group of armature conductors are symmetrically placed with respect to the pole, in that the center line of the pole is identical with the center line of slot *6*, midway between the limit lines *a b* and *c d* of the group of conductors. All of the conductors in a group carry current in the same direction, hence these limit lines indicate the positions where the currents in the armature conductors are reversed. The current in an armature conductor reverses only when the commutator segments to which the conductor is connected pass into contact with a brush; and the location of the limit lines *a b* and *c d* is dependent upon the position of the brushes on the commutator. Usually the brushes are arranged so that they may be adjusted; and in machines without commutating poles, the brushes are commonly shifted in position to assist the commutation.

When the brushes of a motor or a generator are shifted to assist commutation, it is always in the direction of the shift of the field flux and opposite in direction to the torque as indicated by the arrow in Fig. 8. Suppose the brushes were given a lead, which for convenience in calculating may be taken as one slot pitch. The lines *a b* and *c d*, Fig. 9, would each be moved one slot to the right. The effect would be exactly the same if the pole on the diagram was shifted one slot to the left as shown by the dotted lines, Fig. 9.

The number of ampere-turns establishing the flux along any tooth in Fig. 9 is 500 ampere-turns greater than that along the next tooth to it on the left; hence, if the pole was shifted from full-line position to the dotted-line position, one tooth pitch to the left, then the ampere-turns that establish flux along each tooth under the pole for the full-line position will be 500 ampere-turns more than are active along a similarly located tooth under the pole for the dotted-line position. If the magnetomotive force of each tooth under the pole is reduced 500 ampere-turns

then that of the whole pole is reduced by a like amount, and the effect is the same as though the excitation of the whole field coil were reduced 500 ampere-turns.

The effect of an angular brush shift of more or less than a tooth will be proportional to the angle of the lead, because the armature in revolving occupies the positions shown in the diagrams only for an instant, and the total magnetic effect must be an average of the effects in a number of successive positions.

31. In Fig. 9 a shift of one tooth pitch in the position of the brushes caused a reduction of 500 ampere-turns in the excitation of the whole field coil. As there are twelve slots per pole, the shift is $\frac{1}{12}$ of the pole pitch. There are 3,000 armature ampere-turns per pole and a reduction of 500 ampere-turns is $\frac{1}{6}$ of the armature ampere-turns per pole. Thus, a shift of $\frac{1}{12}$ of a pole pitch produces a reduction of the excitation of the whole field coil equivalent to $\frac{1}{6}$ of the armature ampere-turns per pole. The fraction $\frac{1}{6}$ is double the fraction $\frac{1}{12}$ that represents the angle of shift, hence it is sometimes stated that the opposing ampere-turns are those that lie "within the double angle of lead."

Since the armature ampere-turns which lie within the double angle of shift of the brushes directly oppose the magnetomotive force of the field flux, they are often called the *back ampere-turns* or *demagnetizing ampere-turns* of the armature.

32. Should the brushes be shifted in the direction of the torque, the armature ampere-turns within twice the angle of the shift would *assist* the field excitation. This assistance will increase and decrease with the armature, or load, currents; and the effect of this brush movement on the field excitation is much the same as an added series-field winding.

33. In machines equipped with commutating poles there is no occasion to shift the brushes, and they are therefore set at the neutral position. The brush shift is zero and there can be no armature back ampere-turns. In machines without commutating poles, the brush shift may be enough to bring the coil under commutation under the weak pole tip. If the percentage of the armature covered by the poles is 75 per cent.,

then the neutral space will be 25 per cent. of the pole pitch. From the neutral position to the pole tip will then be 12½ per cent. of the pole pitch. The brush shift is usually less than from the neutral position to the pole tip, and is not often over 10 per cent. of the pole pitch. With a brush shift of 10 per cent. of the pole pitch, the armature back ampere-turns will be 20 per cent. of the armature ampere-turns per pole.

34. Effect of the Armature Flux on Short-Circuited Coil.—The magnetic fluxes established by the armature currents in the region between the poles are important because of their influence on those coils that are short-circuited by the brushes.

In Fig. 10 are shown two main poles *N* and *S*, each having a field coil in which a current is established, there being no cur-

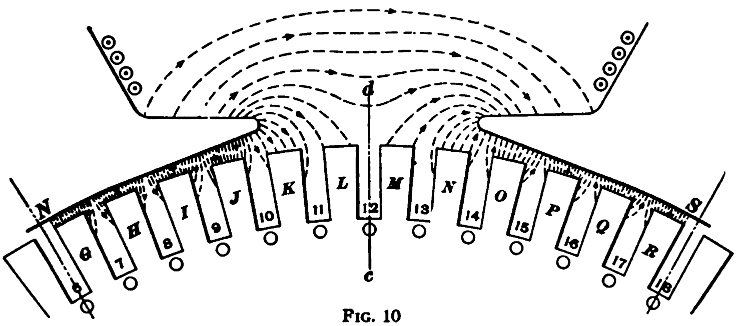


FIG. 10

rent in the armature conductors at this time. Fig. 11 shows the same poles, but the armature conductors are now represented as carrying current, except the conductors in slot *12*, which form part of a coil or parts of coils short-circuited by the brushes. By comparing these two diagrams, particularly in reference to the flux established in the teeth *L* and *M*, it will be noticed that in Fig. 10 tooth *L* receives a small number of lines from the pole *N* while an equal number of lines pass from the tooth *M* to the pole *S*.

In the rotation of the armature, conductors moving in the region from slot *11* to slot *13*, Fig. 10, would encounter few lines of force, and these would be half downwards and half upwards,

so that their effects would be largely neutralized. The slot *12*, therefore, may be considered as in a true neutral region.

35. Under the conditions shown in Fig. 11 there will be established, by the combined action of the currents in the field coil and in the armature conductors, a considerable number of magnetic lines passing downwards through the teeth *L* and *M*. The currents in the conductors in slots *9*, *10*, and *11* also tend to send lines of force across these slots and down tooth *L*. The flux established across the slots *15*, *14*, and *13* is reversed in direction, because the currents in the conductors are opposite to those in the conductors *9*, *10*, and *11*; there-

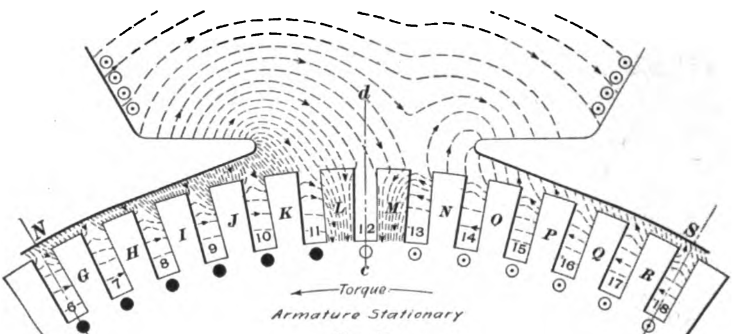


FIG. 11

fore, the slot flux on the right of slot *12* passes downwards through the tooth *M*. Thus, in Fig. 11 there is a considerable downward flux in teeth *L* and *M*.

In Fig. 11, the armature is assumed to be stationary and the current to be supplied to it from an outside source. In such a case the coils short-circuited by the brushes would have no current in them and there would be no current in the conductors in slot *12*.

36. If the armature is now assumed to move, the flux through teeth *L* and *M*, indicated in Fig. 11, in passing into the teeth that come successively into those positions will cut across, or be cut by, the conductors moving through the position now occupied by slot *12*. An electromotive force will be generated in the conductors of slot *12*, and the current

established in these short-circuited conductors will force some flux across slot *12*.

The direction of the currents established in the conductors of slot *12* will depend on the direction of rotation of the armature. In Figs. 12 and 13 is shown a complete magnetic circuit of which a part is indicated in Fig. 11. In Fig. 12 the rotation is to the right or opposite in direction to that of the torque; hence, mechanical energy is required to drive the armature and the *action is that of a generator.*

The conductors of a short-circuited coil that are located on or near the limit line *c d* will have downward electromotive

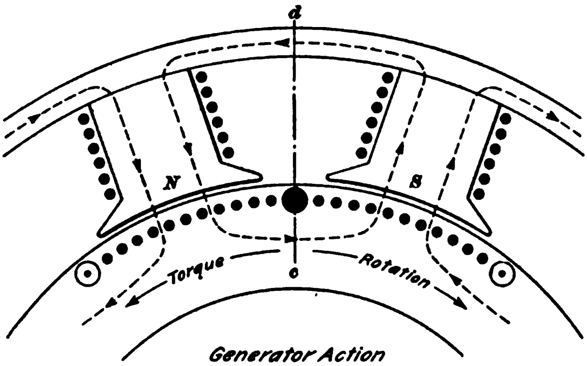


FIG. 12

forces induced in them which will establish downward currents as indicated by the conductor on the line *c d*, Fig. 12.

In Fig. 13 the direction of rotation is the same as that of the torque; therefore, the machine is developing mechanical energy and is *acting as a motor.* The electromotive forces now induced and the currents established in the conductors of the short-circuited coil that are located on or near the center line, are now upwards, as indicated by the conductor on line *c d*, Fig. 13.

37. Effects of the Current in the Short-Circuited Coil.—The magnetic effects which the currents in the short-circuited coils exert on the flux from the main poles may be noted from Figs. 12 and 13. One complete mean magnetic

circuit, as indicated by the dotted lines, includes the conductors on one side of each of the two field coils and nine slot groups of armature conductors. In all of these conductors the currents that are upwards in direction tend to drive the flux along the mean path in the direction indicated by the arrowheads on the dotted lines; the downward currents in the other conductors within the path oppose the main flux. It should be noted that in the case of the generator, Fig. 12, the currents in the short-circuited conductors, represented by large circles, tend to oppose and to weaken the main flux, while in the motor,

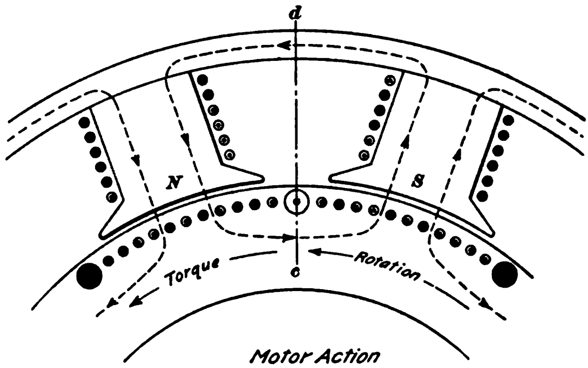


FIG. 13

Fig. 13, the currents in the corresponding conductors tend to increase the flux.

It may be stated, therefore, that the currents in the armature coils of a *generator*, when they are short-circuited by the brushes in a position midway between pole tips, tend to weaken the main field. In the case of a *motor* the currents in the coils short-circuited by the brushes tend to strengthen the main field.

38. The currents established in the short-circuited coils are objectionable; and it is desirable to prevent them as far as possible. They are induced when the short-circuited coil encounters flux that is established in the locality of these coils by the action of the magnetomotive forces of the armature coils. In Fig. 11, the objectionable flux is that shown passing downwards in the teeth L and M. The flux established by the main

field coils is passing upwards out of the teeth *N*, *O*, *P*, etc.; and, if the brushes were shifted, the coil could be short-circuited in the neighborhood of the position of a tooth in which the upward flux due to the action of the main field coils would develop an electromotive force that would assist in neutralizing the electromotive force caused by the objectionable flux.

The objectionable flux is established by the armature currents; therefore, the effects increase or decrease as the load increases or decreases when the main-field excitation is a constant or a nearly constant quantity.

With very small or no armature currents, no objectionable flux is established and the true neutral is on the center line *c d*,

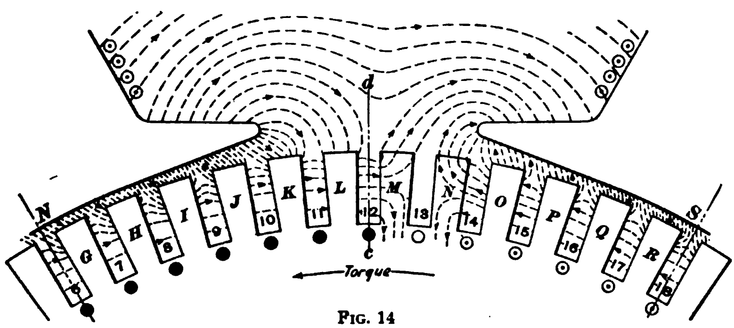


FIG. 14

Fig. 10. As the armature currents increase, the effect of the flux they establish can be neutralized by greater brush shift so as to cause the coils to be short-circuited farther from the center line.

39. In Fig. 14 it is assumed that the brushes are shifted so that the short-circuited coil occupies the position of slot *13*, and that the load is such that the effect of the flux established by the armature currents is just neutralized by the flux passing from the teeth *M* and *N* to the pole *S*. It will be seen that, as the armaure rotates, conductors in the position of slot *12* while passing to the position of slot *14*, or in the opposite direction, will encounter but comparatively few lines of force.

It should be noticed that the flux passing from the teeth *M* and *N* to the pole *S* must be established upwards against the

full downward magnetomotive force of the armature. This flux cannot be so established unless the field magnetomotive force between the pole S and the teeth M and N is greater than the total armature magnetomotive force for that particular load. It is consequently necessary in designing machines intended to operate without commutating poles to select a length of air gap that will require a proper expenditure of field magnetomotive force between the pole and the armature.

40. The effect of the objectionable flux on the short-circuited coil may be entirely neutralized by suitably arranged commutating poles, and in such a machine the magnetic effects of the short-circuited armature currents may be disregarded. In machines not equipped with commutating poles, however, short-circuited currents will be encountered at some loads, even though the brushes are shifted. With varying loads, it is not considered practical to shift the brushes with every change in the load, but the practice is to set the brushes permanently in a mean position such that the short-circuited currents encountered will not be excessive at any load. In such a machine, short-circuited currents whose magnetic effect on the main field is as great as from 5 per cent. to 10 per cent. of the magnetomotive force of the armature may be expected. When the magnetomotive force of the short-circuited currents is greater than 10 per cent. of the armature magnetomotive force sparking at the brushes may be encountered.

41. Summary.—The four important effects of the armature currents upon the main field flux are as follows:

1. The inclination of the magnetic lines of force in the air gap under the main poles. This action causes the torque, or twisting force, of the armature, and it is the only effect of the armature reaction that is essential to the proper action of the machine.

2. The shifting of the magnetic flux under the main poles. This may be prevented by compensating-pole-face windings or it may be reduced by introducing a reluctance in the magnetic circuit at or near the air gap of the main poles. Such a reluctance requires a greater field excitation and increases the cost of the machine.

3. The demagnetizing action of those armature conductors which lie within the double angle of the brush shift. This action is not present in commutating-pole machines, because the brushes are not shifted. The demagnetizing ampere-turns are proportional to the load current, and where it is necessary to compensate or counteract them series turns may be added to the main-field coils. Excepting some slight magnetic leakage effects, these demagnetizing turns can in this way be entirely nullified.

4. The magnetizing or demagnetizing action of the short-circuited currents. These do not exist where properly adjusted commutating poles are used. In motors, these currents assist the main-field excitation, while in generators they oppose it.

COMMUTATING POLES

42. Purpose of the Commutating Poles.—If the machine is equipped with commutating poles, the effect of the undesirable flux established by the armature magnetomotive force may be nullified through a wide range of load, without shifting the brushes. These poles are of smaller dimensions than the main poles and are placed in the middle of the neutral space between the main poles. They are each provided with a field coil which is connected in series with the armature in such a manner that the magnetomotive force the coil develops will directly oppose that of the armature in the region of the short-circuited coils.

In Fig. 15 is shown a diagram of a commutating pole S' midway between the main poles N and S. This pole is supposed to be supplied with a coil connected in series with the armature so that its magnetomotive force will increase and decrease exactly as does the armature magnetomotive force. Further, the number of turns of wire of the commutating-pole coil is such that it will have a somewhat greater magnetomotive force than that of the armature. If the armature has 12 slots per pole and on full load there are 500 ampere-conductors per slot, the armature magnetomotive force for this group of slots will be 3,000 ampere-turns per pole.

The full-load excitation of the commutating pole should be more than the armature magnetomotive force of 3,000 ampere-turns per pole. Assume a value of 3,500 ampere-turns per commutating pole; of these, 3,000 ampere-turns oppose and nullify the 3,000 ampere-turns per pole of the armature, and prevent the armature magnetomotive force from establishing any flux in the region of the short-circuited coil. An excess of 500 ampere-turns is left to establish an equivalent and upward flux in teeth L and M, Fig. 15, to nullify the effects of the self-induction flux that crosses the slots and passes downwards through these teeth.

43. Armature-Inductance and Self-Induction Fluxes. It is well to distinguish between the flux established by the arma-

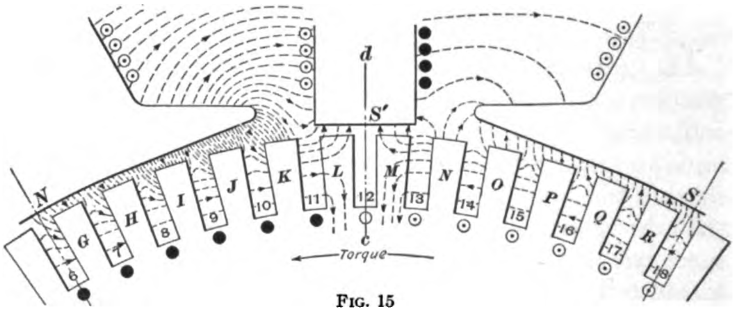


FIG. 15

ture magnetomotive force as a whole and that which is established by a single coil. The armature magnetomotive force *as a whole* does not change as the armature rotates, because as conductors are taken from one end of the group others are added just as fast to the other end; but it does change as the armature currents increase and decrease. Any flux which the armature magnetomotive force establishes may be called the *armature-inductance flux.*

Any flux, as, for example, that which crosses the slot and which reverses as the current in the coil of that slot reverses during the process of commutation is established by the magnetomotive force of a single coil and may be termed the *inductance flux of a coil*, or the *self-induction flux.*

Some of the lines of the flux of self-induction which cross a slot do not encircle all of the conductors in the slot, and, therefore, in reversing, these lines do not develop so great an electromotive force as those lines which encircle all conductors. It is for this reason that the flux established by the commutating pole is *equivalent* and opposite to rather than *equal* and opposite to the self-induction flux.

It should be noted in Fig. 15 that conductors in the position of slot *11* may pass to the position of slot *13* without encountering any large number of magnetic lines.

44. Comparative Lengths of Commutating Poles and Armature Core.—If the commutating pole covers the entire armature length, the armature cannot then establish any armature-inductance flux in the region of the coils under commutation, and the commutating pole need only establish sufficient flux to nullify the effect of the self-induction flux that is established by the changing current in the coil during commutation. However, the presence of the commutating pole immediately over the tops of the teeth, so reduces the reluctance as to permit the coils in the slots under the commutating poles to establish comparatively large inductance fluxes.

It is not absolutely necessary for the commutating poles to cover the whole length of the armature core, but this is sometimes done. If, however, the poles are shorter than the core, armature-inductance flux will be established in the uncovered part, and the commutating poles must then establish not only enough flux to nullify the effect of the self-induction flux, but also enough to counteract the effect of such armature-inductance flux as is established. At the same time this greater flux from the commutating pole must be established through a smaller area because of the reduced length of the pole; hence, the excitation on the commutating pole must be increased considerably.

If the armature coils are drum-wound, or of any other type having conductors spaced one pole pitch apart, alternate commutating poles may be omitted if the remaining poles are able

to neutralize the effect of the self-induction of the complete coil and to neutralize the effect of the armature-inductance flux established in the neutral space where the commutating pole is omitted.

45. Excitation for Commutating Poles.—The armature- and the coil-inductance fluxes are established through comparatively long paths in air without appreciable magnetic saturation in steel. These fluxes will vary strictly in proportion to the excitation establishing them. Also, if the section of the commutating pole be made large enough so that it does not saturate, the correcting flux established by this pole will be strictly proportional to its excitation. But the excitation of the commutating pole is proportional to that of the armature, because they are connected in series and the same current provides both excitations; hence, if the commutating pole establishes exactly the correct flux to nullify the effect of the armature- and self-inductance fluxes at some one load, it will be correct for them at all other loads up to the point where the commutating pole saturates and its flux is no longer proportional to its excitation.

46. Percentage of Compensation of Commutating Poles.—The excess in excitation of the commutating poles over that required for armature reaction is sometimes referred to as that required to *compensate* for the inductance fluxes. If there are 3,000 armature ampere-turns per pole and 3,500 ampere-turns per commutating pole, the 500 ampere-turns are those required for compensation. These are often referred to in per cent. of the armature reaction. In this case the compensation would be $\frac{500}{3{,}000} \times 100 = 16\frac{2}{3}$ per cent.

47. To approximate roughly the compensation, in per cent., required by commutating poles, the following formula may be used:

$$J = \frac{120}{\frac{3}{16} + g_c} \times \frac{g_c}{d_a} \times \frac{l_a}{l_p} \times p_m,$$

in which J = compensation, in per cent.;

g_c = length of the air gap under the commutating poles, in inches;

d_a = diameter of the armature, in inches;

l_a = length of the armature core, in inches;

l_p = length, parallel to the shaft, of the commutating pole, in inches;

p_m = number of main poles.

If there is only one commutating pole for every pair of main poles, then the value of J must be doubled.

To illustrate the use of the formula, the following case will be considered: A generator armature has a diameter of 36 inches and a length of 10 inches. There are 96 slots with four coils per slot, or 384 coils. Each coil has one turn and the coils form a parallel winding. The machine has 6 main poles; the air gap is $\frac{1}{4}$ inch; and the commutating poles cover 8 inches of the 10-inch length of the armature. What is the required compensation, expressed in per cent.?

Substituting values in the formula,

$$J = \frac{120}{\frac{3}{16}+\frac{1}{4}} \times \frac{\frac{1}{4}}{36} \times \tfrac{10}{8} \times 6 = 14.3 \text{ per cent.}$$

There must be placed on each commutating pole one turn to overcome the armature reaction and 14.3 per cent. of a turn to compensate for armature inductance, for every equivalent turn on the armature. There are 384 turns on the armature and six poles, or 64 turns per pole. As there are 6 paths, each armature turn will carry but a sixth of the total current, and a sixth of 64 or $10\frac{2}{3}$ is the equivalent turns per pole when each turn carries the complete armature current. Adding 14.3 per cent. to $10\frac{2}{3}$ turns will equal $10.67 \times 1.143 = 12.2$ turns. It will be necessary to put on 13 turns per commutating pole, and if the effect of the pole is too strong, some of the current may be shunted around the coils.

48. Surface of Armature Core Covered by Commutating Poles.—Authorities differ as to the amount of armature surface that should be covered circumferentially by the commutating poles. It seems advisable, however, to make the

commutating poles wide enough so that the coils short-circuited by the brushes will be beneath the commutating poles during the entire period of short circuit. It should be sufficient to make the angle covered by the commutating pole 50 per cent. more than the angle spanned by the brush. In *Design of Direct-Current Machines*, Part 1, it was suggested that the brushes for a 125-volt machine should not cover more than 10 per cent. of the annular space from one neutral point to the adjacent neutral point, or its annular equivalent, the pole pitch; hence, the polar angle of the commutating poles will be ample if made 15 per cent. of the pole pitch.

49. Total Flux of Commutating Pole.—The total flux in the commutating pole is made up of leakage from the sides of the poles and flux from the end. The leakage flux from the sides is established by the total magnetomotive force of the coil on the commutating pole, but the flux from the end is established by the compensating ampere-turns only.

The leakage flux may be estimated by allowing from 6 to $7\frac{1}{2}$ magnetic lines per inch length of pole per ampere-turn and per side, as explained in Art. **18.** An illustration of these calculations will be given later in connection with the design of the 150-kilowatt belted generator.

50. Advantages of Neutralizing Armature Inductance.—The commutation has been observed to be better when there are as many commutating poles as there are main poles and where the commutating poles cover all or nearly all of the armature length. That is to say, it is well to prevent any armature-inductance flux from being established in the region of the short-circuited coil.

COMMUTATION

PURPOSE OF COMMUTATION

51. Commutation is the process that occurs when the commutator segments in which the ends of a coil terminate come into electrical contact with a brush. The process lasts, so far as any particular coil is concerned, as long as both of these segments are in contact with the brush. During commutation, one or more coils of the armature winding are short-circuited by each of the brushes, and the currents in these coils are reversed as they pass from one side of the brush to the other. The brushes also serve to transmit current from the armature windings to the line, or from the line to the armature windings.

During the time a coil is short-circuited by a brush, if its face conductors or end connections cut across magnetic flux, an electromotive force will be induced within the coil. If commutating poles are used and their excitation is properly adjusted, an electromotive force may be induced in the coils encountering the commutating-pole flux which will be equal and opposite to the electromotive forces induced by other fluxes encountered by these coils. Therefore, the electromotive force developed by the movement of the short-circuited coil will be very low or zero.

COMMUTATION WITH UNIFORM CURRENT DENSITY IN BRUSH CONTACT

52. Process of Commutation.—A portion of a parallel ring winding is shown in Fig. 16, but the following explanation applies also to a parallel drum winding. Several coils are shown connected to commutator segments *a* to *f*, the ends of each coil being connected to adjacent commutator segments. A brush *g* bears on segments *b*, *c*, *d*, and *e* and the three coils *b–c*, *c–d*, and *d–e* are being commutated, but in this case are *not generating electromotive forces*.

Current $+i$ from the group of coils on the left passes through coil *a–b* and then divides, part passing directly through

segment b and brush g, and the remaining part through coil b–c and segment c to brush g. The current divides in accordance with Ohm's law and in inverse proportion to the resistance encountered in the two paths. In a similar manner the current $-i$ from the group of coils on the right will divide between the parallel paths e to g and coil e–d to g.

Carbon, graphite, or other high-resistance brushes are used, and, therefore, the drop in voltage in a single coil of an armature under full-load conditions is much less than the drop in voltage between a commutator segment and a brush, because the resistance of a coil is very small in comparison with the contact resistance between a segment and the brush. The coil resistance, therefore, will have but little influence on the distribution of current over the contact surface of the brush, and the current density in that surface will be uniform, which is the ideal condition for perfect commutation.

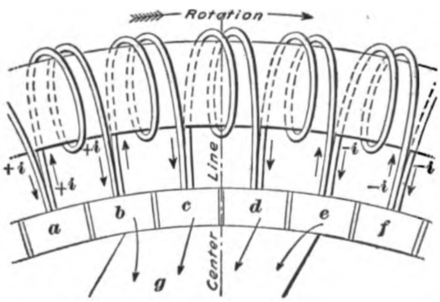


FIG. 16

53. In Fig. 16 a uniform current density in the brush contact surface is assumed, and, at the instant shown, the mica segment between c and d is on the center line of the brush contact surface. The coil a–b, at the instant shown, is not under commutation, because both of the segments a and b in which it terminates are not in contact with the brush. A current of $+i$ amperes is in coil a–b and of $-i$ amperes in coil f–e; the + and – signs simply indicate that there is a change in the direction of flow of electricity in the coils as they pass from one side of the brush to the other. Electricity, is, however, flowing toward the brush from the group of coils on either side of it, and the current in the brush contact surface is $2i$.

Since the current density is uniform in the contact surface, i amperes pass through the left half of the brush contact area; hence, the sum of the currents from segments b and c to the

brush must be equal to i, and on the right half of the brush contact area the sum of the currents from segments d and e must also be equal to i. At the instant shown there is no current in the coil c–d.

Thus, when the mica segment between the terminals of a coil is at the entering edge of the brush, the current in the coil is $+i$ amperes; it is zero when the mica segment is on the center line, and $-i$ amperes when the mica segment is at the leaving edge of the brush. In fact, when the current density in the brush contact surface is uniform, the current in any coil such as b–c

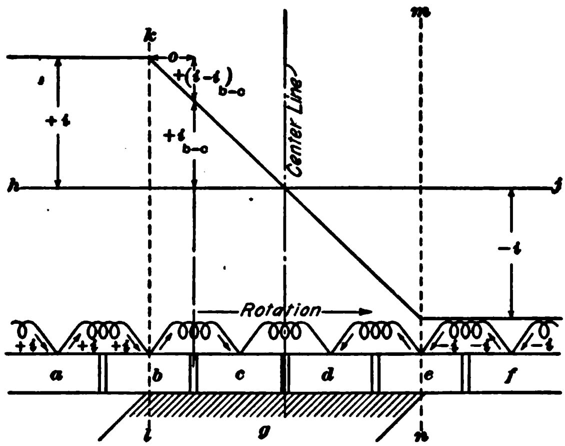


Fig. 17

will decrease at a uniform rate from the time short circuit begins until the current in the coil is zero, which, in this case, is when the mica segment between its terminals is on the center line of the brush contact surface; the current will then increase at a uniform rate in the opposite direction until it is full value when the mica segment passes out from under the brush and the short circuit ceases.

54. Curve Showing Uniform Current Changes.—In Fig. 17 the variation of the current in a coil during commutation is plotted graphically to scale. Above an axis line $h\ j$, a horizontal line is drawn at a distance representing the current $+i$

amperes. The coils pass from left to right and the load current in any coil of a group is assumed to be uniform until short-circuiting begins, which occurs when the mica segment between its terminals reaches the line $k\,l$. The short circuit continues until the mica segment passes beyond the line $m\,n$. The lines $k\,l$ and $m\,n$ are separated by a distance equal to the thickness of a brush, less the thickness of one mica segment. During the short circuit, the current steadily decreases, as indicated by the diagonal line, from $+i$ on the line $k\,l$ to zero on the center line of the brush contact surface, where it reverses and increases steadily until a value of $-i$ is reached on the line $m\,n$.

The value of the current in any coil under short circuit may be found by erecting a perpendicular through the mica segment between its ends and noting where it cuts the diagonal line. For example, the current in the coil *b–c* is indicated by the line $+i_{b-c}$. Since the current entering the segment *b* from the coil *a–b* is $+i$ amperes and the current leaving segment *b* through the coil *b–c* is i_{b-c}, then the current passing from segment *b* to the brush *g* must be $i-i_{b-c}$.

The current $i-i_{b-c}$ passes through a brush area equal to the product of the distance *o* and the width of the brush parallel to the shaft, or perpendicular to the paper in the diagram. The current density in this brush surface will be proportional to $\frac{o}{i-i_{b-c}}$. If the current line between the lines $k\,l$ and $m\,n$ is straight, as shown in Fig. 17, then the length of the line *o* will always be proportional to $i-i_{b-c}$, no matter where the coil may be, and the current density, which is proportional to $\frac{o}{i-i_{b-c}}$, will be the same over the whole surface of the brush.

In a diagram such as Fig. 17, the condition of perfect commutation, or of uniform current density in the brush contact surface, is that the current line shall be straight during the period of short circuit.

COMMUTATION WHEN CURRENT DENSITY IN BRUSH CONTACT IS NOT UNIFORM

55. Effect of Generation of Electromotive Force in the Coil Under Commutation.—When the armature coils which are under short circuit develop an electromotive force, its direction is usually such as to tend to delay or to prevent the reversal of the current. Thus, in Fig. 12, the current in the short-circuited conductors on the center line *c d* is in the same direction as that in the conductors to its left. But in rotating, these short-circuited conductors came from the group

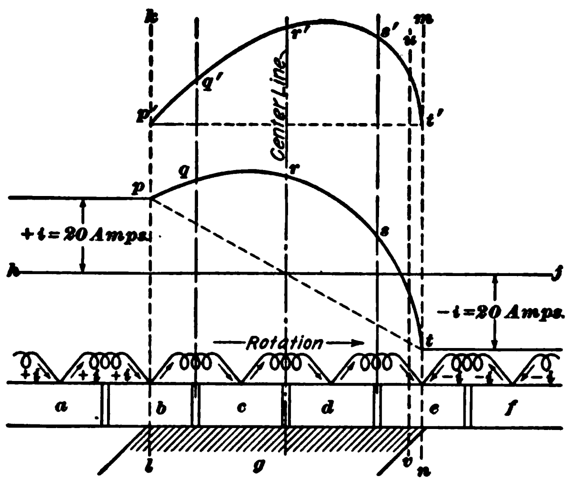


FIG. 18

to the left, hence the short-circuited current is in the same direction as the main working current before short circuit started. In Fig. 13, also, the current in the short-circuited conductors is in the same direction as that in the group of conductors to their right, from which these conductors came.

Fig. 18 shows the current curve *p–q–r–s–t* for a coil under commutation and in which an electromotive force is generated. Had there been no short-circuited currents, the current line would have been straight between the points *p* and *t*. For purposes of discussion, that part of the current in the coils that

is represented for the various positions of the coil by the vertical distances between the line $h\,j$ and the straight line $p\,t$, is called the *working current* and that part represented by the vertical distances between the lines p–q–r–s–t and $p\,t$ is called the *short-circuit current* which is superimposed upon the working current. The short-circuit current at any point is measured by the vertical distance from $p\,t$ to the current line p–q–r–s–t. The short-circuit current is plotted as the curved line p'–q'–r'–s'–t', every point of which is the same distance vertically above $p'\,t'$ as p–q–r–s–t is above the straight line $p\,t$.

56. Variation of Current Density in Brush Contact. The presence of the short-circuited currents lowers the current density in the contact surface on one side of the brush and raises it on the other. This may be more easily understood if numerical values are assigned to the conditions indicated in the diagram, Fig. 18. Let the current i be 20 amperes; the current in the brush is $2\,i$, or 40 amperes. Let the brush measure $\frac{3}{4}$ inch between the lines $k\,l$ and $m\,n$ and $1\frac{3}{4}$ inches parallel to the shaft. The *average* current density in the brush contact surface will then be $\frac{40}{\frac{3}{4}\times 1\frac{3}{4}}$, or 30.5 amperes per square inch.

The combined working and short-circuit current in the coil b–c at the instant shown may be assumed to be 25 amperes, or 5 amperes more than the current in the coil a–b. This extra 5 amperes must, therefore, pass from the brush g into the segment b and thence into the coil b–c, where it joins the 20 amperes from the coil a–b. Let it be assumed that the commutator segments are each $\frac{1}{4}$ inch wide and the brush covers just one-half of the segments b and e. The contact area between b and the brush is then $\frac{1}{8}\times 1\frac{3}{4}$ inches $=.219$ square inch. With 5 amperes passing through this area, the current density is $5\div .219=22.8$ amperes, but since the current at this point is between the brush and the segment, whereas the load current is between the segment and the brush, the density may be considered negative, or -22.8 amperes per square inch.

57. The current in the coil c–d at the instant shown is about 26 amperes, or 1 ampere more than in the coil b–c. This

ampere of current must have passed from the brush to the segment *c*, and the area of this segment in contact with the brush is $\frac{1}{4}\times1\frac{3}{4}$ inches = .4375 square inch. The current density is again negative and is $\frac{1}{.4375}$, or -2.3 amperes.

The current in the coil *d–e* is 10 amperes, or 16 amperes less than in the coil *c–d*. This 16 amperes must have passed *into* the brush through the segment *d*, hence the current density through this contact area is $\frac{16}{.4375}$, or $+36.6$ amperes per square inch.

The current in the coil *e–f* is -20 amperes, while in the coil *d–e* it is $+10$ amperes, or a *change* of 30 amperes. Or it might be stated that the coil *d–e* has 10 amperes passing in the direction of *d* to *e* and the coil *e–f* has 20 amperes passing in the direction *f* to *e*, hence out of *e* a current of 30 amperes must pass onto the brush *g*. The current of 30 amperes passes through a contact area of .219 square inch, hence the density is $\frac{30}{.219}$, or $+137$ amperes per square inch.

58. The addition of short-circuited currents to the working current has something of the same effect on the current density in the brush contact surface as the addition of armature-inductance flux has upon the magnetic density in the air gap under the main poles, in that the condition of uniform density is destroyed. The maximum current density is encountered at the leaving edge of the brush.

Consider the case of a coil whose mica segment has reached the position of the line *u v*, Fig. 18, which is $\frac{1}{32}$ inch from the line *m n*. The current in this coil will be -5 amperes in the *u v* position and -20 amperes in the *m n* position, a change of 15 amperes in $\frac{1}{32}$ inch. This 15 amperes must pass through an area equal to $\frac{1}{32}\times1\frac{3}{4}$ inches = 055 square inch, and the current density is $\frac{15}{.055}$, or $+$*273 amperes per square inch*. Thus, the density becomes higher and higher as the segment is leaving the brush.

59. Effects of Excessive Current Density in Brush Contact.—When the current density in the edges of the brushes and segments exceeds the carrying capacity of the materials of which they are made, these edges are damaged or destroyed. Sometimes the heat generated actually burns away the carbon brush, the current is conducted through the heated gases at the edge of the brush, and *sparking* is visible. At other times the brush is disintegrated by the heat and the commutator surface is smeared with a black deposit, or the carbons may become red hot in spots and *glow*.

In some cases the copper at the edges of the segments is melted and rubbed off by the brush, and copper particles are noticed as a fine powder around the machine. These copper particles often collect under the brushes, and the brushes are then said to *pick copper*. If the brushes pick copper, and a coating is formed over the brush contact surface, the contact resistance of the brush is greatly reduced and short-circuited currents are increased. The sparking which results is often greenish in spots, indicating that the current density at these spots has reached a value sufficiently high to volatilize the copper.

The burning away of the edges of the brushes and the copper segments reduces the contact area and increases the current density so that the rate of destruction of the brush and commutator segments is increased. Eventually, the surface of the copper segments is so destroyed as to leave the mica segments projecting above the surface of the copper. The commutator is now said to have *high mica* and the brush contact with the copper is so poor that arcing occurs. The brushes will then be worn away rapidly and the commutator will wear in deep grooves.

COMMUTATION WITH SEVERAL COILS PER SLOT

60. In the discussion of the current line, Figs. 17 and 18, it was assumed that the armature winding was single parallel for simplicity in explaining the division of the currents. Practically the same considerations apply to other windings, with such modifications as are required by the peculiarities of the

individual windings. It has been further assumed that all of the armature coils are exactly alike, and therefore they will all have the same current line during short circuit.

The action of the coils during commutation will be alike only when there is but one coil per slot. In Fig. 19, four coils per slot are indicated, and there will be four commutator segments *a*, *b*, *c*, and *d* per slot pitch. A case is first considered when the machine is not equipped with commutating poles. Let it be assumed that the teeth *e* and *f* are, at the instant shown, in such position that the armature and coil-inductance fluxes for some particular load are exactly neutralized by the fluxes from the pole tips *N* and *S*, as explained in connection with Fig. 14. The brush, Fig. 19, represented for clearness as covering but a single segment, is shifted until it covers the segment *c* at the instant shown. If the position of the brush is correct for, say, the coil connected between segments *b* and *c*, the teeth *e* and *f* will have moved out from the exactly correct neutralizing position when the other coils in this slot are commutated. While the electromotive force due to the movement of a short-circuited coil may be zero for one of the four coils, it will not be zero for the others. The current line of one coil will show no short-circuited current, while the lines of the other coils will. In fact, a machine having four coils per slot will probably have four distinctly different current lines for the coils under short circuit, for each load, instead of one line for all coils. Under full-load conditions one of these current lines will show a greater short-circuited current than the others, and this will probably result in the burning or otherwise marking of the edge of every fourth commutator segment.

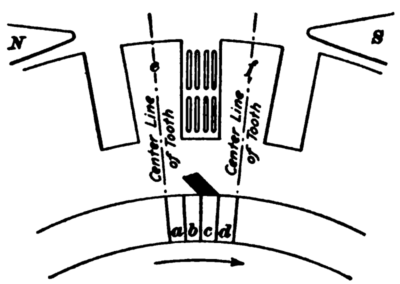


FIG. 19

When there are several sides of coils in a slot and commutating poles are used, the poles must be wide enough so that each coil is commutated while under the pole.

HINTS ON COMMUTATION

61. The commutator of a machine that is operating properly should have and maintain a smooth burnished surface. Any improper commutation, whether accompanied by visible sparking or not, which causes roughening of the commutator surface or the burning of the brushes, is undesirable, in that it increases the attention required to maintain these parts in a proper condition to carry successfully the load currents demanded of the machine.

To obtain good commutation, short-circuited currents should be avoided or at least limited as much as possible. To avoid short-circuited currents altogether, the coils under commutation must not develop any electromotive force; but, since the conductors are moving, this condition can only be obtained by opposing the fluxes established by the coil and the armature. This can best be done by the use of commutating poles or a compensating pole-face winding. When commutating poles are not used, short-circuited currents to some extent at some loads may be expected.

62. The short-circuit current curve p'-q'-r'-s'-t', Fig. 18, starts at zero, rises to a maximum and decreases again to zero within the space spanned by the brush. This current is equal to the electromotive force of short circuit divided by the resistance of the short-circuited path. The electromotive force of short circuit will be small if the turns per coil are few, if the speed of the machine is low, if the armature and coil magnetomotive forces are low, or if the reluctances offered to these magnetomotive forces are high. The short-circuit current will be small if the resistance of the short-circuit path is high. The resistance of a single armature coil is usually a very small quantity and in order to introduce sufficient resistance in the short-circuit path, brushes of comparatively high-resistance materials, such as carbon and graphite, are used. At the beginning and end of the short circuit, the area of contact between the brush and the segment just entering or just leaving is but a line, and the contact resistance is high. It is this high resistance at the

beginning and end of short circuit which causes the short-circuit current to begin and end at zero.

When both the segments in which a coil terminates are in complete contact with the brush, such as coil *c–d*, Fig. 16, the resistance offered by the brush contact surface is the least possible, and this resistance remains constant as long as both segments have complete contact with the brush.

63. The complete calculations of the commutating conditions, especially where short-circuit currents exist, is extremely complicated, if not impossible. It involves the calćulation of the armature and coil inductance fluxes for both the slot and end portions of the coils, the flux from pole to armature teeth in the space between the poles, and the calculation of the resistances of the brush and of the short-circuited coil. From these, the turns per coil, and the speed, the short-circuit current line can be approximated, and the current density in the edges of the brush and commutator segments can then be computed. Approximate or partial calculations are often used, but they lack reliability except where experience teaches their limitations.

64. In designing machines to be operated without commutating poles it is best to avoid high speeds, to place but few coils in a slot, to make coils of few turns each, to use brushes of high resistance, and to expend more magnetomotive forces between the pole and the armature teeth per pole than is the armature reaction per pole. It is also well to reduce the armature coil inductance by using a short armature core, with shallow and not too narrow slots.

DESIGN OF DIRECT-CURRENT MACHINES
(PART 3)

MAGNETIC-CIRCUIT DESIGN PROBLEMS

MAGNETIC CIRCUIT FOR A 5-HORSEPOWER MOTOR

1. The following problem relates to the magnetic circuit of the 5-horsepower motor, the armature of which was designed in *Design of Direct-Current Machines*, Part 1. In the development of this problem, values and general data included in Parts 1 and 2 are used and frequent reference should be made to these parts. A sketch of the armature, drawn to scale and based on the dimensions previously determined, should be made as indicated in Fig. 1. The poles are to cover 70 per cent. of the pole pitch and are assumed to be made of punchings. It is best to make the poles the same length, parallel to the shaft, as the armature core, or $5\frac{1}{2}$ inches. The magnetic density in the pole core should be about 90,000 lines of force per square inch.

The magnetic leakage constant may now be estimated. Assume a leakage of $4\frac{1}{2}$ magnetic lines per ampere-turn of the coil, per side of pole, and per inch of length of pole parallel to the shaft plus the distance between the adjacent pole tips of a pair of poles. As determined by scale from Fig. 1, the distance between pole tips will be $2\frac{1}{4}$ inches. The length of the pole parallel to the shaft will be $5\frac{1}{2}$ inches, and the corrected length

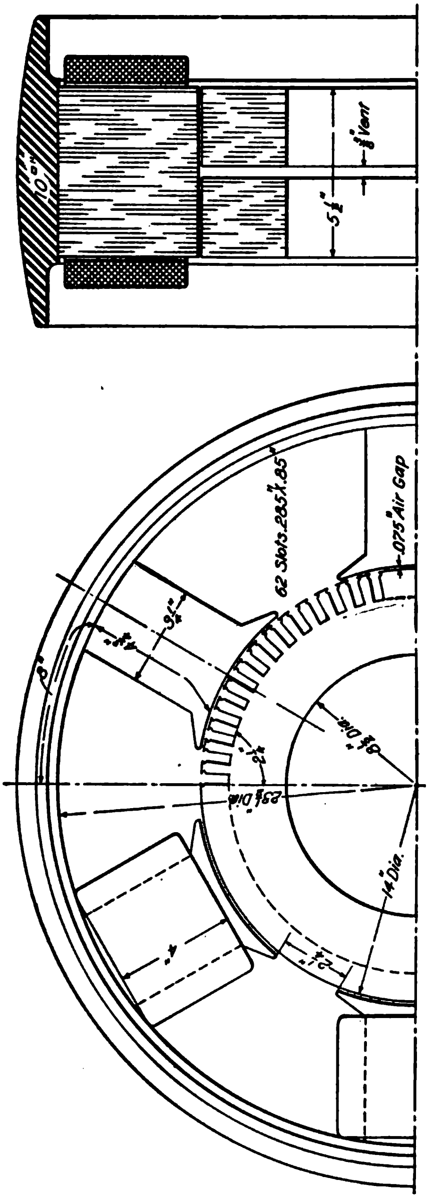


FIG. 1

for leakage calculations will be $5\frac{1}{2}+2\frac{1}{4}=7\frac{3}{4}$ inches. The number of lines established per ampere-turn per coil for each side of a pole will be $4\frac{1}{2}\times7\frac{3}{4}=34.9$. As there are 6 poles and 12 sides of poles, the leakage flux established per ampere-turn per coil will be $34.9\times12=418.8$ lines, or approximately *420 lines*, which will be the leakage constant of this magnet frame.

2. The armature reaction per pole equals the product of the armature turns per pole and the current per conductor. The number of slots is 62 and there are 12 conductors per slot, or a total of 744 conductors. With 6 poles and 2 conductors per turn, there will be $\frac{744}{6\times2}=$ *62 turns per pole*. The total armature current at full load is 39.8 amperes for the two paths of

the winding, or 19.9 amperes per path and per conductor. The armature reaction will be $62\times19.9=$ *1,234 ampere-turns per pole.*

The field ampere-turns establishing magnetic leakage may be taken roughly as 25 per cent. more than the armature reaction, or from 1,500 to 1,600 ampere-turns. These figures will be modified later.

3. The total leakage flux will be the product of the leakage constant, 420, and the magnetomotive force establishing leakage, taken for trial purposes as 1,600 ampere-turns, or 672,000 lines. The total flux required by the armature is 7,980,000 lines; hence, the total flux in all of the field-magnet cores will be 7,980,000+672,000=8,652,000 lines.

The field flux per pole will be 8,652,000÷6=1,442,000 lines; hence, if the density in the poles is assumed to be 90,000 lines per square inch, the cross-sectional area of each pole core will be 1,442,000÷90,000=16 square inches. The length of the pole pieces, parallel to the shaft, is 5½ inches, and, since the poles will be made of punchings, only 90 per cent. of this length will be net steel. The width of the pole core will be $\frac{16}{5.5\times.9}=3.23$ inches, or 3¼ inches, approximately. The area of all poles for carrying the flux will be $6\times3\frac{1}{4}\times5\frac{1}{2}\times.9=$ *96.5 square inches.*

4. The radial field-coil space for a 6-pole machine should be from 50 to 60 per cent. of the pole pitch measured on the armature surface. The pole pitch previously determined, measured on the armature circumference, is 7.33 inches, and 55 per cent. of this will be $7.33\times.55=4$ inches. To obtain a 4-inch coil space, Fig. 1, the diameter of the bore for seating the pole pieces on the yoke casting will be about 23½ inches.

The pole flux of 1,442,000 lines divides at the yoke; therefore, 721,000 lines will pass each way. If the yoke is to be made of cast steel, a density of from 65,000 to 80,000 lines per square inch would be economical. If the cross-sectional area of the yoke be made 10 square inches, the resulting density of 72,100 lines will be satisfactory. The yoke section may be made into any convenient shape. In Fig. 1 it is shown as spreading out over the field coils to protect them, and the

outside is rounded to improve the appearance of the machine. The total yoke section required to carry the total field flux will be $12 \times 10 = $ *120 square inches.*

5. The effective area of the air gap is that area which with a uniform density in the gap will be equivalent magnetically to the actual area of the gap and the actual densities in it. The value of the effective area is somewhat difficult to calculate owing to the surface of the armature core being cut away by the slots and air vents. The magnetic lines are

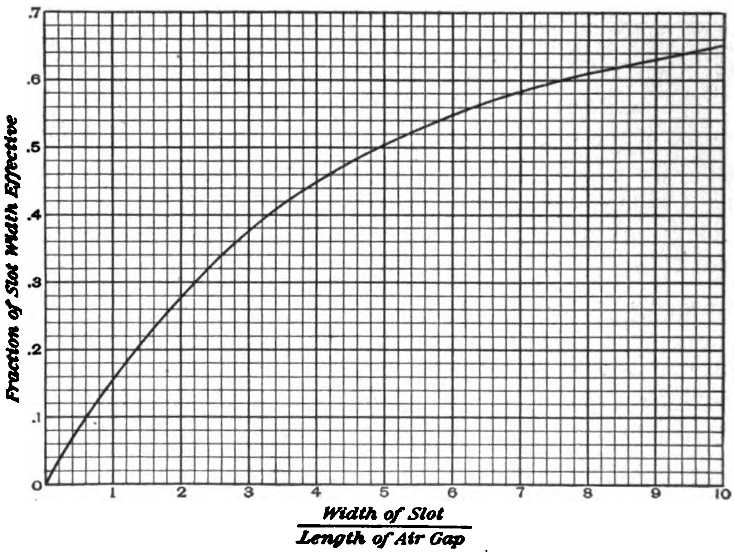


FIG. 2

established from the sides of the teeth as well as from their ends, and the density in the pole face directly over a slot is quite high, though never as high as it is directly over a tooth, because the length of the magnetic path in air, and, therefore, the reluctance, is greater over the slot than over the tooth.

If the slot is very wide and the air gap is relatively very short, the difference in the length of the magnetic lines over the tooth and those over the slot will be considerable and, therefore, the difference in the density will also be considerable. When the air gap is as large as the opening of the slot, the lines

spread so as nearly to fill the region over a slot, and the difference in density over a slot and over a tooth is small.

6. The reduction in the actual area of the air gap to be made in obtaining the effective area depends, therefore, not only upon the number and width of the slots, but also upon the relative value of the width of the slot to the length of the air gap.

Fig. 2 shows a curve that indicates the percentage of the slot that may be considered as removed entirely, to obtain the effective circumference of the area of the air gap, for different values of the ratio of the width of the slot to the length of the air gap.

In the case of this motor, commutation should be satisfactory if the ampere-turns expended in the air gap are as great as the armature ampere-turns per pole. The armature ampere-turns per pole are 1,234. The armature dimensions are 14 inches diameter by $5\frac{1}{2}$ inches long. The poles are to cover 70 per cent. of the cylindrical surface. The area of all pole faces will be $3.1416 \times 14 \times 5.5 \times .7 = 169.3$ square inches. The total flux required for all of the air gaps is 7,980,000 lines and, therefore, the *average* magnetic density in the air gap will be $7{,}980{,}000 \div 169.3 = 47{,}100$ lines per square inch.

In the preliminary calculations for the armature dimensions, a density of 40,000 lines per square inch was assumed. The final dimensions chosen call, however, for a somewhat higher density. The effective area of the air gap will be less than 169.3 square inches; therefore, the density will be higher than 47,100 lines per square inch.

A trial value of density, 52,000 lines per square inch, will be assumed for the determination of the effective area of the air gap. The number of ampere-turns $I\,T$ required to force a flux having a density of 52,000 lines per square inch across an air gap of length l will be $I\,T = .313 \times l \times B = .313 \times l \times 52{,}000$. If a magnetomotive force of 1,234 ampere-turns is substituted, as previously suggested, and the equation is solved for l, the length of the air gap will be $\frac{1{,}234}{.313 \times 52{,}000} = .076$ inch. Assume the length of the air gap to be *.075 inch.*

The width of the armature slots is .285 inch, and the value of the abscissa, Fig. 2, will be $\frac{.285}{.075}=3.8$. The corresponding ordinate will be .435, which is the fraction of the slot width that is effective. There are 62 slots each .285 inches wide, which gives a total actual slot width of $62\times.285=17.67$ inches, and a total effective width of $17.67\times.435=7.69$ inches.

The armature circumference will be $14\times3.1416=43.98$ inches, and deducting 7.69 inches will leave 36.29 inches as the net effective circumference.

The length of the armature is $5\frac{1}{2}$ inches, but this includes one $\frac{3}{8}$-inch air vent which, for determining the effective width of the air gap, may be treated as a slot. The abscissa, Fig. 2, will be $\frac{3}{8}\div.075=5$, and the corresponding ordinate will be .5. The effective width of the air vent will be $\frac{3}{8}\times.5=.19$ inch, and the net effective length of the armature will be $5.5-.19=5.31$ inches. The effective circumference is 36.29 inches; the effective length, 5.31 inches; and 70 per cent. of this area is under the pole; therefore, the effective area of the air gap will be $36.29\times5.31\times.7=$ *134.9 square inches.*

7. The effective air-gap density will be $\frac{7{,}980{,}000}{134.9}=$ *59,200 lines per square inch*, and the magnetomotive force required to maintain this density over the air gap will be $.313\times.075\times59{,}200=$ *1,390 ampere-turns*. This is considerably more than 1,234 ampere-turns, because the density estimate of 52,000 lines per square inch was too low. The air gap could be reduced to .07 inch, which would lower the value of the magnetomotive force to 1,297 ampere-turns, but the longer air gap, .075, will aid somewhat in commutation and may, therefore, be used. The ampere-turns per inch will be $\frac{1{,}390}{.075}=18{,}500$.

8. It is necessary to estimate the average lengths of the parts of the magnetic circuit, and this may best be done by sketching in the average magnetic line on a drawing of the machine made to scale, as indicated in Fig. 1, and measuring this line. In a yoke section, such as shown, the magnetic

MAGNETIC CIRCUIT

Part	Area Square Inches	Length Inches	Density Lines per Square Inch	Ampere-Turns per Inch	Ampere-Turns	Material
Armature core	105	2.25	76,000	16.5	37	Annealed sheet steel
Armature teeth (roots)	67.8	.85	117,700	128.75	109	Annealed sheet steel
Armature teeth (tops)	85		93,900			
Effective air gap	134.9	.075	59,200	18,500	1,390	Air
Pole piece	96.5	4.75	90,500	39	185	Unannealed sheet steel
Magnet yoke	120	8	72,800	22	176	Cast steel
Total ampere-turns for magnetic circuit					1,897	
Ampere-turns to overcome effects of armature reaction					250	
Total excitation per pole					2,147	

lines spread out over the complete section and in doing so their average length is increased. An allowance for this feature was made in Fig. 1 in the estimated length of 8 inches.

9. The data for the magnetic circuit of the 5-horsepower motor are given in the accompanying tabulation. The data in the second column relate to the total areas through which the total flux for the various parts of the magnetic circuit pass; for example, 134.9 square inches is the sum of the areas of 6 air gaps; and 120 square inches is 12 times the area of the yoke section, since there are two paths through the yoke for the flux passing through each of the 6 poles. The total flux, 7,980,000 lines, divided by the areas of the armature core and air gap gives the density in lines per square inch in those parts. The methods of determining the densities in the teeth, pole pieces, and yoke are explained later. The data in the fifth column, except that for the air gap, are obtained from Fig. 1, *Design of Direct-Current Machines*, Part 2. The data in the sixth column are the products of the data in the third and fifth columns.

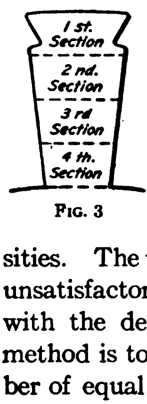


FIG. 3

The density at the tooth roots is 117,700 and at the tops is 93,900 lines per square inch, while at intermediate points there are corresponding densities. The use of the average of the top and bottom densities is unsatisfactory, because the ampere-turns do not vary uniformly with the density where saturation is approached. A better method is to divide the length of the tooth, Fig. 3, into a number of equal parts, for example, four, find the density in each, and then average the excitation required for these densities. The difference in density between the bottoms and the tops of the teeth is $117,700-93,900=23,800$ lines, and it may be considered that this density changes in four equal increments of 5,950 lines per square inch. The density at the top of the tooth is 93,900 lines, and at the top of the next section it will be $93,900+5,950=99,850$ lines. The average density for the first section will be $\frac{93,900+99,850}{2}=96,875$ lines. The average density in the second section will be $96,875+5,950=102,825$ lines;

in the third section 5,950 more, or 108,775; and in the fourth section 5,950 more than in the third section, or 114,725 lines. The ampere-turns per inch for these sections are found from Fig. 1, *Design of Direct-Current Machines*, Part 2, to be 51, 80, 139, and 245; and the average of these will be $515 \div 4 = \mathit{128.75}$. This value of the average ampere-turns per inch is used in column five of the data table. For a length of tooth of .85 inch, 109 ampere-turns will be required.

10. The magnetomotive force establishing magnetic leakage between the pole pieces must be estimated before the flux in the poles and yoke can be determined. To the ampere-turns required for the armature core, the teeth, and the air gap should be added the excitation required to overcome the magnetic effects of armature reaction and of the short-circuited armature currents. Some of the magnetic effects of armature reaction are to distort the flux in the pole face, air gap, and teeth, and to cause saturation where the density in the steel is forced to a high value, thus increasing the reluctance of the magnetic circuit and weakening the flux. The allowance to offset these effects is usually not more than 10 per cent. of the armature reaction. In this case, the armature reaction is 1,234 ampere-turns, and 8.1 per cent. of this, or 100 ampere-turns, will be allowed to compensate for the distortion in the flux.

The polar angle is 70 per cent.; hence, the neutral space will be 30 per cent. of the pole pitch. If the brushes were placed in the exact neutral point under no-load conditions, they could be shifted nearly 15 per cent. of the pole pitch backwards, which is in the direction to improve commutation of a motor, before the coils under commutation would be under a pole tip. If it is assumed that a shift of 10 per cent. is required for commutation, then an allowance of 20 per cent. of the armature reaction, or 250 ampere-turns, will be necessary to overcome the back ampere-turns.

The magnetic effects of the short-circuited currents in a motor armature *assist* the field excitation and these may amount to from 5 per cent. to 10 per cent. of the armature

reaction. If this effect is assumed to be 100 ampere-turns, this will balance the allowance of 100 ampere-turns made for the effects of distortion and saturation. The net effect of the armature reaction, therefore, is to require 250 additional ampere-turns.

11. The field ampere-turns available for establishing leakage are 37 for the armature core, 109 for the teeth, 1,390 for the air gap, and 250 to overcome effects of armature reaction, making a total of 1,786 ampere-turns. The leakage constant of this frame is 420; therefore, the leakage flux will be $1,786 \times 420 = 750,000$ lines.

The total flux in the magnet cores and yoke will be $7,980,000 + 750,000 = 8,730,000$ lines. This flux, divided by the total cross-sectional area of the pole pieces, gives the density in those parts, or $8,730,000 \div 96.5 = 90,500$ lines per square inch. The density in the magnet yoke will be $8,730,000 \div 120 = 72,800$ lines per square inch. The corresponding values of ampere-turns per inch, Fig. 1, *Design of Direct-Current Machines*, Part 2, will be 39 and 22; and of ampere-turns $4.75 \times 39 = 185$, and $8 \times 22 = 176$. The total excitation per pole will be 2,147 ampere-turns.

FIELD COILS FOR A 5-HORSEPOWER MOTOR

12. Since this is to be a shunt-wound motor, the 6 field coils in series are to be connected across the 115-volt supply circuit. Each coil will have $\frac{1}{6}$ of 115 volts impressed across its terminals. The resistance of each coil and the number of turns in it must be such that when 115 volts is impressed on the series of 6 coils, the current maintained multiplied by the total number of turns will produce in each field coil approximately the 2,147 ampere-turns previously calculated for the excitation of each pole. Also, the coils must not overheat when carrying the proper current. In this case, assume that the heating must not cause a rise in temperature of more than 40° C. above the temperature of the surrounding air.

The energy loss in the field coils is entirely due to the $I^2 R$ loss and the heat thus set up is chiefly dissipated by air, circulated

by the armature, coming in contact with the outside of the coils. Heat is also conducted from the coils, by the steel of the poles, to the yoke and to the pole tips, where it is carried away by the air from the armature coming in contact with these uncovered surfaces. The cooling of the field coils is, therefore, dependent to some extent on the peripheral velocity of the armature.

13. There are a number of methods employed for insulating and protecting the field coils of motors and generators. Some of these result in thorough protection for the coils, but do not allow of satisfactory heat dissipation. Coils that are more nearly bare dissipate heat more rapidly. Coils may be wound double or triple; that is, one coil outside of another with an air space, or vent, between. The conductors of the coil are immersed in the magnetic flux of the leakage field and there is some force acting on these conductors tending to push the coils out against the yoke. It is necessary, therefore, to construct the coils in a substantial manner, and to support them so that they cannot move, because the insulation is easily destroyed by rubbing.

One of the simplest methods of making coils is to wind the conductors on metal spools which are thoroughly insulated before receiving the wire. The flanges of the spool cannot be made of iron or steel because they would increase the magnetic leakage between poles. Spools with sheet-steel bodies and flanges made of insulating material are used extensively. The outside of such a coil may be left bare or it may be wound with small rope for mechanical protection. Such a coil may be impregnated with an insulating compound so that oil and moisture cannot penetrate it. Coils to be impregnated are placed in a heated vessel and the air is pumped out. The hot insulating compound is then pumped in under pressure. In a coil so treated there are no air particles in the spaces between the cotton fibers on the wire, and the heat from the wire is rapidly conducted to the surface of the coil.

Some coils are wound on a frame which is afterwards removed, and the conductors of the coil are then bound together with cord or tape. Such a coil may be impregnated, but the tape

insulation added to the outside of the coil somewhat interferes with heat dissipation.

14. The dissipation of heat is considerably affected by the presence of other coils. In commutating-pole machines, the coils are necessarily close together and the space between them is in some cases so limited as to prevent sufficient air from passing over their surfaces to remove the heat. The temperature rise of the coils with the same loss is, therefore, greater in commutating-pole machines than in those without these poles.

With similar peripheral velocities, armatures of short length will fan the field coils better than those of greater length. If the yoke casting is shaped so as to protect the armature and field coils, the temperature rise of the field coils may be increased.

15. Fig. 4 shows curves indicating approximately the relation between peripheral velocity of the armature in feet per minute and the degrees centigrade rise in temperature of the field coil for a radiation of 1 watt per square inch of circumferential surface of field coils. The data for these curves were taken from observations on average machines made with yokes similar to that shown in Fig. 1. Curve A, Fig. 4, indicates the heating that may be expected from a main field coil that is taped all over and is placed adjacent to commutating-pole coils. Curve B indicates the heating of the same coil if the commutating poles and coils were removed; this curve is also practically correct for a bare main ooil placed adjacent to commutating poles, or for a bare commutating-pole coil. Commutating-pole coils are often wound with *bare copper on edge*, so arranged that air may readily circulate around the turns; such a commutating-pole coil or one with air vents through it will dissipate heat so as to conform nearly with curve C. Curve D indicates the probable heating of coils when they are cooled by means of a fan placed on the armature shaft.

The curves, Fig. 4, are not to be considered as absolutely reliable, and should be used only until the heating of the first motors constructed of a new type has been checked. The heating of individual frames is often quite different from that of others apparently of the same general type and style; and

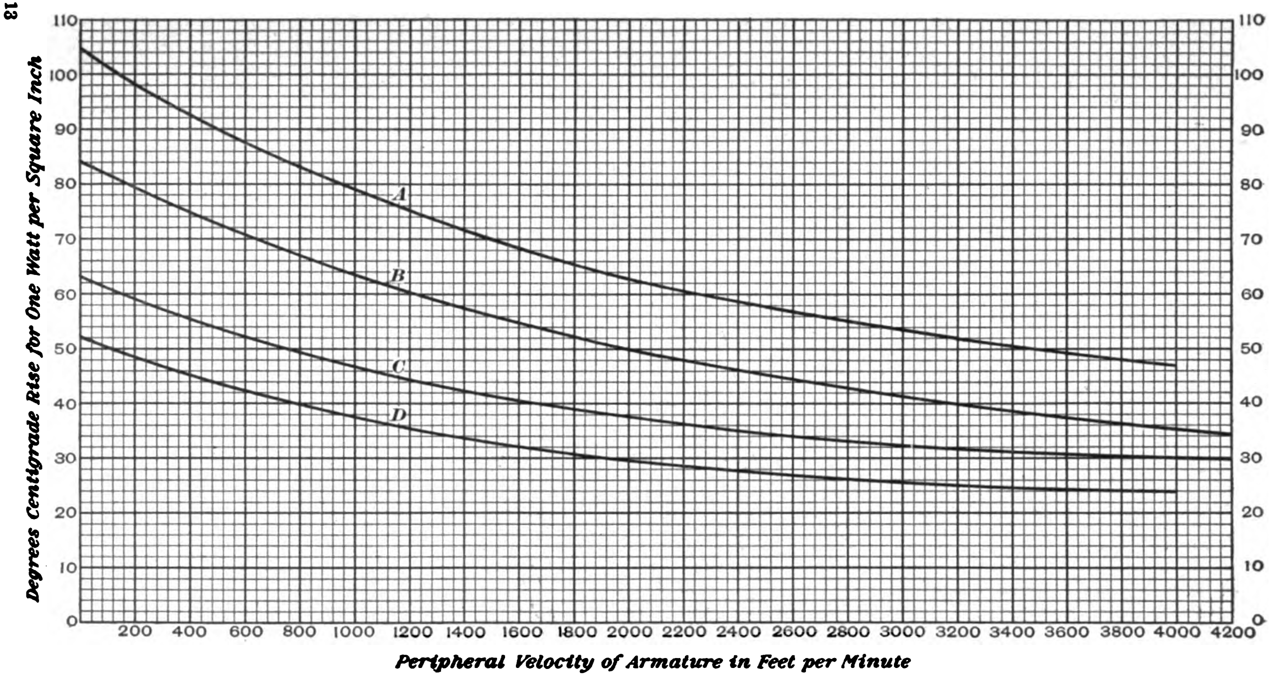

Degrees Centigrade Rise for One Watt per Square Inch
0
10
20
30
40
50
60
70
80
90
100
110
A
B
C
D
200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400 3600 3800 4000 4200
Peripheral Velocity of Armature in Feet per Minute

FIG. 4

the heating of motors made from the same lot of castings may differ, due possibly to variations in the movements of air set up by the armature windings against the field coils.

The cost of the field coils is an important item of the construction of a motor or generator, and, therefore, to keep down the cost, as small an amount of copper as is practicable should be used in these coils. This requires that the coils shall operate with their temperature close to the allowable heating limit, but not exceeding it; therefore, a close study of heating curves is essential.

16. The method of calculating the watts per square inch of circumferential field-coil surface is in general similar to that explained for armatures in *Design of Direct-Current Machines*, Part 1. The heat generated within a field coil of the form shown in Fig. 5 is assumed to pass out through its circumferential surface, the end surfaces of the coil not being considered for heat radiating purposes. This assumption is made because in an ordinary coil there is a square inch of coil surface midway between the ends of the coil that is so far away from either that the heat generated in or near that area would have to be conducted through too great a distance within the coil for much of it to reach the end surfaces. The heat developed within this square inch would, therefore, have to be dissipated either outward to the surrounding air or inward to the steel of the pole. Machines are usually guaranteed not to exceed a given temperature in any part, and the maximum temperature will be found where the maximum heat dissipation is taking place. This maximum dissipation cannot be greater than the amount calculated by the assumption that all the watts lost pass out through the circumferential surface of the coil.

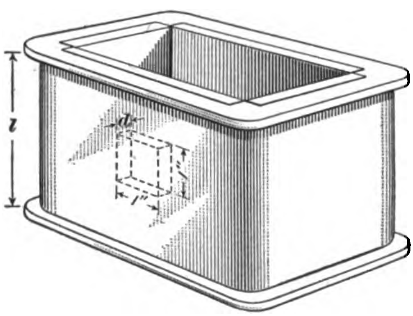


Fig. 5

17. Consider the square inch of surface indicated by the dotted lines, Fig. 5. If there are $I\,T$ ampere-turns in the length l, expressed in inches, of the coil, then the ampere-conductors per inch of length of coil will be $\frac{I\,T}{l}$, and the total current in all of the conductors in the square prism formed by the dotted square inch on the surface, a corresponding square inch on the inner surface of the coil, and the connecting lines, as indicated in Fig. 5, will be $\frac{I\,T}{l}$ amperes.

The resistance of a circular mil-foot of copper wire that is 40° C. hotter than the ordinary room temperature may be taken as 12.5 ohms, or $12.5 \div 12 = 1.04$ *ohms per mil-inch.* This resistance constant is a little greater than that used for armatures, because field coils usually operate at higher temperatures. For totally enclosed motors, the resistance constant may be as high as 1.08 or 1.1 ohms per mil-inch.

If there are T turns in the coil, there will be $\frac{T}{l}$ wires in the square prism the ends of which are formed by the dotted square inch on the surface and the corresponding square inch on the inner surface of the coil. If each wire has a cross-sectional area of a circular mils, then the total area of the combined bundle of $\frac{T}{l}$ wires will be $\frac{Ta}{l}$, and the resistance of a 1-inch length of the bundle will be $\dfrac{1.04\times 1}{\frac{T\,a}{l}}$, or $\frac{1.04\,l}{T\,a}$.

The loss in watts p in a 1-inch length of the bundle is equal to the product of the square of the total current and the resistance, or $p=\left(\frac{I\,T}{l}\right)^2\times\frac{1.04\,l}{T\,a}=\frac{1.04\,I^2\,T}{l\,a}$.

$$p=\frac{1.04\,I\,T}{l}\times\frac{1}{\frac{a}{I}}$$

Rule.—*The number of watts lost per square inch of circumferential surface of the field coil is equal to 1.04 times the ampere-conductor per inch of length of coil divided by the circular mils per ampere in the conductor.*

18. In the formula of Art. **17,** the known quantities ordinarily are the ampere-turns $I\,T$ and the length l. The watts per square inch p may also be known or may be calculated by the formula. The number of turns T may be calculated, if the size of wire in circular mils and watts p per square inch are known; thus, $p=\frac{1.04\,I^2\,T}{l\,a}=\frac{1.04\,I^2\,T^2}{l\,a\,T}=\frac{1.04\,(I\,T)^2}{l\,a\,T}$, and

$$T=\frac{1.04\,(I\,T)^2}{l\,a\,p}$$

19. In a shunt-wound field coil, the size of wire must be so selected that the resistance of the coil will allow the correct current to produce the required number of ampere-turns.

Let e = the impressed electromotive force on a coil;
M = the mean length of one turn, in inches;
T = the number of turns;
I = the current in the coil;
r = the resistance of the coil.

Then $I=\frac{e}{r}$, but $r=\frac{1.04\,M\,T}{a}$. Substituting the value of r,

$I=\frac{e}{\frac{1.04\,M\,T}{a}}=\frac{e\,a}{1.04\,M\,T}$, or

$$a=\frac{1.04\,M\,I\,T}{e}$$

20. The shortest length of a turn S may be closely estimated from the dimensions of the pole piece or by measurements of the winding form. The mean length of a turn M is greater than the shortest length S by the addition of a circle the diameter of which is the depth of the winding d, or

$$M=S+\pi\,d$$

The depth will depend on the number of wires that can be wound in a square inch of winding space. Table I shows data

TABLE I

MAGNET WIRE, BROWN & SHARPE GAUGE

Gauge Number	Diameter, in Mils				Area Circular Mils	Ohms Per 1,000 Feet at 20° C. 68° F.	Pounds per 1,000 Feet Bare
	Bare	Single-Cotton Covered	Double-Cotton Covered	Triple-Cotton Covered			
0000	460.0			478	211,600	.049	640.50
000	409.6			428	167,805	.062	508.00
00	364.8			383	133,079	.078	402.80
0	324.9			343	105,534	.098	319.50
1	289.3			307	83,694	.124	253.30
2	257.6			276	66,373	.156	200.90
3	229.4			247	52,634	.197	159.30
4	204.3		216	220	41,742	.248	126.40
5	181.9		194	198	33,102	.313	100.20
6	162.0		174	178	26,250	.394	79.46
7	144.3		156	160	20,816	.497	63.02
8	128.5		140	144	16,509	.627	49.98
9	114.4		126	130	13,094	.791	39.63
10	101.9	108.0	112	116	10,381	1.000	31.43
11	90.7	97.0	101	105	8,234	1.257	24.93
12	80.8	87.0	91	95	6,529	1.586	19.77
13	72.0	78.0	82	86	5,178	1.999	15.68
14	64.1	71.0	75	79	4,107	2.521	12.43
15	57.1	63.0	67	71	3,257	3.179	9.86
16	50.8	55.0	59	63	2,583	4.009	7.82
17	45.2	49.0	53	57	2,048	5.055	6.20
18	40.3	44.0	48	52	1,624	6.374	4.92
19	35.9	39.0	43	47	1,288	8.038	3.90
20	32.0	36.0	40	44	1,022	10.140	3.09
21	28.5	32.0	36	40	810	12.780	2.45
22	25.3	29.0	33	37	642	16.120	1.94
23	22.6	27.0	31	35	509	20.320	1.54
24	20.1	24.0	28	32	404	25.630	1.22
25	17.9	22.0	26	30	320	32.310	.97
26	15.9	20.0	24		254	40.750	.77
27	14.2	18.0	22		202	51.380	.61
28	12.6	17.0	21		160	64.790	.48
29	11.3	15.0	19		127	81.700	.38
30	10.0	14.0	18		100	103.000	.30
31	8.9	12.5			80	129.900	.24
32	8.0	11.5			63	163.800	.19
33	7.1	10.5			50	206.600	.15

relating to single-, double-, and triple-cotton covered magnet wires for the range of sizes ordinarily used in field coils of motors and generators.

21. Dividing both sides of the formula of Art. **18** by l gives $\frac{T}{l}=\frac{1.04\,(I\,T)^2}{l^2\,a\,p}=\frac{1.04}{a}\times\left(\frac{I\,T}{l}\right)^2\times\frac{1}{p}$. The value of the fraction $\frac{T}{l}$ is the number of wires per inch of length of coil space, therefore, if the coil is uniformly wound and $\frac{T}{l}$ is divided by K_w, the number of wires of that particular size that can be wound in 1 square inch of cross-section of the coil, the result will be the depth d of the winding; then $d=\frac{T}{l}\times\frac{1}{K_w}$, or by substitution of the value of $\frac{T}{l}$,

$$d=1.04\left(\frac{1}{a\,K_w}\right)\times\left(\frac{I\,T}{l}\right)^2\times\frac{1}{p}$$

The values of $\frac{1}{a\,K_w}$ are given in Table II.

22. To obtain the size of wire from the formula of Art. **19,** the value of M must be known, while to obtain the depth d of the winding from the formula of Art. **21,** the size of the wire must be known. The value of $\frac{1}{a\,K_w}$, on which the depth depends, does not change much from size to size of wire; hence, a guess can be made as to the value of M; and, the size of wire a approximated and this approximation corrected later if necessary.

23. The formulas just developed are applied to the design of the field coils for the 5-horsepower motor as follows: The perimeter of the pole piece, Fig. 1, is 5.5+3.25+5.5+3.25 =17½ inches. Let it be assumed that the coil will be taped, and that, to allow for clearance and insulation between the pole and the wire, .9 inch more will be required. Then, the shortest

TABLE II

DATA ON MAGNET WIRE, BROWN & SHARPE GAUGE

Gauge Number	K_w = Number of Wires per Square Inch		$\frac{1}{aK_w} = \frac{1}{\text{(Area in Circular Mils)} \times \text{Number of Wires per Square Inch}}$	
	Single-Cotton Covered	Double-Cotton Covered	Single-Cotton Covered	Double-Cotton Covered
4		22.2		1.08×10^{-6}
5		27.7		1.09×10^{-6}
6		34.7		1.10×10^{-6}
7		43.4		1.105×10^{-6}
8		54		1.12×10^{-6}
9		67		1.14×10^{-6}
10	87.5	84	1.1×10^{-6}	1.15×10^{-6}
11	109	104	1.115×10^{-6}	1.17×10^{-6}
12	135	129	1.13×10^{-6}	1.185×10^{-6}
13	168	161	1.15×10^{-6}	1.20×10^{-6}
14	207	198	1.175×10^{-6}	1.23×10^{-6}
15	256	246	1.2×10^{-6}	1.25×10^{-6}
16	315		1.23×10^{-6}	
17	388		1.26×10^{-6}	
18	478		1.29×10^{-6}	
19	585		1.33×10^{-6}	
20	715		1.37×10^{-6}	
21	875		1.41×10^{-6}	
22	1070		1.46×10^{-6}	
23	1290		1.52×10^{-6}	
24	1570		1.58×10^{-6}	
25	1880		1.66×10^{-6}	
26	2250		1.75×10^{-6}	
27	2690		1.84×10^{-6}	
28	3240		1.93×10^{-6}	
29	3890		2.03×10^{-6}	
30	4700		2.13×10^{-6}	

length of a turn S will be $17.5+.9=$ *18.4 inches*. The coil space, Fig. 1, will be 4 inches and the net length l for winding the wire will be taken as *3.9 inches*. For the depth of the winding d, a preliminary estimate of .6 inch will be made.

The mean length of a turn will be the perimeter, 18.4 inches, plus the circumference of a circle having a diameter d or a radius $\frac{d}{2}$; as there are four 90° bends in one turn, and each bend of a mean turn has a radius of $\frac{d}{2}$ inches, then, $M=S+\pi d=18.4+3.1416\times.6=20.3$ inches. The size of wire, in circular mils, calculated by the formula of Art. **19**, when $I\,T$ is taken as 2,147, Art. **9**, and when a voltage e of $\frac{115}{6}$ is impressed on each coil of the six field coils, will be

$$a=\frac{1.04\times20.3\times2{,}147}{\frac{115}{6}}=2{,}365 \text{ circular mils}$$

Reference to Table I shows that a wire having an area of 2,365 circular mils would be between the sizes 16 and 17 B. & S. gauge.

24. The peripheral velocity of the armature is $\frac{14\times3.1416\times200}{12}=733$ feet per minute. Reference to Fig. 4, curve B, shows that the indicated rise in temperature for 1 watt per square inch for a taped main field coil on a frame without interpoles is 69° C. For a rise of 40° C. in temperature, the watts per square inch will be $\frac{40}{69}=.58$.

25. The depth of the winding d may now be calculated. From Table II the value of $\frac{1}{a\,K_w}$ for No. 17 B. & S. single-cotton covered wire is $\frac{1.26}{10^6}$. Substitution of the numerical values

determined in the formula of Art. **21** gives $d=1.04\times\frac{1.26}{10^6}\times\left(\frac{2,147}{3.9}\right)^2\times\frac{1}{.58}=.68$ inch.

The corrected value of M in the formula of Art. **20** will be $M=18.4+3.1416\times.68=20.5$ inches.

The corrected value of the size of wire a, expressed in circular mils, as calculated by the formula of Art. **19** will be $a=\frac{1.04\times20.5\times2,147}{\frac{115}{6}}=2,388.$

The nearest size of wire is No. 16 B. & S., which has an area of 2,583 circular mils, or 8 per cent. more than is required. If No. 16 B. & S. wire is used, the coils will maintain 8 per cent. more ampere-turns than 2,147, or $2,147+2,147\times.08=2,319$ ampere-turns. If this value is not considered satisfactory it will be necessary to wind the coils partly of No. 16 B. & S. single-cotton covered wire and partly of No. 17 B. & S. single-cotton covered wire.

26. When making up coils of two sizes of wire in series it is customary to wind the larger wire in which for equal lengths the least loss takes place underneath and the smaller size outside. When using two sizes of wire in this way there is a slight loss in efficiency in the performance of the coils, and to make an approximate allowance for this, the ampere-turns should be increased 2 per cent. and the watts per square inch 4 per cent. When these corrections are made, the ampere-turns will be $2,147+2,147\times.02=\mathit{2,190}$; and the watts per square inch will be $.58+.58\times.04=.603$. In this case the change in the depth of winding d will be negligible.

27. The corrected size of wire, in circular mils, will then be $a=\frac{1.04\times20.5\times2,190}{\frac{115}{6}}=\mathit{2,436}$ *circular mils.*

From the formula of Art. **18**, the number of turns per coil will be $T=\frac{1.04\times2,190^2}{3.9\times2,436\times.603}=\mathit{871}.$

To determine how many of the 871 turns are to be formed of No. 16 B. & S. gauge wire and how many of No. 17 B. & S. gauge wire, use a modification of the formula of Art. **19** solving for $I\,T$ thus, $I\,T = \frac{a \times e}{1.04 \times M}$. First substitute the value of a for No. 16 B. & S. wire and then the value of a for No. 17 B. & S. wire and determine the ampere-turns for each case.

When the coils are wound entirely with No. 16 B. & S. wire,

$$I\,T = \frac{2{,}583 \times \frac{115}{6}}{1.04 \times 20.5} = 2{,}322$$

When the coils are wound entirely with No. 17 B. & S. wire,

$$I\,T = \frac{2{,}048 \times \frac{115}{6}}{1.04 \times 20.5} = 1{,}841$$

28. For 2,190 ampere-turns, there must be a greater number of turns of No. 16 B. & S. than of No. 17 B. & S. wire. The difference in the area of the two wires is $2{,}583 - 2{,}048 = 535$ circular mils, while the difference in ampere-turns is $2{,}322 - 1{,}841 = 481$.

It may be said that if 100 per cent. of the turns are changed from No. 16 B. & S. to No. 17 B. & S. wire the reduction in ampere-turns would be 481. It is desired to reduce the ampere-turns from 2,322 to 2,190, or 132 ampere-turns; hence, if changing 100 per cent. reduces the ampere-turns 481, a reduction of 132 ampere-turns will be accomplished by changing $\frac{132}{481} \times 871 = 239$ turns from No. 16 B. & S. to No. 17 B. & S. wire. There will be, therefore, *239 turns* of No. 17 B. & S. wire and $871 - 239 =$ *632 turns* of No. 16 B. & S. wire.

Another method of proportioning the turns for each size of wire is as follows: The area of No. 16 B. & S. wire is 2,583 circular mils; the desired area is 2,436 circular mils; the area of No. 17 B. & S. wire is 2,048 circular mils; and $2{,}583 - 2{,}436 = 147$; $2{,}583 - 2{,}048 = 535$; $2{,}436 - 2{,}048 = 388$; $\frac{147}{535} \times 871 = 239$

turns of No. 17 B. & S. wire; $\frac{388}{535}\times 871=632$ turns of No. 16 B. & S. wire.

29. The calculated number of turns may be checked as follows: The value K_w, Table II, indicates that 315 No. 16 B. & S. single-cotton covered wires may be wound in 1 square inch. For 632 turns, $\frac{632}{315}=2$ square inches, are required. The net length of winding space is 3.9 inches; therefore, the depth of the winding for the No. 16 B. & S. wire will be $\frac{2}{3.9}=.513$ inches.

The shortest length of a turn is 18.4 inches, Art. **23**, then the mean length of a turn for the No. 16 B. & S. wire will be $18.4+3.1416\times .513=20.01$ inches.

The resistance of 632 turns of No. 16 B. & S. wire in each coil (Art. **19**) will be $r=\frac{1.04\times 20.01\times 632}{2{,}583}=5.09$ *ohms*.

The length of the outside turn of the No. 16 B. & S. wire winding will be $18.4+3.1416\times(2\times .513)=21.6$ inches, and this is also the length of the shortest turn of the No. 17 B. & S. wire winding. The indicated value of K_w for a No. 17 B. & S. single-cotton covered wire, Table II, is 388 wires. The cross-sectional area required for 239 turns will be $\frac{239}{388}=.616$ square inch. The depth of the winding will be $\frac{.616}{3.9}=.158$ inch. The depth for both windings will be $.513+.158=.671$ inch. This is very close to the result of the preliminary calculation of .68 inch, Art. **25.**

30. The mean length of a turn of the No. 17 B. & S. wire winding will be $18.4+3.1416\times\left[2\times\left(.513+\frac{.158}{2}\right)\right]=22.12$ inches. The resistance of the No. 17 B. & S. wire coil will be

$$r=\frac{1.04\times 22.12\times 239}{2{,}048}=2.7 \text{ ohms}$$

The total resistance per coil will be $5.09+2.7=7.79$ ohms, and for the six coils in series the resistance will be 46.74 ohms. The current in the shunt field will be $\frac{115}{46.74}=2.46$ *amperes.*

31. The actual excitation will be $2.46\times871=2{,}143$ *ampere-turns;* whereas, as calculated in Art. **11**, 2,147 ampere-turns was desired. Motors manufactured in quantities often run 2 to 3 per cent. above or below their average speed; and a change in the excitation of 4 or 5 per cent. is necessary to cause a change of 2 or 3 per cent. in the speed. Therefore, the excitation just calculated may be considered as close enough.

32. The actual watts per square inch for the No. 16 B. & S. wire winding may be calculated from the formula

$$p=\frac{1.04\ I^2\,T}{l\,a}=\frac{1.04\times2.46^2\times632}{3.9\times2{,}583}=.395\text{ watt}$$

For the No. 17 B. & S. wire winding, $p=\frac{1.04\times2.46^2\times239}{3.9\times2{,}048}$ $=.188$ watt.

The total loss per square inch will be $.395+.188=.583$ watt instead of the estimated value .58 watt, Art. **24**.

33. It was assumed, Art. **26**, that the excitation of 2,147 ampere-turns would fall short by 2 per cent. and the value of .58 watt per square inch would fall short by 4 per cent. On this account the calculations following were made with the values of 2,190 ampere-turns and .603 watt. The final calculations are 2,143 ampere-turns and .583 watt. The first is $\frac{2{,}190-2{,}143}{2{,}190}$ $\times100=2.1$ per cent. less than the estimated value of 2,190 ampere-turns, and $\frac{2{,}147-2{,}143}{2{,}147}=.19$ per cent. less than the original value 2,147 ampere-turns.

The value .583 watt is $\frac{.583-.58}{.58}\times100=.52$ per cent. more than the calculated value .58 watt; therefore, the allowance

of 4 per cent., making the value .603 watt, was greater than necessary.

An allowance is only necessary when there are two sizes of wire in the coil, and the amount to allow may be determined by experience. In any case the allowance is small.

MAGNETIC CIRCUIT FOR A 150-KILOWATT GENERATOR

34. The following problem relates to the magnetic circuit of the 150-kilowatt, six-pole generator, the armatures of which were designed in *Design of Direct-Current Machines*, Part 1.

There will be a commutating pole for every main pole and it will be best to make these poles as long, or nearly as long, as the armature core. The armature is 7 inches in length, and for trial purposes the length of the commutating pole will be taken as 6 inches. For such a generator an air gap of $\frac{3}{16}$ inch or $\frac{1}{4}$ inch would be suitable, and a length of air gap of $\frac{3}{16}$ inch for the commutating pole will be assumed.

By substitution of values in the formula of Art. **47**, *Design of Direct-Current Machines*, Part 2, the compensation will be,

$$J = \frac{120}{\frac{3}{16}+\frac{3}{16}} \times \frac{\frac{3}{16}}{27} \times \frac{7}{6} \times 6 = 15\tfrac{5}{9} \text{ per cent.}$$

35. The approximate value of the total ampere-conductors for the whole armature core was estimated as 64,000. Dividing the value by 6, which is the number of commutating poles, and by 2, the number of conductors to a turn, gives $\frac{64{,}000}{6\times 2} = 5{,}330$, which is the approximate value of the armature reaction. The number of ampere-turns that must be put on each commutating pole is $15\frac{5}{9}$ per cent. more than the value of the armature reaction, or $5{,}330 \times 1.156 = 6{,}160$ ampere-turns, approximately. All of these ampere-turns tend to set up leakage flux in the sides of the commutating poles.

The coils on the commutating poles are so connected as to oppose the armature magnetomotive force, and the difference

between 6,160 and 5,330, or 830 ampere-turns, is the magnetomotive force that establishes flux through the ends of the commutating poles.

36. The commutating pole should span 15 per cent. of the pole pitch, as stated in Art. **48,** *Design of Direct-Current Machines*, Part 2, or its width should be $\frac{3.1416\times 27}{6}\times .15 = 2.12$ inches.

The main poles cover 70 per cent. of the pole pitch, which is the space between centers of main poles, and 15 per cent. of this space is taken up by the commutating poles, leaving only $\frac{30-15}{2}=7.5$ per cent. uncovered space on either side of the commutating pole. As 15 per cent. is equivalent to 2.12 inches, 7.5 per cent. will be equivalent to 1.06 inches, which is the distance between the main pole and the commutating pole.

37. The commutating-pole leakage may be taken as occurring along the 6 inches that the pole actually occupies, plus 1 inch, or a total of 7 inches. The distance to the main poles is 1 inch, which will be the allowance for the spreading of the flux.

If the leakage is assumed as 7 lines per ampere-turn, per inch, and per side, the leakage flux per commutating pole will be $7\times 6{,}160\times 7\times 2=604{,}000$ lines.

38. The number of ampere-turns $I\,T$ required to force a flux having a density B across an air gap of length l will be $I\,T=.313\times l\times B$; then $B=\frac{I\,T}{.313\times l}$. At the end of the commutating pole the ampere-turns available are 830 and the length of the air gap is $\frac{3}{16}$ inch; then the density in the gap at the ends of these commutating poles will be $B=\frac{830}{.313\times\frac{3}{16}}=14{,}100$ lines per square inch.

While the density should be corrected as explained in Art. **6,** this uncorrected value is sufficiently accurate for determining the size of the commutating pole. The actual area of the end

of the pole will be 2.12×6 square inches, but it will be necessary to allow, say, the length of the air gap all around the pole for the fringing flux at the edges. The area will then be $[2.12+2(.1875)]\times[6+2(.1875)]=2.5\times6.375=15.94$ square inches, and the flux through this area will be $15.94\times14{,}100=225{,}000$ lines.

The total flux in a commutating pole will be the sum of the end flux and the leakage flux, or $225{,}000+604{,}000=829{,}000$ lines. If the pole is made straight, that is, with parallel sides, so that its section will be $2.12\times6=12.72$ square inches, the maximum density in the commutating pole will be $\frac{829{,}000}{12.72}=65{,}000$ lines per square inch.

39. The flux in the commutating poles varies with the load. If the poles are of steel, they will reach saturation at about 100,000 lines per square inch; and with a full-load density of 65,000 lines per square inch, the saturation point would be reached at about 50 per cent. overload. When these poles become saturated their regulating function is impaired and sparking at the brushes will probably ensue. The overload margin allowed should be sufficient for an ordinary machine. If this allowance is considered too great, the size of the commutating pole may be reduced, thus reducing the margin. A reduction in the size of the pole will reduce the leakage area and thus, because of the reduced leakage flux, will not greatly increase the density.

Whenever the density in the commutating pole is very low, the pole may be made smaller than its end; that is, the pole may be made with a shoe on the end, shaped much like that on a main pole, for spreading the flux over the armature surface.

The commutating poles could be made smaller for the 250- and 550-volt windings because the brushes in these cases are narrower and span a smaller angle, but to preserve uniformity of the frames for the three types of armatures all of the poles will be made alike.

40. The value of the leakage constant for this magnet frame may be approximated by using the same leakage for

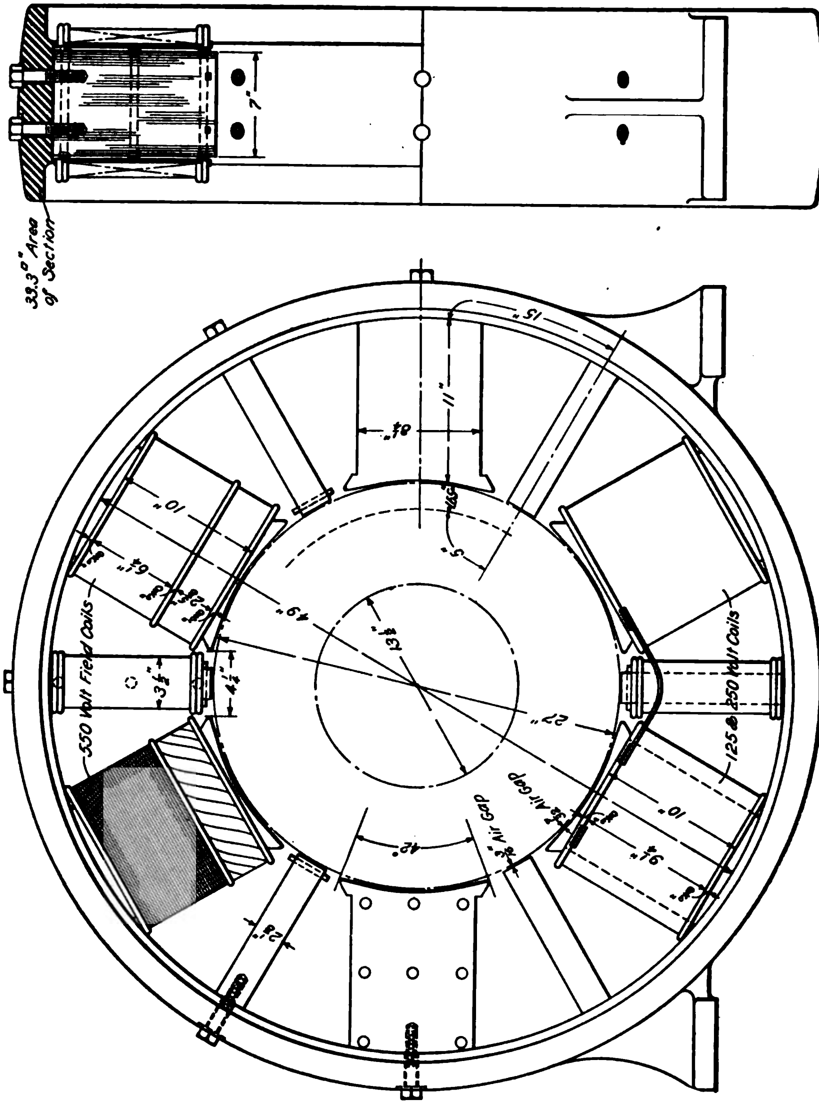

33.3□" Area of Section
550 Volt Field Coils
125 & 250 Volt Coils
Air Gap
Air Gap

Fig. 6

the main pole as for the commutating pole for the region opposite the commutating pole and 4½ lines per ampere-turn per inch and per side for the leakage from main pole to main pole direct.

The main poles will be made the same length parallel to the shaft as the armature, or 7 inches. From a sketch of the poles, to scale, as Fig. 6, the main-pole tips will be found to be 4¼ inches apart. Allowing 4¼ inches over the length of 7 inches, for the spreading out of the leakage flux, makes 11¼ inches. Of this length, 7 inches was allowed for the commutating pole; hence, over the 4¼ remaining inches the leakage of 4½ lines per ampere-turn per side and per inch exists. The total leakage flux per ampere-turn is then, $[(4\tfrac{1}{2}\times4\tfrac{1}{4})+(7\times7)]\times12=817\tfrac{1}{2}$ lines per ampere-turn. Allow, say, *820* for the leakage constant of this frame.

41. The considerations determining the length of the air gap are mechanical clearance and voltage regulation. Anything more than ⅛ of an inch should be sufficient for mechanical clearance. If the air gap is too small with respect to the rest of the magnetic circuit of a generator, the voltage will change considerably with changes of load. As this effect is caused by the load currents, it is well, as in the case of motors, to proportion the ampere-turns expended in the air gap to the armature reaction per pole. The gap ampere-turns for average generators are from 70 to 125 per cent. of the armature reaction, the latter value giving the better voltage regulation. In this problem, assume the gap ampere-turns to be from 80 to 90 per cent. of the armature reaction. The armature reaction is about 5,330 ampere-turns per pole; therefore, the gap ampere-turns should be from 4,300 to 4,800 ampere-turns.

42. In *Design of Direct-Current Machines*, Part 1, the armature flux was estimated as 24.7 megalines for the 125-volt machine, 25 megalines for the 250-volt machine, and 25.4 megalines for the 550-volt machine.

For 25 megalines, the average density under the main pole piece, as calculated from a modification of formula 1 of Art. 4, *Design of Direct-Current Machines*, Part 1, will be $B=\frac{p\,\Phi}{\%\,\pi\,d\,l}$

$= \frac{25{,}000{,}000}{.7 \times 3.1416 \times 27 \times 7} = 60{,}200$ lines per square inch. The effective air-gap density will be greater than this. Assume a value of 70,000 lines. With a gap density of 70,000 lines and a length of air gap of $\frac{7}{32}$ inch, the gap ampere-turns will be $I\,T = .313 \times \frac{7}{32} \times 70{,}000 = 4{,}800$.

43. The 125-volt armature has 84 slots, each .351 inch wide. The fraction $\frac{\text{width of slot}}{\text{length of air gap}} = \frac{.351}{\frac{7}{32}} = 1.6$. By reference to Fig. 2, it is seen that if the abscissa is 1.6, the ordinate will be .23.

The circumference of the armature $= 27 \times 3.1416 = 84.82$ inches. The effective circumference is less than this by $.23 \times 84 \times .351 = 6.78$ inches, and, therefore, is $84.82 - 6.78 = 78.04$ inches.

The actual length of the armature is 7 inches, but there is one air vent $\frac{1}{2}$ inch wide. Then $\frac{1}{2}$ divided by $\frac{7}{32} = 2.3$, which will be the value of the abscissa, Fig. 2, corresponding to an ordinate of .31. The effective width of the air vent will be $.31 \times .5 = .155$ inch. The effective length of the armature will be $7 - .155 = 6.845$ inches.

For a 70-per-cent. polar angle, the effective area of the air gap will be $78.04 \times 6.845 \times .7 = 374$ square inches.

The effective gap density for 24.7 megalines is $24{,}700{,}000 \div 374 = 66{,}000$ lines per square inch, and the ampere-turns required for a $\frac{7}{32}$-inch air gap will be $I\,T = .313 \times \frac{7}{32} \times 66{,}000 = 4{,}519$. As this value is between 4,300 and 4,800 ampere-turns, it will be considered satisfactory.

The values, $\frac{7}{32}$ inch for the length of the air gap for the main poles, and *374* square inches for the effective area of the air gap, are also considered satisfactory.

44. The 250-volt armature has 110 slots, each .268 inch wide. The abscissa, Fig. 2, will be $\frac{.268}{\frac{7}{32}} = 1.23$, and the corresponding ordinate will be .19.

The circumference of the armature should be reduced $.19 \times 110 \times .268 = 5.6$ inches; therefore, the effective circumference will

be $84.82-5.6=79.22$ inches. The effective length of the armature is 6.845 inches, and the effective area of the air gap will be $79.22\times6.845\times.7=380$ square inches.

45. The 550-volt armature has 59 slots, each .5 inch wide. The abscissa, Fig. 2, will be $\frac{.5}{\frac{7}{32}}=2.29$, and the corresponding ordinate will be .305. The circumference of the armature should be reduced $.305\times59\times.5=9$ inches; therefore, the effective circumference will be $84.82-9=75.82$ inches. The effective length of the armature is 6.845 inches, and the effective area of the air gap will be $75.82\times6.845\times.7=363$ square inches.

46. When calculating the sizes of the various armature slots the assumption was made that the same amount of steel was to be punched out in each of the three cases; therefore, the total cross-sectional area at the tooth roots and at the tooth tops will be the same for the three types of armature. It is not necessary that this be done, but it is sometimes desirable to do so, because the only difference between the magnetic circuits of the three windings is in the teeth and the air gaps, and if these are nearly alike, the magnetic circuits of the three types of machines may be considered as identical. In this case the effective areas of the air gaps are 374, 380, and 363 square inches, and the average will be *372 square inches*.

The errors in using this average value for the area of the air gap and considering the magnetic circuits as identical are $\frac{2}{374}\times100=.53$ per cent. plus; $\frac{8}{380}\times100=2.1$ per cent. plus; and $\frac{9}{363}=2.5$ per cent. minus for the gap ampere-turns. For very careful calculations these errors might be considered too great, but ordinarily it is expensive to construct a machine of this size so that its air gap is within .01 inch of its correct length. The length .01 inch corresponds to $\frac{.01}{.22}\times100=4.5$ per cent. of the length of the air gap, so that the error of averaging

the air-gap areas is small with respect to the probable error in the gap itself.

47. The excitation available for establishing magnetic leakage is that required for the armature core, the teeth, the air gap, and the allowance to overcome the effects of armature reaction. The effective area of the armature core was found to be 358 square inches, and the length of this part of the magnetic circuit is 5 inches, as indicated in Fig. 6. For a flux of 25 megalines, the density in the core will be $\frac{25{,}000{,}000}{358} = 70{,}000$ lines per square inch.

Reference to Fig. 1, *Design of Direct-Current Machines*, Part 2, shows that a density of 70,000 lines in annealed sheet steel requires 12 ampere-turns per inch, so for 5 inches, *60 ampere-turns* will be required. The densities at the tooth roots and tops for a flux of 25 megalines and for the areas of *184* and *227* square inches, as previously determined, will be, respectively, $\frac{25{,}000{,}000}{184} = 136{,}000$, and $\frac{25{,}000{,}000}{227} = 110{,}100$, lines per square inch.

48. The tooth may be considered as being divided into four parts, as in the case of the 5-horsepower motor problem, Art. **9.** The difference in density between the bottoms and tops of the teeth is $136{,}000 - 110{,}100 = 25{,}900$ lines, and it may be considered that this density changes in four equal increments of $\frac{25{,}900}{4} = 6{,}475$ lines per square inch.

The density at the top of the tooth is 110,100 lines, and at the top of the next section it will be $110{,}100 + 6{,}475 = 116{,}575$ lines. The average density for the first section will be $\frac{110{,}100 + 116{,}575}{2} = 113{,}338$ lines. The average density in the second section will be $113{,}338 + 6{,}475 = 119{,}813$ lines; in the third section 6,475 more, or 126,288; and in the fourth section 6,475 more than in the third section, or 132,763 lines per square inch. The ampere-turns per inch for these sections are found

from Fig. 1, *Design of Direct-Current Machines*, Part 2, to be 215, 380, 550, and 750 ampere-turns, and the average of these will be $\frac{215+380+550+750}{4}=474$. The length of the teeth is 1.65 inches; therefore, the ampere-turns required for the teeth will be $474\times1.65=$ *782*.

The density for the $\frac{7}{32}$-inch air gap, found by using the average values 25,000,000 lines and 372 square inches, will be $\frac{25{,}000{,}000}{372}=67{,}200$; and the ampere-turns for the air gap will be $\frac{7}{32}\times67{,}200\times.313=$ *4,601*.

49. This generator is equipped with commutating poles, and the brushes are not shifted from the true neutral position; therefore, there will be no back ampere-turns. Because of the action of the commutating poles, there should be no short-circuited currents in the coils undergoing commutation. There remains only the weakening effect due to the shifting of the flux under the main poles.

With an armature reaction of 5,330 ampere-turns, gap ampere-turns 4,601, and a ratio $\frac{5{,}330}{4{,}601}=1.16$, the armature interference may be taken as about 10 per cent. of the armature reaction, or 533 ampere-turns.

50. The excitation establishing magnetic leakage will be the sum of 60 ampere-turns for the armature core, 782 for the teeth, 4,601 for the gap, and 533 for the armature interference caused by the armature reaction, or a total of 5,976 ampere-turns.. With a leakage constant of 820, Art. **40,** the leakage flux for 5,976 ampere-turns will be $820\times5{,}976=4{,}900{,}000$ lines.

The armature flux was taken as 25 megalines; therefore, the maximum flux in the cores of the field magnets will be $4.9+25=29.9$ megalines, or $29.9\div25=1.196$ times the armature flux. This ratio, 1.2 approximately, of the field flux to the armature flux is the coefficient of magnetic leakage and will remain nearly constant for changes of flux, but will change for changes in the length of the air gap.

51. If the armature flux is 25 megalines and the leakage coefficient is 1.2, the field flux will be $25 \times 1.2 = 30$ megalines for the six poles, or 5 megalines per pole.

The poles are to be of sheet-steel punchings, 90 per cent. of which may be considered as metal and 10 per cent. as scale and air space. A density of about 95,000 lines per square inch should be used, so for 5 megalines, $\frac{5,000,000}{95,000} = 52.6$ square inches of metal, or $\frac{52.6}{.9} = 58.4$ square inches of punchings, will be required.

The main poles will be made 7 inches long parallel to the shaft; therefore, for an area of 58.4 square inches, the pole section should be 7 inches by 8.34 inches. If the section is made 7 inches by $8\frac{1}{4}$ inches, the density for the 5-megaline field flux will then be $\frac{5,000,000}{7 \times 8.25 \times .9} = 96,000$ lines per square inch. The metal area of the six poles will be $7 \times 8.25 \times .9 \times 6 =$ *312 square inches.*

52. The radial length of the space for the field coil may be taken as from 60 to 75 per cent. of the pole pitch. If a value of 65 per cent. is assumed, the coil space will be $\frac{27 \times 3.1416}{6} \times .65 = 9.19$ inches, or practically $9\frac{1}{4}$ inches.

It is assumed that the coils will be wound on spools with insulating heads. These heads will be at least $\frac{3}{8}$ inch thick so that the spool will measure about 10 inches over all. To allow room for such a spool, the bore of the seat of the poles on the frame, Fig. 6, will be about 49 inches. The length of the average magnetic path in the pole may be measured as 11 inches, as indicated in Fig. 6. The ampere-turns per inch required for a density of 96,000 lines per square inch in unannealed steel, Fig. 1, *Design of Direct-Current Machines*, Part 2, is 51.5 and the magnetomotive force for 11 inches will be *567 ampere-turns.*

53. The flux in the yoke will be one-half of that in the pole, or 2.5 megalines. For a density of 75,000 lines per square inch

in cast steel, the yoke section will be $\frac{2{,}500{,}000}{75{,}000}=33.3$ square inches. The area required for the total flux is 12 times this, or *400 square inches*. The length of the magnetic path in the yoke, as measured in Fig. 6, will be about $12\frac{1}{2}$ inches; but since the yoke spreads out over the field coils, the length of the path is taken as *15 inches*. For a density of 75,000 lines per square inch, the corresponding ampere-turns per inch in cast steel will be 24, and for 15 inches the required magnetomotive force will be $24\times15=$ *360 ampere-turns*.

54. The data for the magnetic circuit may be recorded as shown in the tabulation of data for the magnetic current.

MAGNETIC CIRCUIT

Part	Area Square Inches	Length Inches	Material
Armature core......	358	5	Annealed steel
Tooth roots........	184	1.65	Annealed steel
Tooth tops.........	227		
Effective air gap....	372	$\frac{7}{32}$	Air
Magnet pole........	312	11	Unannealed steel
Magnet yoke.......	400	15	Cast steel

55. As there are a number of different fluxes to be considered with the various windings, it will be best to compute and plot a saturation curve. Let it be assumed that the successive armature fluxes are 10, 20, 25, and 28 megalines, then the densities and corresponding ampere-turns may be computed as shown in the tabulation of data for saturation curve.

56. The values of B and $I\ T$ determined for the 25-megaline flux are inserted in columns 6 and 7. The values of B for the other total fluxes will be proportional to these fluxes directly and may be so calculated. Thus, for the 10-megaline flux, divide the armature-core density, 70,000, by $2\frac{1}{2}$, which equals

DATA FOR SATURATION CURVE

Part	10 Megalines		20 Megalines		25 Megalines		28 Megalines	
	B	IT	B	IT	B	IT	B	IT
Armature core ...	28,000	20	56,000	38	70,000	60	78,400	95
Tooth roots.......	54,400	10	108,800	108	136,000	782	152,300	1,584
Tooth tops........	44,000		88,000		110,100		123,200	
Effective gap	26,900	1,840	53,800	3,680	67,200	4,601	75,300	5,152
Magnet poles	38,400	85	76,800	231	96,000	567	107,500	1,375
Magnet yoke	30,000	98	60,000	219	75,000	360	84,000	549
		2,053		4,276		6,370		8,755

28,000 lines per square inch. For the 20-megaline flux, the value of B for the armature core will be twice 28,000, or 56,000 lines per square inch. For the 28-megaline flux, the value of B for the armature core will be $28{,}000 \times 2.8 = 78{,}400$ lines per square inch. When computing the density in the poles and yoke the leakage coefficient of 1.2 must be taken into consideration as is done in the values in the table.

For each value of density the ampere-turns per inch are obtained from Fig. 1, *Design of Direct-Current Machines*, Part 2, and this is multiplied by the length of the path through which the flux of this density is to be maintained, in order to obtain the value of IT for the table. The ampere-turns for the teeth are found by the method explained in Arts. **9** and **48**.

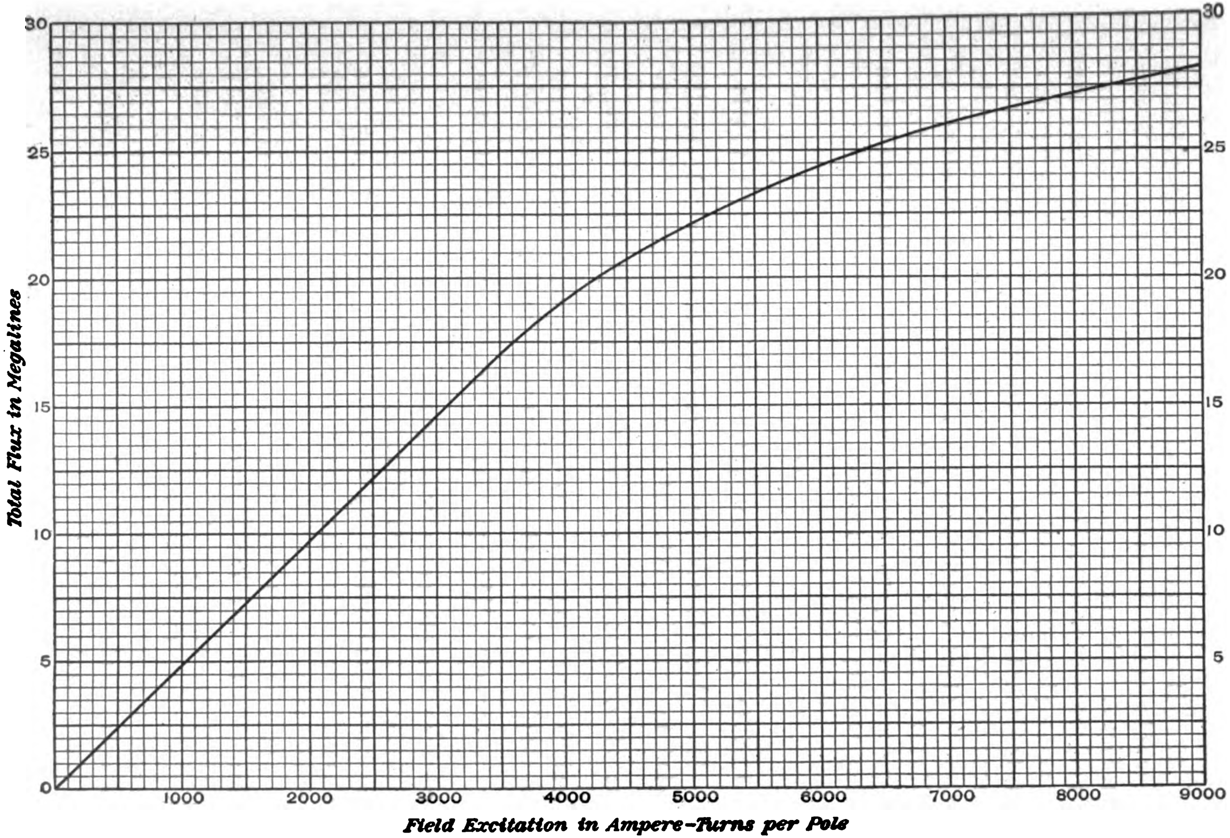

Total Flux in Megalines
0
5
10
15
20
25
30
1000
2000
3000
4000
5000
6000
7000
8000
9000
Field Excitation in Ampere-Turns per Pole

FIG. 7

57. If the total flux is plotted against the ampere-turns per pole, a saturation curve of the machine is obtained, as shown in Fig. 7. This curve may be checked experimentally when the machine is completed, by operating it at no load at a constant speed and observing the terminal volts with varying currents in the shunt-field windings. The flux may then be calculated from the voltage, the speed, and the number of conductors in the armature winding, by the formula of Art. **2**, *Design of Direct-Current Machines*, Part 1; and the field excitation expressed in ampere-turns is the product of the current in the field winding and the number of turns per coil.

FIELD COILS FOR A 150-KILOWATT GENERATOR

58. The 125-volt armature has one turn per coil, parallel winding, with 168 coils and commutator segments. The number of armature turns per pole will be $\frac{168}{6}=28$, but since this is a six-path winding, only one-sixth of the total armature current is established in each of these 28 turns. The armature ampere-turns per pole will be equivalent to those in a coil of $\frac{28}{6}=4\frac{2}{3}$ turns carrying the total armature current.

In Art. **34** the compensation for the commutating-pole winding was calculated as $15\frac{5}{8}$ per cent. Each commutating pole should have $15\frac{5}{8}$ per cent. more turns than $4\frac{2}{3}$, or $4.66\times1.156 = 5.4$ turns.

The number of turns on these poles must be nearly correct; 5 turns is too few and 6 turns too many. Two methods may be utilized: one, to put 5 and 6 turns on alternate commutating poles; and the other, to put 6 turns on all poles and, by means of a low-resistance conductor connected across the series of coils, to shunt a part of the current to reduce the ampere-turns. The latter method will now be considered.

59. The current output of the 125-volt machine is 1,200 amperes. To obtain the same value of ampere-turns for the

6-turn coil as the 5.4-turn coil will have with the full current of 1,200 amperes, the current must be reduced 10 per cent. by shunting $\frac{1}{10}$ of the total current around the coil. The reduced current in the 6-turn coil will be $1,200 \times .9 = 1,080$ *amperes*.

The peripheral velocity of the armature is 4,060 feet per minute. Reference to curve *c*, Fig. 4, shows that one watt per square inch for a bare commutating-pole coil and a peripheral velocity of 4,060 feet will produce a rise in temperature of about 30° C. For a 40° C. rise, $40 \div 30 = 1.33$ *watts per square inch* may be dissipated.

60. The length of the commutating pole, Fig. 6, is $\frac{49-[27+(2\times\frac{3}{16})]}{2}=10\frac{13}{16}$, or $10\frac{3}{4}$ inches, approximately. Allowance of $\frac{1}{2}$ inch at the armature end for supporting the coil and two $\frac{3}{8}$-inch insulating washers leaves $9\frac{1}{2}$ *inches net coil space*. In this space there are to be 6 turns, each carrying 1,080 amperes.

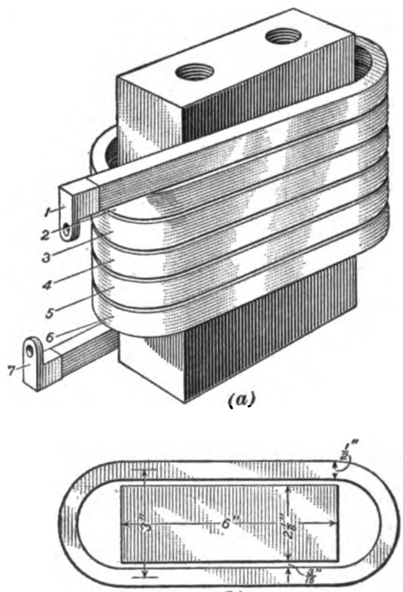


Fig. 8

When the leads are brought out as shown in Fig. 8, and the turns are wound spirally, there will be on the narrow side of the core near the terminals, five loops, numbered *2*, *3*, *4*, *5*, and *6* and two terminal conductors numbered *1* and *7*. The distance parallel to the pole from the upper side of conductor *1* to the lower side of *7* is to be 9.5 inches. These conductors are equally spaced and there are *7* of them, one more than the number of turns because of the spiral method of winding, each carrying a current of 1,080 amperes, between the extreme top and the bottom of the coil; that is, within the space of $9\frac{1}{2}$ inches. The ampere-conductors per inch, upon which the heating

depends will be $\frac{1{,}080\times 7}{9.5}=796$. In this coil there are six turns, but $\frac{I\,T}{l}$ does not represent the correct ampere-conductors per inch unless T is taken as seven.

61. From the formula of Art. **17**, the circular mils per ampere of the conductor will be $\frac{a}{I}=1.04\frac{I\,T}{l}\times\frac{1}{p}=1.04\times 796\times\frac{1}{1.33}=622$. For 1,080 amperes, $622\times 1{,}080=672{,}000$ circular mils will be required. This is equivalent to $672{,}000\times\frac{3.1416}{4}=528{,}000$ square mils, or .528 square inches.

62. Seven conductors will be spread in a length of 9.5 inches, or the space allowance for each conductor will be $9.5\div 7=1.36$ inches. Allowing $\frac{1}{8}$ inch for insulation and clearance, the copper conductor could be $1.36-.125=1.235$ inches wide by $\frac{.528}{1.235}=.428$ inch deep. If the conductor is made .5 inch deep, it should be made $.528\div .5=1.056$ inches wide. Let it be decided to build the conductor of 10 strands, each .11 inch wide and .5 inch deep. The total width will be $10\times .11=1.1$ inches, and the cross-sectional area of the conductor will be $10\times .11\times .5=.55$ square inch. The general appearance of the coil without spool heads and supports will be as indicated in Fig. 8.

63. Allowance of a clearance of $\frac{3}{16}$ inch between the pole and the copper, for insulation, makes the shortest distance from copper to copper on opposite sides of a pole that is $2\frac{1}{8}$ inches thick equal to $2\frac{1}{8}+\frac{3}{16}+\frac{3}{16}=2\frac{1}{2}$ inches. The distance from center to center of copper is $2\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=3$ inches. The ends of the coils are half circles of 3-inch mean diameter. The mean length of a turn will be $(2\times 6)+3.1416\times 3=21.42$ inches.

The area of the conductor is .55 square inch = 550,000 square mils = $550{,}000\div .7854=$ *700,000 circular mils*. Each coil consists of 6 turns, each 21.42 inches long, plus two terminal leads

each about 3 inches in length. The total length of the coil will be $3+3+6\times21.42=134.5$ inches.

As explained in Art. **19,** the resistance of six coils in series, when hot, will be $r=\frac{1.04\times134.5}{700{,}000}\times6=$ *.0012 ohm.*

As explained in Art. **17,** the loss in watts per square inch of copper in this coil will be $p=\frac{1.04\times1{,}080^2\times7}{9.5\times700{,}000}=1.277.$

64. The armature winding for the 250-volt generator has 110 coils of one turn each arranged in two parallel paths. The number of armature turns per pole will be $\frac{110}{6}$, and this divided by 2, the number of paths in the series armature winding, is $\frac{110}{6\times2}$, the number of turns per commutating pole that are the equivalent of the armature turns per pole. The addition of $15\frac{5}{8}$ per cent. for compensation makes the number of turns $\frac{110}{6\times2}\times1.156$ $=10.6$ turns. It will be necessary to wind 11 turns for each commutating-pole coil.

If the copper is wound edgewise and spirally, as indicated in Fig. 8, allow in the heating calculations for one more conductor than the number of turns, or 12 conductors per coil space, each carrying the full-load current of 600 amperes. The ampere-conductors per inch of the $9\frac{1}{2}$-inch coil space will be $\frac{600\times12}{9.5}=758.$

65. For 1.33 watts per square inch, the circular mils per ampere will be $1.04\times758\times\frac{1}{1.33}=593.$ For 600 amperes, the conductor should have a sectional area of $593\times600=355{,}800$ circular mils, $355{,}800\times.7854=279{,}000$ square mils $=.279$ square inch. If the same size of strip is used as in the case of the 125-volt generator, five strips in parallel would be employed, and the sectional area will be $5\times.11\times.5=.275$ square inch. This is equivalent to $\frac{.275\times10^6}{.7854}=$ *350,000 circular mils.*

The total length of the coil will be $3+3+11\times21.42=241.6$ inches; and the resistance of the six coils in series, when hot, will be $\frac{1.04\times241.6}{350,000}\times6=$ *.00431 ohm.*

66. The armature winding for the 550-volt generator has 236 coils of 1 turn each and they are arranged in two paths. The number of turns per commutating pole, $15\frac{5}{8}$ per cent. compensation being allowed as before, will be $\frac{236}{6\times2}\times1.156=22.74$. It will be necessary to wind the coils with 23 turns each.

The full-load current is 273 amperes; therefore, the amperes per inch on the commutating pole will be $\frac{273\times(23+1)}{9.5}=$ *690.*

67. For 1.33 watts per square inch, the circular mils per ampere should be $1.04\times690\times\frac{1}{1.33}=540$. For 273 amperes, the cross-sectional area should be $273\times540=147,000$ circular mils, $147,000\times.7854=115,500$ square mils, or .1155 square inch. If $\frac{1}{2}$-inch copper strip is used, the total thickness of the conductor should be about $.1155\div.5=.231$ inch thick It is probable that two strands of $.11''\times.5''$ copper, or $\frac{2\times.11\times.5\times10^6}{.7854}$ = *140,000 circular mils*, would be sufficient, and there would be the advantage of using the same stock of copper as for the commutating coils of the 125-volt and 250-volt generators. If, upon trial, the conductor is heated more than is desirable, the new coils can be made of larger stock.

The total length of the coil will be $3+3+23\times21.42=498.7$ inches, and the resistance of the six coils in series, when hot, will be $\frac{1.04\times498.7}{140,000}\times6=$ *.0222 ohm.*

68. Generators are usually required to compound; that is, to develop some desired voltage at no load and some other voltage, greater than the first, at full load. Also, there is often a decrease in the speed of a generator as it is loaded, and the action of the field windings must cause the generator to

compound in spite of the change in speed. Assume that the generators of the three specified voltages will be driven at 600 revolutions per minute at no load, and at 575 revolutions per minute at full load.

69. Let it be required that the 125-volt machine shall compound from 120 volts at no load to 125 volts at full load, both being terminal voltages.

At no load, there is practically no drop of voltage in the armature windings; therefore, the internal voltage developed is the same as the terminal voltage. The flux required to generate 120 volts, as calculated by a modification of the formula of Art. 2, *Design of Direct-Current Machines*, Part 1, will be

$$p\,\Phi = \frac{E_i \times m \times 60 \times 10^8}{f S} = \frac{120 \times 6 \times 60 \times 10^8}{168 \times 2 \times 600} = 21.4 \text{ megalines.}$$

70. At full load the internal voltage must be greater than the terminal voltage 125 by an amount equal to the total drop in voltage in the armature winding, the brushes, the commutating-pole coils, and the series coils. The armature resistance is .00285 ohm and the drop in the armature winding will be $1{,}200 \times .00285 = $ *3.42 volts*. The current density in the brushes is 29.1 amperes per square inch, and the drop in the brushes is 2.41 volts. The resistance of the commutating-pole winding is .0012 ohm, and the full-load current in these coils is 1,080 amperes. The drop will be $1{,}080 \times .0012 = $ *1.3 volts*. The drop in the series-field winding is still unknown, but it is usually so small that an estimate is accurate enough for use in the calculation of the total flux. The winding data may be calculated on the assumption made and then corrected later if an error in the estimate is found. For the series-field winding on commutating-pole machines, assume one-sixth of the armature drop, and for non-commutating pole machines, one-fourth of the armature drop. In this case $\frac{1}{6}$ of $3.42 = .57$ volt.

The generated internal electromotive force will be the terminal voltage plus the drop in voltage in the armature, brushes, commutating coils, and series coils, or $125 + 3.42 + 2.41 + 1.3 + .57 = 132.7$ volts. It will be well to allow 133 volts, so as to leave something for the drop in the leads and connections.

The flux required to generate 133 volts at a speed of 575 revolutions per minute will be $p\,\Phi = \frac{133 \times 6 \times 60 \times 10^8}{168 \times 2 \times 575} = 24.8$ megalines.

71. Reference to Fig. 7 shows that for 21.4 megalines, 4,800 ampere-turns per pole are required, while for 24.8 megalines, 6,300 ampere-turns per pole are needed. At full load, besides the ampere-turns required for the flux used in generating the internal voltage, there are needed, Art. **49**, 533 ampere-turns to overcome the effects of armature reaction, which makes a total of 6,833 ampere-turns.

A shunt-field coil that maintains 4,800 ampere-turns on 120 volts, under no-load conditions, will maintain $4{,}800 \times \frac{125}{120} = 5{,}000$ ampere-turns on 125 volts under full-load conditions. The full-load excitation of *6,833 ampere-turns* will be made up of *5,000 ampere-turns* for the shunt coil and *1,833 ampere-turns* for the series coil.

72. Reference to Fig. 6 shows that the total space for one shunt coil will be 10 inches. Let it be assumed that the coils are to be wound on spools provided with a sheet-iron body and heads of veneer board. The net space for wire will be $9\frac{1}{4}$ inches and there will be 6,833 ampere-turns per coil, or $6{,}833 \div 9.25 = 739$ *ampere-turns per inch of winding space.*

73. These coils will have no wrapping of insulation over the wire; therefore, curve B, Fig. 4, should be used for determining the rate of heat dissipation. The peripheral speed of the armature is 4,060 feet per minute, and for this value 1 watt per square inch will cause a rise of 35° C.; therefore, for a rise of 40° C., $\frac{40}{35} = 1.14$ watts per square inch may be dissipated.

74. The circular mils per ampere required will be $\frac{a}{I} = 1.04 \times 739 \times \frac{1}{1.14} = 674$. The full-load amperes are 1,200; therefore, for a series coil of 1,833 ampere-turns, at least two turns

must be used. Two turns and 1,200 amperes will produce 2,400 ampere-turns; therefore, for 1,833 ampere-turns the current will have to be shunted down to $1,833 \div 2 = 916.5$ *amperes*. For 916.5 amperes, the cross-sectional area of the conductor will be, approximately, $916.5 \times 674 = 618,000$ circular mils, $618,000 \times .7854 = 485,000$ square mils, or .485 square inch. The series coil may be placed either alongside or underneath the shunt coil. In the latter case, a strip conductor is commonly used for the larger machines. In this case, assume the series coil to be beneath the shunt coil and the copper strip to be 9 inches wide and .055 inch thick, or .495 square inch in cross-section.

75. The section of the main pole piece, Fig. 6, is $8\frac{1}{4}$ inches by 7 inches, and the perimeter $2 \times 8\frac{1}{4} + 2 \times 7 = 30.5$ inches. The length of the shortest turn in the field spool will be about 1 inch more than this, or 31.5 inches.

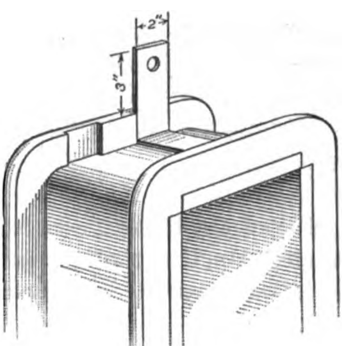


FIG. 9

To insulate the copper of the series winding, let it be wound with a strip of muslin about 15 inches wide wrapped around the copper strip as far as it will go. The muslin is 6 inches wider than the strip, so that it will cover one side and lap 3 inches along both edges on the other side, thus leaving 3 inches near the center of that side bare. Between every layer so wound there will be two thicknesses of muslin near the edges and one thickness near the middle of the copper strip. The thickness of the muslin strip may be taken as .01 inch, which makes the insulation thickness per turn .02 inch. Sheet copper .055 inch thick usually requires a little extra room due to slight irregularities in the surface; therefore, it is well to allow .03 inch for insulation per turn. The depth of the series winding will be $2\,(.055 + .03) = .17$ inch.

To obtain the mean length of a series turn add $3.1416\times.17 = .5$ inch to the length of the shortest turn, 31.5 inches, making *32 inches*. The length of two turns will be *64 inches*.

When winding this coil, a lead $\frac{1}{8}$ inch by 2 inches, Fig. 9, is soldered to one end of the large copper strip. The lead is bent at right angles and one of the spool heads is grooved to receive it. The lead should project about 3 inches from the coil.

76. When calculating the resistance of the series coils, an allowance must be made for the leads and connections between the coils. The length of the connections between the series coils, Fig. 6, is estimated at 10 inches, and the leads may be taken as equivalent to 5 inches each, or 10 inches for both. The allowance per coil for leads and connections will be 20 inches.

Let the connections be made of two strands of $\frac{1}{16}'' \times 2''$ copper which together make the same cross-section as the leads, or $\frac{1}{8}\times 2 = .25$ square inch. The copper in the coil has a cross-section of .495 square inch. A length of 20 inches of the smaller strip has the same resistance as $20\times\frac{.495}{.25} = 40$ inches of the larger strip. Adding 40 inches to 64 inches of winding length makes *104 inches* equivalent length of winding, leads, and connections per coil. A cross-section of .495 square inch is equivalent to 630,000 circular mils. As explained in Art. **19,** the resistance of the six series coils, when hot, will be $\frac{1.04\times 104}{630{,}000}\times 6 = .00103$ *ohm.*

77. As explained in Art. **17,** the watts lost per square inch in the copper of the series coils with a current of 916.5 amperes will be $p = \frac{1.04\times 916.5^2\times 2}{9\times 630{,}000} = .308.$

If the total allowable heat dissipation, in watts per square inch, is 1.14, Art. **73,** and of this the series winding dissipates .308 watt per square inch, the shunt may dissipate .832 watt per square inch.

78. The shunt coils must maintain 5,000 ampere-turns, Art. **71,** with a loss of .832 watt per square inch. The drop of voltage in the shunt coils should be about 80 per cent. of the line voltage, leaving 20 per cent. for the field rheostat; therefore, in this case there should be $125 \times .8 = 100$ volts across the terminals of the six coils in series, or $100 \div 6 = 16.7$ volts across the terminals of each coil.

The shortest turn of the shunt coil will be the same as the greatest turn of the series coil. The shortest turn of the series coil is 31.5 inches, the mean turn, 32 inches, and the greatest turn, 32½ inches. The mean length of the shunt turn may be estimated as 36 inches, approximately.

79. As explained in Art. **19,** the size of the shunt wire required will be $a = \frac{1.04 \times 36 \times 5{,}000}{16.7} = 11{,}200$ circular mils. Reference to Table I shows that the next larger size of wire is No. 9 B. & S., having an area of 13,049 circular mils. If this size is used, the drop in volts in the six coils will be less than 100 volts and, therefore, a greater drop must be absorbed in the field rheostat.

If it is important to maintain just 100 volts on the coils, two sizes of wire may be used to wind the coils; but in generators a single size of wire is frequently used in the shunt coils.

80. As explained in Art. **18,** the number of turns per coil will be $T = \frac{1.04 \times 5{,}000^2}{9\frac{1}{4} \times 13{,}094 \times .832} = 258$. In sizes larger than No. 10 or No. 12 B. & S., it is customary to use double-cotton covered wire for field coils, while for smaller sizes, single-cotton covered wire may be used. No. 9 B. & S. double-cotton covered wire measures .126 inch in diameter. When wound side by side, the wires take up a little more room than their diameter and a practical rule is to allow for the size of double-cotton covered wire when single-cotton covering is to be used, and to allow for the size of triple-cotton covered wire when double-cotton covering is to be used.

No. 9 B. & S. triple-cotton covered wire has a diameter of .13 inch; and in a length of $9\frac{1}{4}$ inches, $9.25 \div .13 = 71$ turns may be placed in a single layer. Four such layers will make *284 turns*, which are more than required, but it will make a better looking coil to complete the last layer.

With 284 turns per coil and 67 turns per square inch, Table II, $\frac{284}{67} = 4.24$ square inches of winding space will be required. If this space is $9\frac{1}{4}$ inches wide, it will be $4.24 \div 9.25 = .46$ inch deep.

81. The shortest turn of the shunt coil is 32.5 inches in length, and the mean turn of the shunt coil will be $32.5 + 3.1416 \times .46 = 34$ inches. As explained in Art. **19,** the resistance of the six shunt coils, when hot, will be $r = \frac{1.04 \times 34 \times 284}{13{,}094} \times 6 = \mathit{4.6}$ *ohms*.

For 5,000 ampere-turns with 284 turns, the current will be $5{,}000 \div 284 = 17.6$ amperes, and the drop of voltage in the six coils will be $17.6 \times 4.6 = 81$ volts, leaving 44 volts for the field rheostat to absorb.

As explained in Art. **17,** the watts per square inch on the shunt coil will be $p = \frac{1.04 \times 17.6^2 \times 284}{9.25 \times 13{,}094} = \mathit{.755}$.

The watts per square inch are .308 for the series coils and .755 for the shunt coils, or a total of *1.063*.

82. For the 250-volt machine, the no-load voltage may be taken as 240; hence, at 600 revolutions per minute, the total flux will be $p\,\Phi = \frac{240 \times 2 \times 60 \times 10^8}{110 \times 2 \times 600} = \mathit{21.8}$ *megalines*. Reference to Fig. 7 shows that for 21.8 megalines, 4,900 ampere-turns per pole will be required.

83. The armature resistance is .0116 ohm, and the drop for the full-load current of 600 amperes will be $.0116 \times 600 = 6.96$ volts. The brush drop is 2.52 volts. The commutating-pole coils have a resistance of .00431 ohm and the drop will be

$.00431 \times 600 = 2.59$ volts. The drop in the series-field coils, including leads and connections, may be taken as 2 volts, approximately. The total drop will be 14 volts and the internal electromotive force at full load will be $250+14=264$ volts.

At the full-load speed of 575 revolutions per minute, the total flux required will be $p\,\Phi = \frac{264 \times 2 \times 60 \times 10^8}{110 \times 2 \times 575} = 25$ *megalines.* Reference to Fig. 7 shows that for 25 megalines, 6,450 ampere-turns will be required. If, as before, 533 ampere-turns are allowed for overcoming the effects of armature reaction, the total full-load excitation will be $6{,}450+533=$ *6,983 ampere-turns.*

The shunt ampere-turns at no load are 4,900 and the terminal voltage 240. For a terminal voltage of 250, the shunt excitation at full load will be $4{,}900 \times \frac{250}{240} =$ *5,100 ampere-turns.* The series coils will provide $6{,}983-5{,}100=1{,}883$ ampere-turns.

84. With the full-load current of 600 amperes, three turns will produce 1,800 ampere-turns, and four turns, 2,400. It will be best to use four turns and shunt a portion of the main current around the series coils to obtain the proper excitation. The current in the series coils should be $1{,}883 \div 4 = 471$ amperes.

85. The length of the winding space for the shunt and the series coils is $9\frac{1}{4}$ inches, there being $6{,}983 \div 9.25 = 755$ ampere-turns per inch. For a rate of heat dissipation of 1.14 watts per square inch, the circular mils per ampere will be $\frac{a}{I} = 1.04 \times 755 \times \frac{1}{1.14} = 689$.

86. The conductor for the series coil should have a cross-section of $689 \times 471 = 325{,}000$ circular mils, $325{,}000 \times .7854 = 255{,}000$ square mils, or .255 square inches. Sheet copper .03 inch thick by 9 inches wide has a cross-sectional area of *.27 square inches*, or 344,000 circular mils, and this is ample. Adding an allowance of .03 inch per turn of conductor for

insulation, Art. **75**, the depth of the series winding will be $.03\times4+.03\times4=.24$ inch.

The shortest length of a turn on the spool is $31\frac{1}{2}$ inches and the mean length of a turn of the series coil will be $.24\times3.1416+31.5=32.25$ inches. The length of copper for four turns will be $32.25\times4=129$ inches. Adding 40 inches per coil, Art. **76**, as an allowance for leads and connections, makes the equivalent length of copper in the coil, as far as resistance is concerned, about $129+40=$ *169 inches*.

87. The resistance of the six series coils, when hot, will be $\frac{1.04\times169}{344,000}\times6=$ *.00307 ohm.*

The watts lost per square inch in the series coil will be $p=\frac{1.04\times471^2\times4}{9\times344,000}=.298$.

The watts per square inch permissible in the shunt coil are $1.14-.298=$ *.842*.

88. The full-load excitation of the shunt coil, 5,100 ampere-turns, is nearly the same as for the 125-volt generator, but the voltage is double; therefore, the cross-sectional area of the shunt-coil wire will be half that of the 125-volt machine, or about 6,500 circular mils. A No. 12 B. & S. wire with a cross-sectional area of 6,529 circular mils will be selected.

The number of turns per shunt coil will be $T=\frac{1.04\times5,100^2}{9\frac{1}{4}\times6,529\times.842}$ $=532$.

Either single- or double-cotton covered wire will be satisfactory. Single-cotton covered wire will be selected and the outside diameter of double-cotton covered wire will be used in determining the number of turns per layer, which will be $9.25\div.091=102$. Six layers will contain *612 turns* and this number will be used. The current for 5,100 ampere-turns will be $5,100\div612=$ *8.33 amperes.*

89. For 612 turns at 135 turns per square inch, Table II, a cross-sectional winding area of $612\div135=4.53$ square inches

will be required. The depth of the winding space will be $4.53 \div 9.25 = .49$ inch.

The shortest length of a turn of the shunt coil will be $.48 \times 3.1416 + 31.5 = 33$ inches; therefore, the mean length of a turn will be $.49 \times 3.1416 + 33 = 34.5$ inches.

The resistance of the six shunt coils, when hot, will be $\frac{1.04 \times 34.5 \times 612}{6{,}529} \times 6 = \textit{20.2 ohms}$. The drop in voltage with 8.33 amperes will be 168, leaving $220 - 168 = 52$ volts for the field rheostat to absorb.

90. Generators developing 550 volts are often required to compound from 500 volts at no load to 550 volts at the terminals for full load. The total flux under no-load conditions will be $p\,\Phi = \frac{500 \times 2 \times 60 \times 10^8}{236 \times 2 \times 600} = \textit{21.2 megalines}$.

Under full-load conditions, the drop in the armature winding will be $.0572 \times 273 = 15.6$ volts; the drop in the brushes, 2.55 volts; the drop in the commutating-pole coils, $.0222 \times 273 = 6.1$ volts; and the drop in the series coils may be taken as 6 volts, approximately. The internal full-load electromotive force will be $550 + 15.6 + 2.55 + 6.1 + 6 = \textit{580 volts}$.

91. The total flux for 580 volts at 575 revolutions per minute will be $\frac{580 \times 2 \times 60 \times 10^8}{236 \times 2 \times 575} = \textit{25.6 megalines}$.

Reference to Fig. 7 shows that for 21.2 megalines, 4,700 ampere-turns per pole, and for 25.6 megalines, 6,750 ampere-turns per pole, will be required. Adding 533 ampere-turns to overcome the effects of armature reaction makes the total full-load excitation $6{,}750 + 533 = 7{,}283$ ampere-turns. On full load the shunt ampere-turns will be $4{,}700 \times \frac{550}{500} = 5{,}170$ The series coils will provide $7{,}283 - 5{,}170 = 2{,}113$ ampere-turns.

92. It is advisable to insulate the series winding from the shunt winding more thoroughly for the 550-volt machine than for those of lower voltages, as there is considerable

danger of a breakdown of the insulation between the two windings.

A method often employed is to place the series and shunt windings side by side on the field spools and separate them by a partition. If this partition is made $\frac{3}{8}$ inch thick and is otherwise similar to the spool heads, the winding space for both coils will be $9\frac{1}{4}-\frac{3}{8}=8\frac{7}{8}$ inches. The series winding will occupy $8\frac{7}{8}\times\frac{2,113}{7,283}=2.57$ inches. A space allowance of $2\frac{5}{8}$ inches will be made and the coil will be wound with copper $2\frac{1}{2}$ inches wide.

The calculated number of turns in the series coil will be $2,113\div273=8$, nearly. In order to allow for variation in the quality of the steel and to provide for the use of a shunt to the series coil if desirable for the regulation of the compounding, *9 turns per coil* will be used.

93. For $2,113\div2.5=845$ ampere-turns per inch and a rate of heat dissipation of 1.14 watts per square inch, the circular mils per ampere will be $\frac{a}{I}=1.04\times845\times\frac{1}{1.14}=771$. For 273 amperes, the conductor will have a cross-sectional area of $771\times273=210,000$ circular mils; $210,000\times.7854=165,000$ square mils, or .165 square inch.

Copper strip $2\frac{1}{2}$ inches wide and .07 inch thick, and having a cross-sectional area of *.175 square inch*, will be used. Adding an allowance of .03 inch per turn of conductor for insulation makes the depth of the series winding $.03\times9+9\times.07=.9$ inch.

94. The space for the shunt winding will be $8\frac{7}{8}-2\frac{5}{8}=6\frac{1}{4}$ inches. In this space 5,170 ampere-turns are to be provided. Allowing 20 per cent. of the full-load voltage for the drop in the field rheostat, the voltage for the six shunt coils in series will be $550\times.8=440$, or $440\div6=73$ per coil.

No. 12 B. & S. wire gave 5,100 ampere-turns with 168 volts in the case of the 250-volt machine; therefore, No. 15 B. & S. wire will give the same number of ampere-turns on twice this voltage, or $168\times2=336$ volts. For 440 volts, a reasonable

estimate will be No. 16 B. & S. wire, having a cross-sectional area of 2,583 circular mils.

The number of turns will be $T = \frac{1.04 \times 5{,}170^2}{6.25 \times 2{,}583 \times 1.14} = 1{,}510.$

95. Single-cotton covered No. 16 B. & S. wire will be employed and the outside diameter of double-cotton covered wire, .059 inch, will be used in the calculation for the number of turns per layer. In a length of $6\frac{1}{4}$ inches, $6.25 \div .059 = 106$ turns may be wound. For 1,510 turns, $1{,}510 \div 106 = 14.25$ layers will be required. Fifteen layers will be wound, making 15×106 = *1,590 turns*. From Table II, $1{,}590 \div 315 = 5.05$ square inches of winding area will be required, and the depth of the shunt winding will be $5.05 \div 6.25 = .81$ inch.

The series coil will be .9 inch deep and the shunt coil .81 inch. If, on account of appearance, these two coils are not considered near enough alike in depth, the series coil could be wound with a little wider and a little thinner copper strip.

EFFICIENCY

EFFICIENCY OF THE 5-HORSEPOWER MOTOR

96. The efficiency of a motor or generator is the ratio of power output to the power input, and is usually expressed in per cent.

The data of the 5-horsepower motor, as developed in this and the previous Sections, will first be considered. The loss in the armature steel is 149 watts; in bearing friction and windage, 40 watts; in brush friction, 19.9 watts; making a total of 208.9, or *210 watts*, approximately, lost mechanically.

The armature $I^2 R$ loss will be $39.8^2 \times .34 = 539$ watts; the $I^2 R$ loss in the brush contact surface is 95.5 watts; the loss in the shunt-field coils will be the product of 2.46, the current in them, and 115, the line voltage, or 283 watts, making the total electrical losses 917.5 watts, approximately. The total losses will be $210 + 917.5 =$ *1127.5 watts*.

The output of the motor will be $746 \times 5 = 3{,}730$ *watts;* the input, $3{,}730 + 1{,}127.5 = 4{,}857.5$ *watts;* and the efficiency at full load, $3{,}730 \div 4{,}857.5 = 76.8$ *per cent.*

97. It is often desired to calculate the efficiencies of motors and generators under various load conditions. These load points are usually half load, three-quarter load, full load, and one-and-a-quarter load.

In a motor, the core loss is very nearly constant under all loads and may be so assumed. The brush friction and bearing friction may be assumed as constant unless there is a very great difference in the speed between no-load and full-load conditions. The shunt-field loss of a motor does not change with the load, since the shunt-field coil is usually connected directly across the circuit and, therefore, the current in the coil remains practically constant. The constant losses are, therefore, $210 + 283 = 493$ watts.

The variable losses are the $I^2 R$ losses in the armature winding and in the brushes. The number of watts converted from electrical into mechanical energy equals the output plus the mechanical losses, both expressed in watts.

The armature currents for the different load conditions are calculated by either formulas **1** or **2** of Art. **32,** *Design of Direct-Current Machines*, Part 1.

To determine the value of E_c, the current density in the brush, and the brush drop, may be calculated preliminarily as if the armature current at fractional loads was equal to the armature current at full load times the fraction of the load. When the value of the armature current is known, the brush drop may be calculated with greater accuracy. If formula **2** is used for the half-load condition, $I = \frac{113}{2 \times .34} - \sqrt{\left(\frac{113}{2 \times .34}\right)^2 - \frac{2{,}075}{.34}} = 19.5$ amperes.

The Sections on the design of the armature and commutator for the 5-horsepower motor in *Design of Direct-Current Machines*, Part 1, should be consulted in connection with the general methods of calculating the other values for the fractional-load conditions.

The data pertaining to the calculations of the efficiency may be tabulated as follows:

	Half Load	Three-Quarter Load	Full Load	One-and-a-Quarter Load
Output, watts	1,865	2,798	3,730	4,663
Mechanical loss, watts	210	210	210	210
Converted watts	2,075	3,008	3,940	4,873
Armature current, amperes	19.5	29.3	39.8	51.3
Current density in brushes, amperes per square inch	13.9	20.8	28.4	36.5
Drop in brushes, volts	2.	2.22	2.4	2.54
I^2R loss in armature, watts	129	292	539	895
I^2R commutator loss, watts	39	65	95.5	130
Total variable losses, watts	168	357	634.5	1,025
Total constant losses, watts	493	493	493	493
Total losses, watts	661	850	1,127.5	1,518
Output, watts	1,865	2,798	3,730	4,663
Input, watts	2,526	3,648	4,857.5	6,181
Efficiency, per cent	73.8	76.7	76.8	75.4

98. In place of the nearly accurate current calculations just made, the armature current for a load condition is sometimes taken as the full-load armature current times the fraction of the load. Thus, if 39.8 amperes is the full-load armature current, the current for the half-load condition may be taken as $39.8 \div 2 = 19.9$ amperes, approximately. Or, the I^2R loss in the armature may be taken as proportional to the square of the fraction of the load. Thus, if 539 watts is the full-load I^2R loss, the loss at half load will be $(\frac{1}{2})^2 \times 539 = 134.8$ watts.

With this approximate method of calculation, the current density in the brush may be assumed to vary directly with the load; that is, at half load the current density will be $28.4 \times \frac{1}{2} = 14.2$ amperes per square inch. The errors caused by these assumptions are usually so small as to permit of this method for efficiency calculations except where accurate determinations are required.

99. There are some losses that occur which are not easily determined. These are sometimes called *load losses*, or *stray losses*. They include the loss due to the short-circuited currents in the armature coils under commutation, the increased core loss, and the pole-face eddy current loss due to the shifting and, therefore, concentrating of the magnetic flux under load.

100. The foregoing general method of calculating the efficiency is known as the *summation of the losses*, or the *separation of the losses*, and does not give exactly the true efficiency. The result is sometimes called the *conventional efficiency*. The only way the efficiency can be accurately determined is to measure the power input and the output by tests on the completed machine.

Since mechanical power is not easily and accurately determined, the separate-loss method is often used. The efficiencies of motors and generators are often guaranteed by the makers to be not less than certain values, which are to be determined by the separate-loss or other specified method.

EFFICIENCY OF THE 150-KILOWATT GENERATOR

101. The separate-loss method as applied to generators differs but little from its application to motors. In generators, the flux is greater at full load than at no load, and the difference is so great that it usually cannot be neglected.

The core loss at no load and at full load is usually estimated by using the no-load and the full-load fluxes to obtain the densities; and the losses at fractional loads and overloads are taken as proportional to those obtained for no load and full load.

102. All losses in shunts and regulating resistances are chargeable to the generators, but bearing friction and windage in engine-driven units where the armature is mounted on the engine shaft directly, are chargeable to the engine. In any case where the generator has separate bearings from the engine, the losses in these bearings, and the windage, are chargeable to the generator.

103. The efficiency of the 150-kilowatt, 250-volt generator will be calculated at half, three-quarters, full, and one-and-a-quarter loads. The flux at no load is 21.8 megalines, Art. **82**, and at full load is 25 megalines, Art. **83**.

The total core loss at full load is *2,970 watts* (Art. **67**, *Design of Direct-Current Machines*, Part 1). The frequency at no load with a speed of 600 revolutions per minute will be $\frac{600}{60}\times\frac{6}{2}$ $=30$ cycles per second and the flux at no load is 21.8 megalines. The no-load core loss of 2,590 watts for these data is determined in the same manner as for the full-load core loss.

At half load, the core loss may be taken as the average of the two load values, or $\frac{2{,}590+2{,}970}{2}=$ *2,780 watts;* at three-quarters load, $\frac{2{,}780+2{,}970}{2}=$ *2,880 watts*, at one-and-a-quarter load, $2{,}970+(2{,}970-2{,}880)=3{,}060$ watts.

104. The armature resistance is .0116 ohm, the commutating-pole coil resistance is .00431 ohm, and the series-field coil resistance is .00307 ohm, making a total of .019 ohm. In the case of the commutating-field and series-field windings some of the main current was expected to be shunted around the coils, so in using the total resistance of these windings the losses are slightly overestimated.

The shunt-field current is 8.33 amperes, and this passes through the armature, commutating coils, and series coils. At full load, the $I^2 R$ loss in the windings, exclusive of the shunt coil, will be $(600+8.33)^2\times .019=$ *7,030 watts;* at half load, $(\frac{1}{2})^2\times 7{,}030$ $=$ *1,760 watts;* at three-quarters load, $(\frac{3}{4})^2\times 7{,}030=$ *3,950 watts;* at one-and-a-quarter load, $(\frac{5}{4})^2\times 7{,}030=$ *11,000 watts*.

The brush drop is 2.52 volts at full load and the commutator $I^2 R$ loss with full-load current of 608.33 amperes will be *1,530 watts*. The current for the various loads may be taken as 304, 456, 608.33, and 760 amperes, respectively. The $I^2 R$ commutator losses should then be determined as explained in Arts. **96** and **97**, *Design of Direct-Current Machines*, Part 1, the losses being approximately *650*, *1,080*, *1,530*, and *2,050 watts*, respectively.

105. The brush-friction loss is *1,310 watts*, and this value may be taken for the various loads.

The full-load, $I^2 R$ loss in the shunt-field circuit, including the rheostat, at 250 volts is $250 \times 8.33 = 2{,}080$ *watts*. At no load the voltage is 240 and the reduction in voltage reduces the current to 8 amperes; therefore, the $I^2 R$ loss will be $240 \times 8 = 1{,}920$ watts. At half load, the $I^2 R$ loss will be $\frac{1{,}920+2{,}080}{2} = 2{,}000$ *watts*; at three-quarters load, $\frac{2{,}000+2{,}080}{2} = 2{,}040$ *watts*; at one-and-a-quarter load, $2{,}080+(2{,}080-2{,}040) = 2{,}120$ *watts*.

106. The friction and windage loss can be approximated from experience with other machines by taking into consideration the size of the bearings, dimensions, and peripheral velocity of the armature, etc.; but the main loss at the required speed is the bearing loss. The loss in ordinary machines rarely exceeds 1 per cent. of the output, and this amount may be used in this case, the loss being $150{,}000 \div 100 = 1{,}500$ *watts*.

The data just determined may be tabulated as follows:

	Half Load	Three-Quarters Load	Full Load	One-and-a-Quarter Load
Core loss, watts	2,780	2,880	2,970	3,060
$I^2 R$ loss in armature, commutating coils, and series coils, watts	1,760	3,950	7,030	11,000
$I^2 R$ commutator, watts	650	1,080	1,530	2,050
Brush-friction loss, watts	1,310	1,310	1,310	1,310
Shunt-circuit loss, watts	2,000	2,040	2,080	2,120
Friction and windage loss, watts	1,500	1,500	1,500	1,500
Total losses, watts	10,000	12,760	16,420	21,040
Output, watts	75,000	112,500	150,000	187,500
Input, watts	85,000	125,260	166,420	208,540
Efficiency, per cent.	88.2	89.8	90.1	89.9

107. A test on the completed machine may serve to check the calculated and estimated data in the tabulation just given. The current output, the current for the shunt-field circuit, and the resistances of the various coils may be measured. The watts required to drive the machine as a motor running free at its normal speed and with its full-load internal voltage may be considered as including the core loss, brush friction, bearing friction, and windage. In this test it is essential that the brushes be in the geometrically neutral position, as otherwise some short-circuited current loss will be included in these *free* losses.

To determine separately the core loss and the brush friction, the machine is usually driven by a small motor and the watts required to drive it with the brushes lifted from the commutator and with the field unexcited are noted. The field is then excited and the increase in power required to drive the machine is taken as the core loss. Again, with the field unexcited, the brushes are put down on the commutator and the increase in power required to drive the machine is taken as the brush-friction loss.

DESIGN OF TRANSFORMERS

FEATURES OF DESIGN

IMPORTANT CHARACTERISTICS

1. A satisfactory design of a transformer is a compromise between certain considerations, notably reliability and cost. Thorough insulation of the windings, high efficiency, good voltage regulation, and moderate temperature under full-load conditions are desirable for safe and economical operation, but to secure these characteristics ample insulating material and low working densities in copper and steel are required, all of which increase the cost of the transformer.

The most essential consideration is safety, and this is almost entirely dependent on the insulation. Insulation between high- and low-voltage windings is of the greatest importance, for in many cases, such as lighting transformers, the low-voltage circuit is freely handled by the customer. Insulation from the high-voltage winding to the case is also very important from the standpoint of safety, for, unless the case of a faulty transformer is thoroughly grounded, the operator may be fatally shocked by touching it.

Reliability is a characteristic next in importance to safety. The transformer must be able to carry its load continuously without attaining temperatures that endanger insulation, and it must possess sufficient mechanical strength to be able to

withstand any stresses imposed on it through sudden changes in load or even short circuit.

Next to safety and reliability, economy of operation is important. The losses must be low and their relative values must be dependent on the character of the service; the regulation must be good and the exciting current low. Some of these characteristics are conflicting, and, therefore, the final design in any case must be a compromise.

FUNDAMENTAL EQUATION

2. The fundamental formula, or equation, used in transformer design was given in a preceding Section and in modified form is repeated here for convenience.

$$\Phi N = \frac{10^8 E}{4.44 f} = \frac{22.5 \times 10^6 E}{f} \qquad (1)$$

in which Φ = the total flux, or the number of lines of force;
N = number of turns in winding;
E = volts induced in winding of N turns;
f = frequency, in cycles per second.

If the flux, expressed in megalines (10^6 lines), is represented by Φ_m,

$$\Phi_m N = \frac{22.5 E}{f} \qquad (2)$$

The two values N and E apply to the same winding; that is, both to the low-voltage (coarse-wire) winding or both to the high-voltage (fine-wire) winding. The product of flux and turns is a constant for any given rating of transformer, and can be determined at once by substituting known values of E and f in the formulas. No definite rules can be laid down for determining the best individual values of flux and turns, but the relative values of the total fluxes in transformers and the ratings as indicated in Fig. 1 represent good practice and will give satisfactory results. Equally good designs, however, will often be found to vary widely in the value of the flux employed.

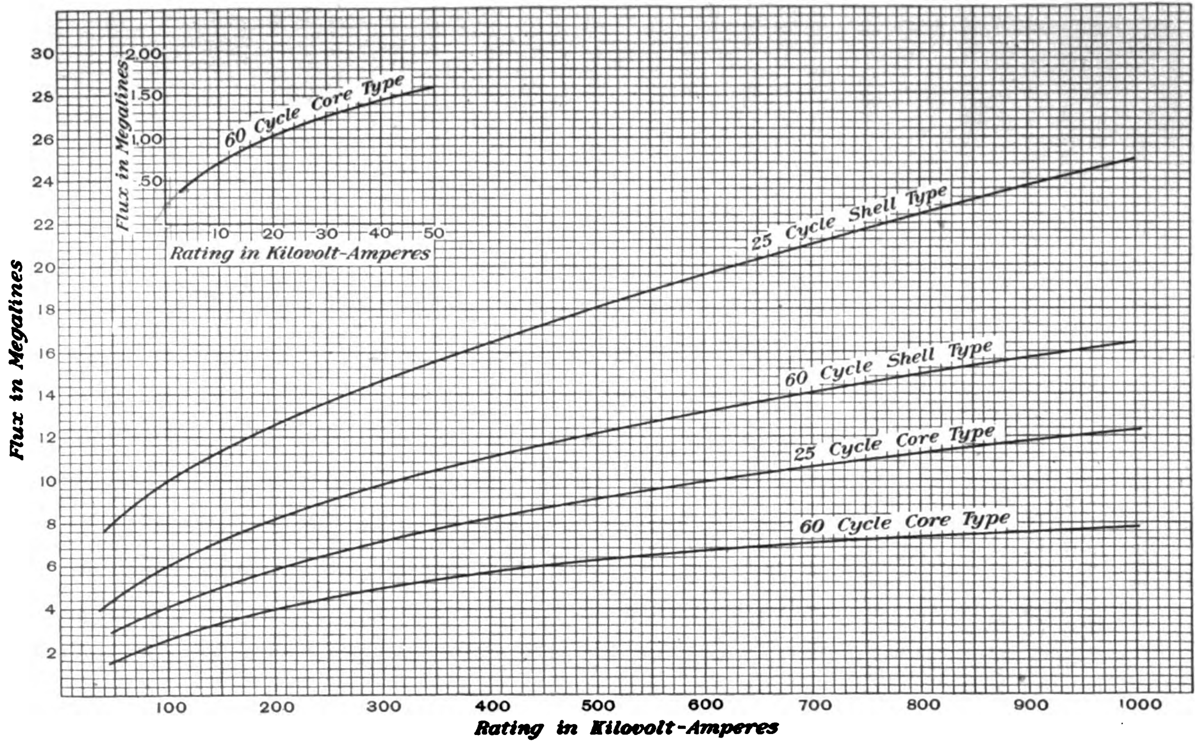

Flux in Megalines
Rating in Kilovolt-Amperes
25 Cycle Shell Type
60 Cycle Shell Type
25 Cycle Core Type
60 Cycle Core Type
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
100
200
300
400
500
600
700
800
900
1000
2.00
1.50
1.00
.50
10
20
30
40
50

FIG. 1

PROPORTIONING DESIGN VALUES

3. Types of Construction.—In selecting the type of construction for any particular design, considerable latitude is allowable. While no arbitrary rules can be laid down, the common practice is to employ the distributed-core type transformer on the smaller sizes, the core type on the medium sizes, and the shell type on the largest sizes. The distributed-core type is a special patented construction. Both the core and the shell types have been advocated for the whole range of capacities.

4. Flux Densities.—A relatively high value of flux density is desirable in order to reduce the cost of the transformer. However, a high flux density involves high core loss, which means reduced efficiency and heating of the core. Also, high flux density requires a large magnetizing current, which is objectionable. Good practice rarely permits higher values for flux density than 80,000 to 90,000 lines per square inch of net cross-section, and in many cases these values must be somewhat reduced. At any given flux density the core losses are lower with low frequency than with high frequency; 25-cycle machines can therefore, in general, employ higher densities than 60-cycle units.

In the absence of exact data as to the particular steel employed and its treatment, the assumption can be made that approximately 10 per cent. of the final core height (perpendicular to the planes of the sheets) will be taken up by the scale or insulating coating of the sheets. The actual cross-section of core therefore necessary will be obtained by adding 11 per cent. to the net section found necessary for the required flux, or, differently expressed, the net section is 90 per cent. of the total section.

5. Relation Between Losses and Efficiency.—Fig. 2 indicates the total losses at full load for various ratings of transformers and can be used in any case where actual efficiencies are not specified. Since the curves indicate the total losses, the division of the loss into core and copper losses must be determined.

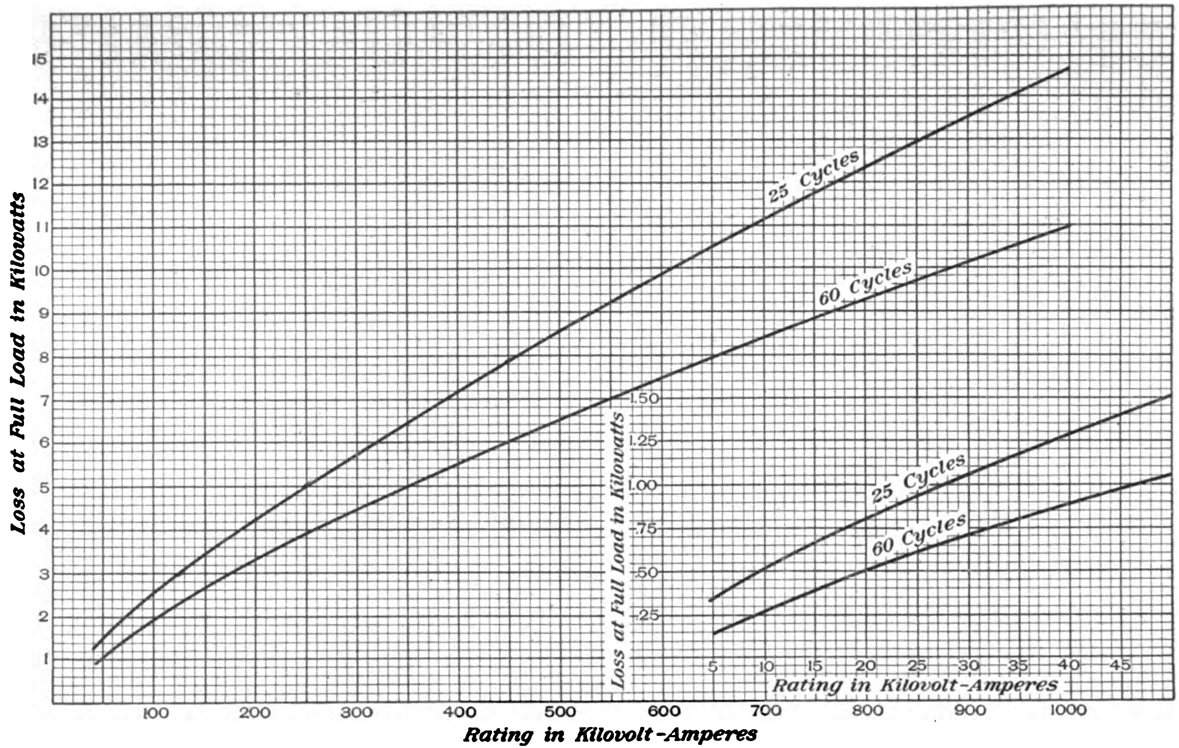

Loss at Full Load in Kilowatts
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
100
200
300
400
500
600
700
800
900
1000
Rating in Kilovolt-Amperes
25 Cycles
60 Cycles
Loss at Full Load in Kilowatts
.25
.50
.75
1.00
1.25
1.50
5
10
15
20
25
30
35
40
45
Rating in Kilovolt-Amperes
25 Cycles
60 Cycles

FIG. 2

To obtain maximum efficiency at any given load, the core and copper losses must be equal at that load. From the standpoint of efficiency alone, this is a desirable condition for power transformers, but is not usually obtained in practice when the percentage exciting current is to be kept at a relatively low value. With improved steel and frequencies of 25 cycles or less, the flux density cannot always be raised enough to produce theoretically correct core loss and at the same time not abnormally increase the exciting current. In 25-cycle transformers the core loss will probably average 40 per cent. of the total loss.

When considering the efficiency of lighting transformers, or of any class where intermittent loading is common, the total energy loss throughout the whole 24 hours of each day must be considered. The ratio of the total useful energy output to the total energy input per day is the *all-day efficiency*, which is a very important consideration. For example, a 50-kilovolt-ampere transformer with 1 kilowatt full-load losses has a full-load efficiency of $\frac{50}{51} \times 100 = 98.04$ per cent. If this transformer is in use at full load only 8 hours per day but is excited all day, its all-day efficiency will be much lower than its full-load efficiency, owing to the constant core loss. If the copper and core losses are equal, each 500 watts, the copper loss in 8 hours will be $8 \times 500 = 4{,}000$ watt-hours and the core loss in 24 hours will be $24 \times 500 = 12{,}000$ watt-hours. The total loss per day will then be 16,000 watt-hours. The useful output in 8 hours will be $8 \times 50{,}000 = 400{,}000$ watt-hours, and the input per day must be $400{,}000 + 16{,}000 = 416{,}000$ watt-hours. The all-day efficiency is then $\frac{400{,}000}{416{,}000} = 96.15$ per cent.

The all-day efficiency of transformers that are continuously excited and intermittently loaded can be improved by making the core losses less than one-half the total losses. For example, if the core losses in the transformer previously referred to were 333 watts and the copper losses 667 watts, the all-day losses would be $24 \times 333 + 8 \times 667 = 13{,}328$ watt-hours, and the all-day efficiency would be $\frac{400{,}000}{413{,}328} = 96.78$ per cent. Increased all-day efficiency can be obtained by still further reducing the

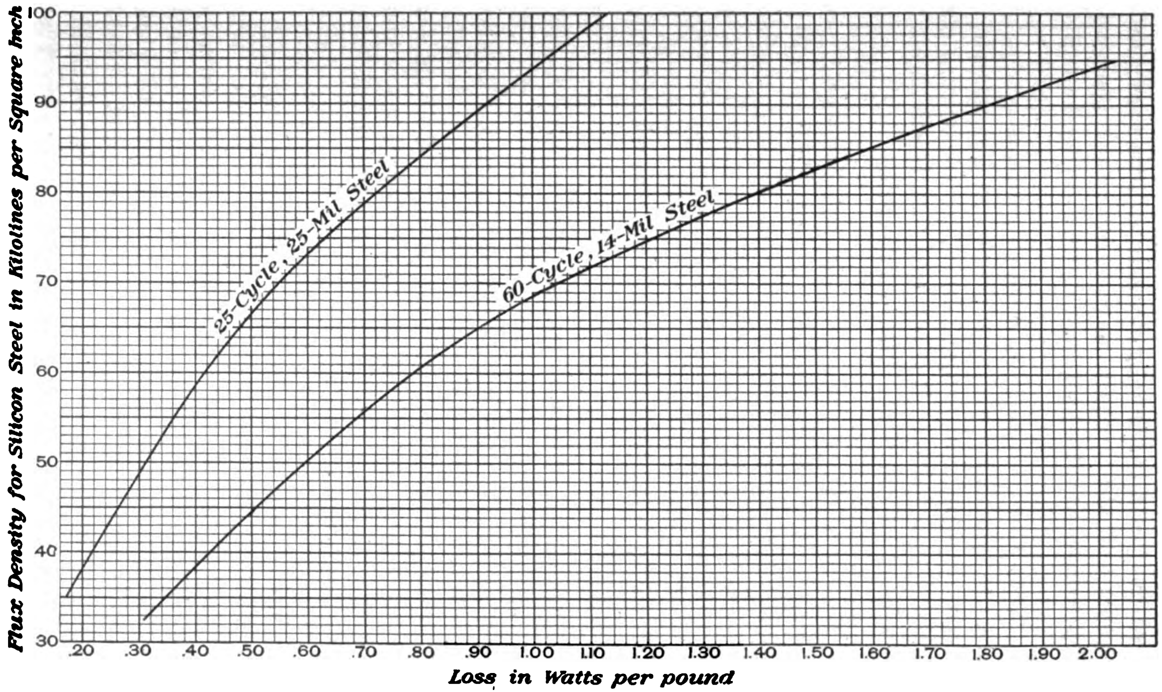

Flux Density for Silicon Steel in Kilolines per Square Inch
Loss in Watts per pound
25-Cycle, 25-Mil Steel
60-Cycle, 14-Mil Steel
100
90
80
70
60
50
40
30
.20
.30
.40
.50
.60
.70
.80
.90
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00

FIG. 3

core loss, but this is, in general, not allowable from an economical standpoint and also because an increase in the copper loss thereby reduces the overload capacity of the transformer.

6. Total Core Loss.—Fig. 3 shows core-loss curves that indicate the value of the watts per pound core loss, considering both hysteresis and eddy-current losses, for silicon-steel cores at various flux densities. It is assumed that 14-mil sheets are used for 60 cycles and 25-mil sheets for 25 cycles.

7. Magnetizing Current.—It is not considered good practice to employ a magnetizing current exceeding 15 to 20 per cent. of full-load current at normal voltage. On large machines a value of from 5 to 10 per cent. represents good practice. Excessive magnetizing currents result in increasing the copper losses of the system, particularly if the power factor of the connected load is less than unity, as is the case with an induction-motor load.

8. Magnetizing Ampere-Turns.—Fig. 4 shows the necessary magnetizing ampere-turns per inch length of the mean magnetic circuit for inducing various flux densities in silicon steel. A value of 100 to 125 ampere-turns will be satisfactory to assume as necessary for forcing flux at 80 to 90 kilolines per square inch across each *joint* of the core structure, if the joints are properly made. If the flux density is lower, the allowance per joint is proportionally lower.

9. Conductors.—Certain practical limits are met with in choosing the size of wire to employ for the windings. When the currents are small, a thin flat strip is usually chosen for transformers of the shell type, and round wire for those of the core type. Conductors much larger than $\frac{1}{2}$ inch to $\frac{3}{4}$ inch wide or $\frac{3}{32}$ inch to $\frac{1}{8}$ inch thick become unwieldy, and for heavy currents it is therefore usual to employ several such conductors in parallel. In general, no part of a coil should be over 1 inch away from the cooling medium, and it is usual to reduce this limit to $\frac{1}{2}$ inch, especially where heavy overload capacity is desired.

10. Current Density in the Conductors.—The current density in amperes per square inch of cross-section of the conductor in either winding is equal to the quotient obtained by dividing the amperes of current in that winding by the square inches of cross-section of the conductor.

The average value of the current densities in both windings in good transformer designs will vary from 800 to 1,400 for

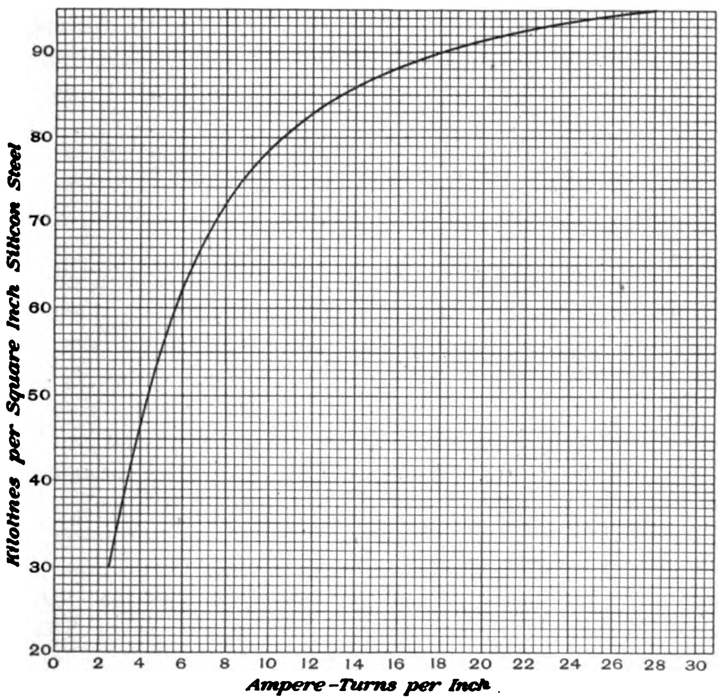


Fig. 4

self-cooled machines, and double this value for water-cooled designs. Constants of 1,200 and 2,400 will probably represent average practice on the two types. These figures should be somewhat reduced for thick coils or in machines subjected to heavy overloads, in order to avoid local heating or hot spots in the interior of the windings; such heat might cause deterioration of the insulation.

11. Cooling Constants.—The heat developed in a transformer due to its losses raises the temperature of the core and windings above that of the cooling medium and the transformer tank. On account of the different temperatures of the parts there is a continuous flow of heat to the tank, where, by convection and radiation, it is carried off into the atmosphere. In water-cooled transformers practically all of the heat is given up to the cooling water, and in machines cooled by an air blast the heat is given up to the cooling air. The rise in temperature of the core and copper above that of the surrounding cooling medium will depend on the loss per unit surface of active material.

In designing a transformer, its temperature rise can be predetermined with a fair degree of accuracy by comparing the calculated watts loss per square inch of surface of core, windings, and tank exposed to the cooling medium, or washed by it, with values for which the heating has been determined by experiment with other transformers. Considerable judgment is necessary in determining what these washed surfaces are. For example, a surface bounding an oil space of less than $\frac{3}{16}$ inch in width can hardly be considered as effective for cooling; neither can a surface covered by a heavy pad of insulation. An allowance of .7 watt per square inch on the core for dissipating the core loss should not, in general, be exceeded, and .5 watt will represent good practice. With a low-frequency transformer, a flux density of comparatively low value may be necessary in order to keep the magnetizing current at a moderate value; therefore, the rate of energy dissipation may be below .5 watt per square inch. On large machines, ducts in the core may be found necessary in order to secure added radiating surface.

12. Fig. 5 shows heating curves that indicate the rise in temperature of the coils in oil-cooled transformers for values of loss in watts per square inch of the radiating surface of the coils and with various sheet-steel tank radiating constants. Data curves, or lines, for other tank constants can be readily drawn; for example, for a .15 tank constant, a straight line midway between the .12 constant and the .18 constant

data lines should be drawn. For cast-iron tanks, 15 per cent. increase in loss in watts per square inch will give the same temperature rise. Core radiation is approximately ⅓ watt per square inch.

The values of the watts per square inch of the coil radiating surface are based on the resistance of the copper at 75° C. The curves represent average results that will be secured on this class of transformers. Whether or not a relatively high loss per unit surface of tank shall be employed depends to a large extent on the facilities at hand for providing the proper tank

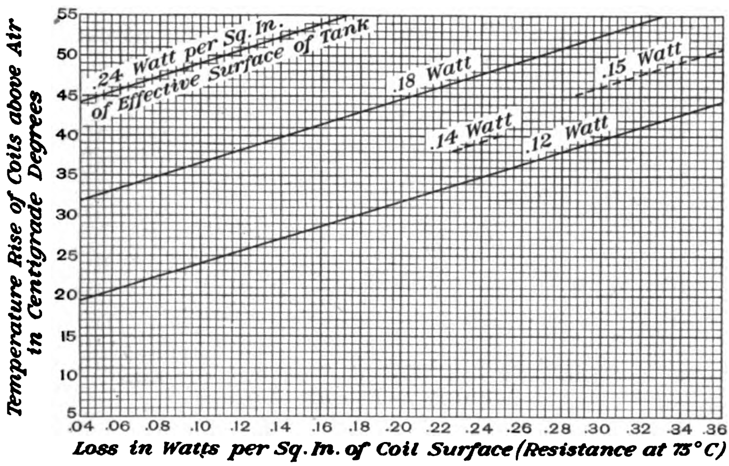


Fig. 5

radiating surface. On small machines, a plain surface will be found sufficient, but as the size increases the provision of proper surface becomes increasingly more difficult.

13. The actual coil temperature rise for which a transformer should be designed is dependent on several conditions, the most important of which is the maximum temperature that will be attained under regular operation. This temperature depends not only on the rise in temperature of the coil and core above the cooling medium, but also on the temperature of the cooling medium itself. With ordinary insulations, coil temperatures exceeding 100° C. are not advisable for any considerable

period. Hot-spot temperatures may be assumed to be at least 10° C. in excess of the average existing in the windings, thus giving about 90° C. for the actual average allowable temperature of the windings. In the absence of actual data for any particular case, 40° C. may be assumed for the maximum temperature of the surrounding air in which the machine may be required to operate. This assumption will give an allowable rise of 50° C. for the windings, which represents average practice for small transformers.

In the case of power transformers where considerable overload capacity is almost a necessary feature, common practice is to design for a 40° C. rise on the continuous normal load

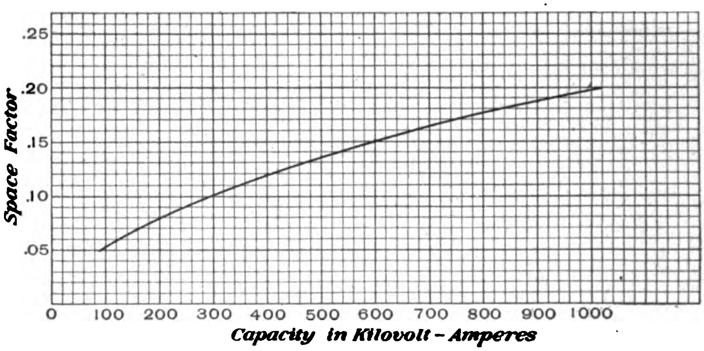


FIG. 6

operation. A knowledge of all the conditions surrounding the operation of a particular transformer will often allow variations from the values mentioned, but for general design temperature rises of from 40° or 50° C., as the case may be, are well chosen.

The loss per unit coil surface that may be allowed is less in value if the heat is obliged to pass through a large amount of insulation. In placing the insulation, this point should be kept in mind. In many cases on high-voltage windings at least a 10-per-cent. increase in temperature over that indicated in Fig. 5 must be assumed, for the reason just mentioned.

14. Space Factor.—A very useful method of arriving quickly at an approximate design involves the consideration of ampere-turns per unit length of coil, and also of the constant

commonly termed the space factor of the windings. By **space factor** is meant the ratio of the space filled by copper to the total area of the winding space within the core structure. This ratio varies with the kilovolt-ampere capacity and with the voltage. The high-voltage winding of a transformer will have a lower space factor than the low-voltage winding, because of the relatively larger space required for insulation.

Fig. 6 shows the effect of capacity in kilovolt-amperes on average space factor of transformers of given voltage, and Fig. 7 shows the effect of voltage on space factor of transformers of given capacity in kilovolt-amperes; both of these curves apply

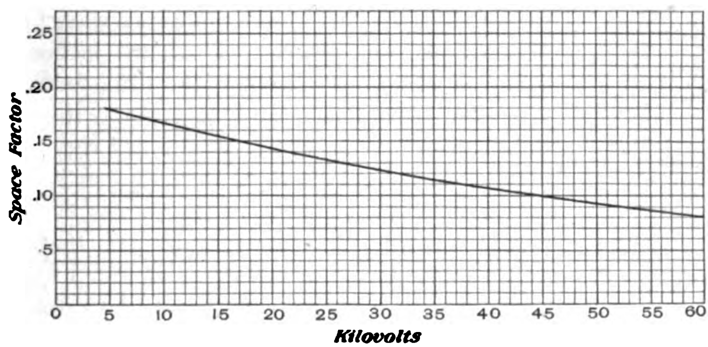


FIG. 7

to one particular line of core-type transformers. In order to determine approximately the necessary wire space area for any given rating of transformer, such curves can often be used to great advantage.

15. Ampere-Turns per Inch.—If the allowable current densities and the watts per square inch of cooled surface of coil are selected for a given design, the ampere-turns per inch length of the washed surface of the coil can then be determined.

Fig. 8 shows a section of one side of one winding of a core-type transformer. The total watts loss in the coil with the resistance value calculated for an assumed temperature of 75° C. is found by the formula

$$P = \frac{9.94\, l\, N\, I^2}{10^6\, a} \qquad (1)$$

in which P = loss in watts;

l = mean length of a turn in feet;

N = the number of turns in series on the coil;

I = the number of amperes of current in the coil;

a = cross-sectional area of the conductor, in square inches.

The constant $\frac{9.94}{10^6} = .00000994$ is the resistance in ohms at 75° C. of a bar of copper 1 foot long and 1 square inch in cross-section. The watts loss per square inch of radiating surface of the coil is equal to the value of the total watts loss divided by the number of square inches of effective radiating surface. The total cylindrical area, both inside and outside of the coil shown in Fig. 8, is $12\, l L$; here l has the value used in formula **1**, and L, expressed in inches, is the combined lengths of inside and outside surfaces of the coil parallel to its axis that are washed by the oil, as indicated in Fig. 8. The thickness, or depth, of the winding is usually so small that it need not be considered in the calculation.

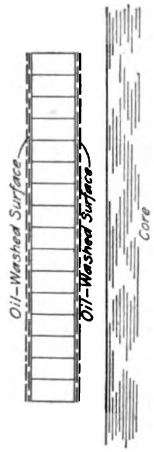


FIG. 8

The average watts loss per square inch of radiating surface of a coil is $P_i = \frac{P}{12\, l L}$, or by substituting for P the value found by formula **1**,

$$P_i = \frac{.828\, N I^2}{10^6\, a L} \qquad (2)$$

Formula **2** may also be written

$$\frac{I N}{L} = \frac{a}{I} \times \frac{10^6\, P_i}{.828} \qquad (3)$$

In formula **3**, $\frac{I N}{L}$ is the ampere-turns per inch length of the inside and outside washed surfaces of the coil, and $\frac{a}{I}$ is the reciprocal of the current density. If the current density is

assumed at 1,200 amperes per square inch of cross-section and the radiation P_i at .24 watt per square inch of washed surface, $\frac{a}{I}$ becomes $\frac{1}{1{,}200}$ and $\frac{I\,N}{L}=\frac{1}{1{,}200}\times\frac{10^6\times .24}{.828}=240$, approximately; and $240\,L$ is then the approximate total number of ampere-turns in the coil.

The value of $\frac{I\,N}{L}$ will be increased or decreased in direct proportion to the allowable value of P_i and inversely to the value of the current density. Also, if the insulation between turns is a large proportion of the length L, as on large high-voltage transformers, the value $\frac{I\,N}{L}$ should be somewhat reduced.

16. Arrangement of Windings.—The amount of leakage flux, and, therefore, the reactance drop in volts of a transformer, depends on the relative positions of the high- and the low-voltage windings. Formerly efforts were made to keep the percentage reactance down to as low a limit as was consistent with reasonable cost. In more recent practice, the allowable reactance has been increased in order to limit the current that is established when the transformer is short-circuited by faults on the line, thus reducing the resulting mechanical stresses so that the windings can withstand them.

In order to reduce magnetic leakage to a reasonable value, the primary and secondary windings must be placed close together. The leakage flux in any gap is proportional to the ampere-turns in the exciting coil and inversely proportional to the length of the leakage path. In the core-type construction, the length of the leakage path is relatively long, and if primary and secondary windings are equally distributed on the two legs of the core, each in one group per leg, the reactance will usually not be excessive.

In the shell-type construction, in which the length of the leakage path is relatively low, the windings must be divided into several groups, the primary and secondary being *sandwiched* together. This reduces the exciting ampere-turns at

each gap, thereby cutting down the leakage flux. On the other hand, such a subdivision of the windings materially lowers the space factor below that of the core type.

One rule in particular must be observed in determining the arrangement of windings, namely, that under all possible connections the total ampere-turns of one winding acting at any time must be distributed practically equally *over* the ampere-turns of the other winding opposing them at that instant. For instance, in a Scott-connected core-type transformer each half of the main winding must be distributed over each leg of the core, as these halves oppose each other in so far as the teazer current in them is concerned. Likewise, in autotransformers, care is necessary in distributing all portions of the secondary over the primary parts of the winding.

17. Reactance.—In general, reactance drop is due to the voltage induced in the primary winding by flux which does not link the secondary. This flux exists between the primary and secondary windings, and is caused by the combined action of the primary and secondary currents, which, taken together, constitute a magnetizing action, driving flux between the windings. The amount of this flux depends on the current, the number of turns in the coils, and the reluctance of the path through which the flux passes. The reluctance of the leakage path is difficult to determine accurately, because the flux exists not only between the windings but actually passes through the copper itself. The reluctance depends on the length of the leakage path and its cross-section, and the reactance of a transformer can be made smaller either by making the path longer or by making the spacing between primary and secondary coils less. The reactance can also be reduced by reducing the ampere-turns effective across the leakage path. The number of ampere-turns can be reduced by either decreasing the total number of turns in the transformer or by dividing the winding into a number of groups, as will be shown in later examples.

The reactance of a transformer may be obtained from the formula

$$X = 275 \times \frac{f\,p\,N^2\,l}{10^8\,l_1} \times \left(\frac{t_1+t_2}{3}+t\right)$$

in which X = reactance in ohms;

f = frequency, in cycles per second;

p = number of leakage paths;

N = number of high-voltage turns effective in establishing leakage flux across one leakage path;

l = mean length of turn, in feet, average of primary and secondary windings;

l_1 = length of leakage path, in inches. In core-type transformers this is the length of the coil; in shell-type transformers it is the width of the wire space;

t = distance from high- to low-voltage coils or group of coils, in inches;

t_1 = thickness of high-voltage coil or group of coils, in inches;

t_2 = thickness of low-voltage coil or group of coils, in inches.

18. Proportions of Wire Space.—In Fig. 9 is shown the outline of the core of a core-type transformer. The figure also indicates the relation between the width, depth, and height of the core and the width and height of the *window*, which is the open space within the core structure reserved for the adjacent sides of the coils that are mounted on the two vertical legs of the core. The stated dimensions relate to a problem given later.

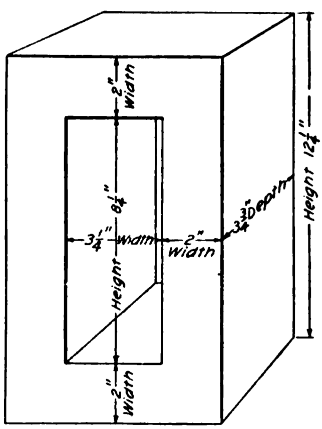


Fig. 9

For highest economy in material, the ratio of the height of the window to its width should be near unity, but in many cases this ratio is impracticable owing to considerations of cooling, mechanical limitations, etc. In both core- and shell-type transformers the usual ratios of the two window dimensions are between the limits 1 to 1 and 1 to 4, the latter

ratio being used only on transformers of high voltage and large capacity.

19. Insulation.—The insulation of the windings in transformers is of primary importance, because safety to life and reliability of operation depend on the quality and arrangement of the insulating material. In many cases a few mils of insulation would give satisfactory dielectric strength, but usually designers materially increase such allowances. This is particularly true when considerable mechanical strength is desired, such as between layers of finely wound wire.

In the modern transformer, there is usually provided between turns, layers, and coils insulations that will withstand many times the stresses imposed by normal operation. This extra insulation is necessitated by the fact that line disturbances resulting from switching, arcing grounds, static discharges, etc. often *pile up* the voltage across a small portion of the winding. For instance, while the normal difference of potential between turns will rarely exceed 100 volts, many of the modern power transformers of medium or high voltage will withstand at these points thousands of volts; in fact, often the whole line voltage.

Double-cotton covering on the conductor is usually employed, with mica, horn fiber, and similar materials added to give the required dielectric strength. The double-cotton covering alone will suffice in many cases. Between layers that have many turns, such as are on lighting transformers, cloth or paper is usually employed. These insulation strips are cut wider than the actual winding layer, in order effectually to prevent leakage between the end turns of adjacent layers. For mechanical reasons, this extension should not be less than $\frac{1}{16}$ inch at either end. Between coils or sections of coils pressboard barriers are usually provided. As the voltage increases, the barrier is thickened until $\frac{1}{4}$ inch to $\frac{3}{8}$ inch is reached; additional insulation is secured by alternating such barriers with oil spaces. Porcelain or treated wood is usually employed to act both as an insulator and for the mechanical support of the coils. These supports engage with the retaining clamps used in drawing

up the core and are thereby capable of resisting the repulsing forces exerted by the windings under short-circuit conditions.

20. The fact that the test voltage is almost always at least double the normal rated voltage influences the selection of all insulating materials. In many cases the insulations have a factor of safety of from three to five times the normal

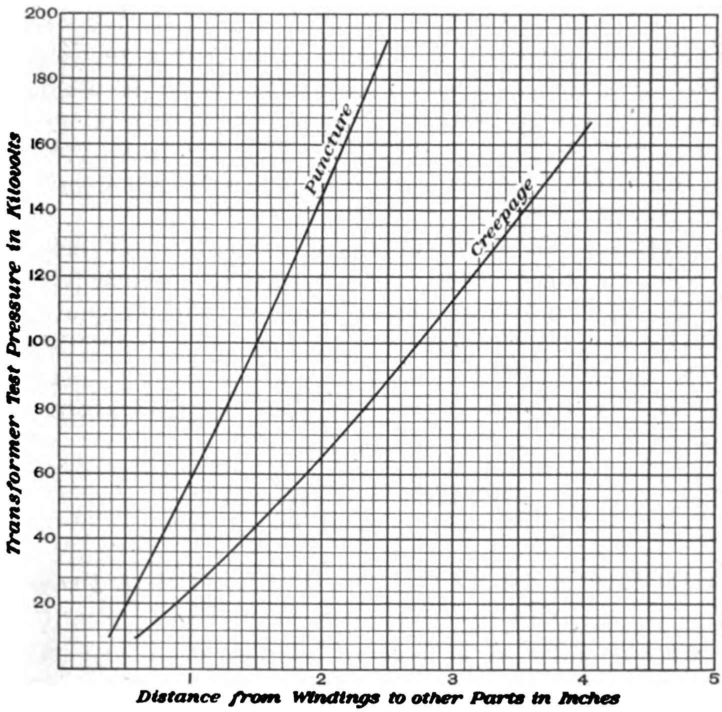


FIG. 10

voltage, in order to take care of variations in the quality of the material and to allow for unusual conditions. In the case of distributing transformers connected directly to the circuits of consumers the transformers are tested with 10,000 volts from the primary coil to the core and the secondary coil combined. The secondary windings are usually tested with 4,000 volts between the secondary coil and the core.

When higher testing voltages than 10,000 are required, the approximate test voltages for various distances of the windings from other parts of the transformer are as indicated in Fig. 10. The upper curve relates to voltages for puncture tests and the lower curve relates to voltages for creepage tests. In some parts of the transformer a breakdown can occur only by puncturing the insulation; in other parts a breakdown may occur by a spark creeping over the surface of the insulation.

21. The insulation in a transformer occupies a very considerable portion of the total space available for the windings, therefore a transformer with a space factor as large as .4 or .5 is a rarity. An inspection of Fig. 7 will show the great decrease in the space factor when the voltage is increased from 5 to 60 kilovolts.

Insulation adds to the cost of a transformer, not only on account of the cost of the insulating material, but also because of the increased dimensions of the transformer required to provide space for the windings. The factor of safety is somewhat reduced on high-voltage transformers, chiefly to avoid using so much insulation with accompanying excessive cost.

DESIGN PROBLEMS

DESIGN OF A 5-KILOVOLT-AMPERE, SINGLE-PHASE, CORE-TYPE TRANSFORMER

22. In order to indicate the application of the design principles just considered, several designs will be made. The first of these problems relates to the design of a 5-kilovolt-ampere, single-phase, 60-cycle, oil-cooled, core-type transformer, with a primary voltage of from 1,100 to 2,200 and a secondary voltage of from 110 to 220.

23. The low-voltage flux turns calculated by formula **2**, Art. **2**, will be $\Phi_m N = \frac{22.5\,E}{f} = 22.5 \times \frac{220}{60} = 82.5$. The value of the flux Φ_m in a 5-kilovolt-ampere, 60-cycle, core-type

transformer is shown by the upper curve of Fig. 1 to be approximately .5 megaline; therefore, $N=82.5\div.5=165$ turns, approximately. As there will be four secondary coils, the number of turns per coil will be 41 and the total turns for the four coils 164. The corrected total flux for 164 turns is $\frac{82.5}{164}$ $=.503$ megaline.

Only two secondary coils are required for ordinary two-wire 110- or 220-volt service; but for three-wire service, in order that each side of the secondary may be evenly distributed over the primary windings, four secondary coils are required. Two secondary coils will be mounted on each vertical leg of the core, Fig. 9, one within the other.

The primary windings have a total of $164\times10=1{,}640$ turns. There will be four primary coils of 410 turns each, two on each vertical leg of the core, and these are mounted on the outside of the secondary coils.

24. For lighting service, relatively low flux densities are usually employed; transformers for such service are often subject to overvoltage, which materially increases the magnetizing current. A density of 75,000 lines (Art. 4) will therefore be chosen, which will require a net cross-section of core of $\frac{503{,}000}{75{,}000}$ $=6.7$ square inches. The gross cross-section (Art. 4) will be $\frac{6.7}{.9}=7.5$ square inches, nearly. A depth of $3\frac{3}{4}$ inches and a width of 2 inches will be selected for the core, as indicated in Fig. 9.

25. On transformers of this size, a tank radiation surface equivalent to .12 to .18 watt per square inch is easily provided; in fact, a plain surfaced tank sufficient to house properly the transformer will usually give more than the necessary surface. For 50° C. rise and a tank radiation of approximately .15 watt, the allowable watts per square inch of coils may be taken as .36. If a .15-tank constant line is drawn in Fig. 5 as directed in Art. **12**, a value of .35 watt is indicated, but the larger value, .36 watt, will be selected for this problem.

Instead of providing a cooling duct between primary and secondary windings, a satisfactory temperature rise can be obtained on the average machine of 5-kilovolt-ampere capacity or under by employing fairly low current densities, particularly on the inner coil. Solid coils are thus obtained for each leg of the core, giving great mechanical strength.

26. An average current density (Art. **10**) of 900 will be assumed, the current density selected for the low-voltage winding being 700 and that for the high-voltage winding being 1,100. The low-current density of 700 was chosen for the secondary winding because that winding will be covered by the primary winding. These values provide ample overload capacity.

27. The full-load high-voltage current will be $\frac{5{,}000 \text{ watts}}{2{,}200 \text{ volts}}$ $=2.27$ amperes, and the full-load, low-voltage current will be 22.7 amperes. The high-voltage ampere-turns $(2.27 \times 1{,}640)$ plus the low-voltage ampere-turns (22.7×164) gives a total of 7,450 ampere-turns, nearly.

28. The dimensions of a section of one side of the coil, Fig. 8, can now be calculated by substituting known values in formula **3** of Art. **15.** Here $I\,N=7{,}450$, $\frac{a}{I}=\frac{1}{900}$, and P_i $=.36$; then, $\frac{7{,}450}{L}=\frac{1}{900}\times\frac{10^6\times .36}{.828}$, and $L=\frac{7{,}450\times 900\times .828}{10^6\times .36}$ $=15.5$ inches. The oil is in contact with both the outside and the inside surfaces of the coil, since a duct is provided between the coil and the core. The value of L in this problem is, therefore, twice the longer dimension of the coil section, and this dimension is $15.5 \div 2 = 7.75$ inches. Each washed surface, Fig. 8, is $\frac{L}{2}$, or 7.75, inches in length.

29. The cross-section of the low-voltage copper will be $\frac{22.7 \text{ (amperes)}}{700 \text{ (current density)}}=.0325$ square inch, approximately; and

of the high-voltage copper, $\frac{2.27}{1,100}=.00206$ square inch, approximately.

The conductors may be round or rectangular in cross-section and they should be wound closely in layers so as to fill the space economically. The secondary conductors are so large that rectangular conductors will be more suitable, and the shape should be selected so that no layers will remain only partly filled. As each low-voltage coil is to contain 41 turns, it will be well to wind 41 turns per layer. The total length of winding space, 7.75 inches, divided by 41 gives .189 inch for copper and insulation. The double-cotton insulation on the conductor will take up .015 inch for both sides of the conductor, leaving .174 inch for copper alone. It will be well to make this dimension .170 inch, leaving the rest for clearance. The other dimension must then be $.0325 \div .170 = .191$ inch. The copper will wind better if two thinner strips are wound together, each, say, .100 inch thick, making the thickness of insulated copper $(.100+.015)\times 2=.230$ inch and the cross-section of bare copper $.170\times .200=.034$ square inch.

30. The high-voltage conductor must have a sectional area of .00206 square inch, or $\frac{.00206\times 1,000,000}{.7854}=2,623$ circular mils. It will be well to use the next larger standard B. & S. wire, which is No. 15, having a sectional area of .002558 square inch, or 3,257 circular mils, a diameter of .057 inch bare and approximately .067 inch over double-cotton insulation.

There must be two high-voltage coils on each leg and they will be placed end to end. The length $7.75 \div 2 = 3.875$, and making a small allowance for coil insulation, the length of the winding space for each coil will be 3.625 inches. The 410 turns of one coil can be arranged in 8 layers of 52 turns each, except the top layer, which will contain only 46 turns. Between layers .010-inch insulation will be used, and it is assumed that this builds up to .012 inch when wound in the coil.

31. In order to determine the dimensions of the *window* of the core structure, the outside dimensions of the primary coil

should be calculated. The *build-up* of the secondary and primary coils will be as follows:

	Depth Inches	Width Inches
Core dimensions (Fig. 9)	3.75	2.00
Core insulation, two sides ($.1\times 2$)	.20	.20
Thickness, two secondary coils, two sides ($.23 \times 2\times 2$)	.92	.92
Insulation for two secondary coils, two sides ($.01\times 2\times 2\times 2$)	.08	.08
Insulation between primary and secondary, two sides ($.125\times 2$)	.25	.25
Over-all dimensions of insulated secondary	5.20	3.45
Primary conductors, eight layers, two sides ($.067\times 8\times 2$)	1.07	1.07
Insulation for layers and outer half-lapped tape, two sides ($.012\times 7\times 2+.008\times 2\times 2$)	.20	.20
Over-all dimensions of insulated primary	6.47	4.72

The portion of the total width of the window that is occupied by conductors and insulation of both primary and secondary windings on both legs of the core will be $\frac{4.72-2 \text{ (the core width)}}{2} \times 2 = 2.72$ inches.

It is probable that the coils will wind larger than calculated, therefore, allowing an increase of 5 per cent., the width of the space occupied will be $2.72+(2.72\times .05)=2.86$ inches. If 3.25 inches is selected for the width of the window, as shown in Fig. 9, the clearance between the adjacent sides of the primary coils on the two cores will be $3.25-2.86=.39$ inch, which is sufficient for the comparatively low voltage employed.

The height of the window will be selected as $8\frac{1}{4}$ inches, which allows for a space of about $\frac{1}{4}$ inch to the core yoke at each end of the secondary coils.

32. The core will have the dimensions indicated in Fig. 9 and its weight may be calculated as follows: The number of cubic inches of the core will be $2(2\times 12.25\times 3.75)+2(2\times 3.25$

$\times 3.75) = 232.5$. Approximately nine-tenths of the core will be of steel (Art. **4**), and the net cubic inches of steel will be $232.5 \times .9 = 209.25$. Allowing .275 pound per cubic inch for steel, the net weight of the core will be $209.25 \times .275 = 57.5$ pounds.

The flux density in the net cross-section of the core will be $\frac{503,000}{2 \times 3.75 \times .9} = 74,500$ lines, or $74,500 \div 1,000 = 74.5$ kilolines, per square inch of area. The laminations will be punched from .014-inch, sheet-steel plates and the core losses can be read from the 60-cycle curve for 14-mil steel in Fig. 3, namely, 1.19 watts per pound for 74.5 kilolines per square inch. The total core loss will be $57.5 \times 1.19 = 68.5$ watts, nearly.

33. The mean length of the magnetic circuit of this core will be $2 \times 8.25 + 2 \times 3.25 + 2 \times 3.1416 = 29.25$ inches, approximately. If straight sheets are used, there will be four joints in the magnetic circuit. From Fig. 4, for a density of 74.5 kilolines per square inch, the value of the magnetizing ampere-turns per inch length of the mean magnetic circuit is 8.6. About 75 ampere-turns (Art. **8**) will be required per joint. The total magnetizing ampere-turns will be $29.25 \times 8.6 + 75 \times 4 = 552$. The primary current is 2.27 amperes (Art. **27**), the number of turns in the primary is 1,640, and the primary ampere-turns at full load is $2.27 \times 1,640 = 3,723$. The magnetizing ampere-turns are, therefore, $552 \div 3,723 = .148$, or 14.8 per cent. of the full-load excitation. If L-shaped laminations are employed to build up the core, the number of joints will be two and the magnetizing ampere-turns will be $\frac{29.25 \times 8.6 + 75 \times 2}{3,723} \times 100 = 10.8$ per cent. of the primary ampere-turns at full load.

34. The mean length of the turns in a coil with rounded corners mounted on a rectangular core can be found by adding to the perimeter of the core the circumference of a circle with a diameter equal to twice the distance from the core to the center of the coil. The data will be taken from Art. **31**, the values now being for one side of the core. The perimeter of the core will be $2(2 + 3.75) = 11.5$ inches. The diameter of the circle

for the secondary coil will be $2\left(\frac{.2}{2}+\frac{.92}{4}+\frac{.08}{4}\right)=.7$ inch; and the circumference, $.7\times3.1416=2.2$ inches. The mean length of the secondary turns will be $\frac{11.5+2.2}{12}=1.15$ feet, nearly.

The straight portions of the primary coil will have practically the same dimensions as the secondary coils, that is, 11.5 inches. The diameter of the circle for the primary coil will be $2\left(\frac{3.45-2}{2}+\frac{1.07}{4}+\frac{.2}{4}\right)=2.09$ inches; and the circumference will be $2.09\times3.1416=6.57$ inches. The mean length of the primary turns will be $\frac{11.5+6.57}{12}=1.5$ feet.

The total length of the secondary copper will be $164\times1.15=189$ feet; and of the primary copper, $1{,}640\times1.5=2{,}460$ feet. In order to allow for leads, cross-connections, and for a possible larger build than calculated, the length value just determined will be increased 5 per cent., making the secondary copper 199 feet, nearly, and the primary copper 2,580 feet, approximately.

The resistance of the windings may be obtained from the preceding data and the instruction of Art. **15.** The secondary conductor has a total cross-section of .034 square inch and a length of 199 feet. The resistance at 75° C. of a copper bar 1 foot long and 1 square inch in cross-section will be $\frac{9.94}{10^6}$ ohm; then the resistance at 75° C. operating temperature of the secondary windings will be $\frac{9.94}{10^6}\times\frac{199}{.034}=.058$ ohm, and of the primary windings will be $\frac{9.94}{10^6}\times\frac{2{,}580}{.002558}=10$ ohms.

The secondary copper loss will be $22.7^2\times.058=29.9$ watts, and the primary copper loss will be $2.27^2\times10=51.5$ watts.

The total copper loss will be 81.4 watts. The combined steel and copper loss will be $68.5+81.4=150$ watts, and the full-load efficiency will be $\frac{5{,}000}{5{,}150}=.971$, or 97.1 per cent.

35. The secondary drop will be $I_s R_s = 22.7 \times .058 = 1.317$ volts $= \frac{1.317}{220} = .6$ per cent. of full-load secondary voltage. The primary drop will be $I_p R_p = 2.27 \times 10 = 22.7$ volts $= \frac{22.7}{2,200} = 1.03$ per cent. of normal primary voltage. The sum of the values of the secondary and primary drop percentages will be $.6 + 1.03 = 1.63$ per cent.

The formula of Art. **17** should be used to determine the reactance of the transformer, as measured on the primary side. In this equation the value of p is 2, since the windings are mounted on the two vertical legs of the core and there are two leakage paths between the primary and secondary windings. The value of N is 820, which is the number of primary turns on each leg of the core. The average value of l is $\frac{1.5 + 1.15}{2} = 1.325$ feet. The value of t_1 is $\frac{1.07 + .2}{2} = .635$ inch. The value of t_2 is $\frac{.92 + .08}{2} = .5$ inch. The value of t is $\frac{.25}{2} = .125$ inch, and the value of l_1 is 7.75. The reactance will be $X = 275 \times \frac{60 \times 2 \times 820^2 \times 1.325}{10^8 \times 7.75} \times \left(\frac{.635 + .5}{3} + .125\right) = 19.1$ ohms.

The reactance drop will be $I_p X_p = 2.27 \times 19.1 = 43.4$ volts $= \frac{43.4}{2,200} = 2$ per cent., nearly, of normal primary voltage.

The impedance drop in percentage of the primary voltage will be $\sqrt{1.63^2 + 2^2} = 2.58$ per cent.

The regulation expressed as a percentage may be determined by the formula

$$\text{per cent. regulation} = \text{total } I\,R \text{ per cent.} + \frac{(I_p\,X_p)^2 \text{ per cent.}}{200}$$

Substituting values, the regulation will be $1.63 + \frac{2^2}{200} = 1.65$ per cent.

36. The weight of copper in the secondary winding, using the value .3212 pound per cubic inch of copper, will be $.034 \times 199 \times 12 \times .3212 = 26$ pounds, and the weight of copper in the primary winding will be $.002558 \times 2{,}580 \times 12 \times .3212 = 25.4$ pounds, a total of 51.4 pounds.

DESIGN OF A 100-KILOVOLT-AMPERE, SINGLE-PHASE, CORE-TYPE TRANSFORMER

37. Let it be required to design a 100-kilovolt-ampere, 25-cycle, oil-cooled, single-phase, core-type transformer for 22,000 volts primary and either 440 or 220 volts secondary. The transformer is to be placed in a corrugated iron tank. The temperature rise, when operating continuously at full load, must not exceed 40° C.

The product of flux and low-voltage turns, from formula **2**, Art. **2**, will be $\Phi_m N = \frac{22.5 \times 440}{25} = 396$. A total flux of 4.1 megalines is indicated in Fig. 1 for the required capacity rating. The trial value for the low-voltage turns will be $\frac{396}{4.1} = 96.6$ turns.

The low-voltage winding is to be designed for both 440 and 220 volts by placing coils in series for 440 volts and in parallel for 220 volts. Such transformers are also often used for three-wire operation, and in such cases each half of the winding must be wound over both legs of the core. This necessitates four coils, therefore, 96 turns, which is a multiple of four, will be selected. The corrected flux will be $\frac{396}{96} = 4.13$ megalines.

The number of high-voltage turns will be $96 \times \frac{22{,}000}{440} = 4{,}800$ turns.

38. The core is made up of silicon sheet steel .025 inch thick. If a flux density of 85,000 lines per square inch is assumed, the effective cross-section of the core will be $\frac{4{,}130{,}000}{85{,}000} = 48.6$ square inches. The space factor of the steel

core being .9, the trial value of the gross cross-section will be $\frac{48.6}{.9} = 54$ square inches.

A double cruciform cross-section will be selected for the two vertical legs of the core, as indicated in Fig. 11. This construction allows of oil ducts between the core and the low-voltage coils, and also allows a large amount of iron to be included within a circle with a limited number of widths of lamination.

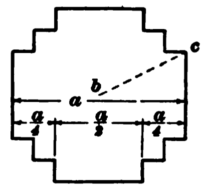


FIG. 11

When the distance between opposite parallel faces of the cross is a inches and the other dimensions are the fractional parts of this distance indicated in Fig. 11, the area of the cross-section can be calculated thus:

$$\text{Area} = a^2 - \left(\frac{a}{4}\right)^2 \times \frac{3}{4} \times 4 = \frac{13a^2}{16}$$

The trial value of the cross-section of the core is 54 square inches; then, $54 = \frac{13a^2}{16}$, or $a = 8.15$; $\frac{a}{2} = 4.08$; $\frac{a}{4} = 2.04$ inches.

Choosing the nearest even dimensions in sixteenths of an inch

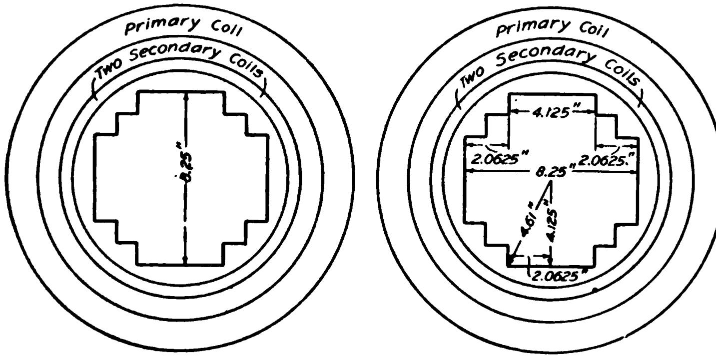


FIG. 12

for $\frac{a}{2}$ and $\frac{a}{4}$, in order that the punching dimensions may be convenient, values of 4.125 and 2.0625 inches will be selected.

The radius of the core, or the distance $b\,c$, Fig. 11, will be $\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{4}\right)^2}$; or, substituting values, $\sqrt{4.125^2+2.0625^2}$ $=4.61$ inches. These dimensions are indicated in Fig. 12, which also shows the relative positions of the cores, the secondary coils, and the primary coils.

With the final dimensions, the core area will be $\frac{13\times 8.25^2}{16}$ $=55$ square inches, and the corrected flux density will be $\frac{4,130,000}{55\times .9}=83.5$ kilolines, approximately.

39. The normal full-load current in the primary windings will be $\frac{100,000}{22,000}=4.545$ amperes; and in the secondary windings, $\frac{100,000}{440}=227.27$ amperes.

If a current density of 1,200 amperes per square inch is assumed, the trial value of the cross-section of the primary copper will be $\frac{4.545}{1,200}=.00379$ square inch; and for the secondary copper, $\frac{227.27}{1,200}=.1894$ square inch.

40. The length of the washed surface of the low-voltage winding may be calculated by substituting values in formula **3** of Art. **15**. The current density assumed is 1,200 amperes per square inch; then, $\frac{a}{I}=\frac{1}{1,200}$. The value assumed for P_t will be .24 watt; the value of $I\,N$ will be $4.545\times 4,800$. Then, $\frac{4.545\times 4,800}{L}=\frac{1}{1,200}\times\frac{10^6\times .24}{.828}$, or

$$L=\frac{4.545\times 4,800\times 1,200+.828}{10^6\times .24}=90.3 \text{ inches.}$$

Some variation from the length L is allowable, and, for reasons that will appear later, 88 inches will be chosen. As

88 inches is the combined length of the inner and outer surfaces, the length of coils parallel to the cores must be 44 inches, and the height of the coil on each vertical core will be 22 inches. The low-voltage winding will be placed next the core and will be surrounded by the high-voltage winding, with oil ducts next the core and between the two windings.

41. The four secondary, or low-voltage, coils must contain 24 turns each. The two coils on each leg will be wound the full distance, 22 inches, one coil over the other. The copper for the conductors of the size required should be in the form of a rectangular strip and if the twenty-four turns are wound in one layer, the width of insulated strip may be $22 \div 24 = .92$ inch. It will be better to use two strips wound side by side in multiple rather than one so wide. Double-cotton insulation on each strip will require .015 inch and for both strips .030 inch, leaving $.92 - .030 = .890$ inch of copper, or $.890 \div 2 = .445$ inch width of each strip bare and .460 inch width of each insulated strip.

The trial cross-section of secondary copper was taken as .1894 square inch, therefore the total thickness of the copper conductor will be $\frac{.1894}{.890} = .213$ inch, approximately. The maximum thickness of copper that may be used to advantage for this purpose being .150 inch, two strands, one wound on the other, will be used, each strand .105 inch thick.

The low-voltage copper conductor will, therefore, consist of four strands wound two wide and two high, each $.445 \times .105$ inch in width and thickness. The final copper cross-section will be $4 \times .445 \times .105 = .187$ square inch. The insulated dimensions of each turn will be: width, $2(.445 + .015) = .920$ inch; thickness, $2(.105 + .015) = .240$ inch.

42. The insulation between the two secondary coils on each leg of the core will be built up to a thickness of .080 inch and will consist of paper and varnished cloth on which the outer coil is wound. The insulation will be made considerably safer than that required for the normal voltage, because mechanical strength is required to withstand the crushing effect of the

outer coil. After both secondary coils are wound they will be bound securely together by tape.

43. The cross-section required for the primary copper being comparatively small, a round wire will be used, as for small conductors it is more convenient and cheaper than rectangular wire. The diameter will be $\sqrt{\frac{.00379 \times 4}{3.1416}} = \sqrt{.004825} = .0695$ inch. The wire selected, therefore, will be .070 inch in diameter and .00385 square inch in cross-sectional area.

Double-cotton insulation on the wire will be used, except that at each end of the primary winding a number of turns equal to 5 per cent. of the total turns will be insulated with four wraps of cotton thread. The turns having extra insulation are formed into separate coils. The diameter outside of the insulation of the main conductor is .080 inch and of the end conductor .087 inch.

In order to reduce the voltage between layers, the high-voltage winding must be arranged in a number of sections. For best economy, the maximum voltage between layers of this type of coil should not exceed 450, and therefore the maximum voltage per layer is 225. The voltage between sections should not be greater than 4,000, necessitating the use of six sections on each leg. As the volts per turn will be 22,000 $\div$ 4,800 = 4.6, the number of turns per layer cannot exceed 225 $\div$ 4.6 = 49. The number of turns on each leg is 2,400, and if 5 per cent. of the total number of turns, 4,800 $\times$ .05 = 240 turns, are made of the heavily insulated wire, formed into a section and placed at the top of each core, where the electric stress is greatest, there will remain 2,400 − 240 = 2,160 turns for the remaining five sections, or 2,160 $\div$ 5 = 432 turns in each section.

The coils will be arranged as indicated in Fig. 13. Insulation is provided between each of the primary sections and at the outer edge of the upper and lower sections, making 7 insulation spaces. The insulating barrier will be .2 inch, but an allowance of .4 inch should be made on account of the layer insulation being folded around the edges of the layers. The

total space allowance should be 2.8 inches. The space for insulated wire will be $22-2.8=19.2$ inches. For preliminary calculations, the end section may be assumed to occupy approximately one-tenth $\left(\frac{240}{2,400}\right)$ of this space, or 1.92 inches, and the other five sections the remainder, 17.28 inches, or 3.46 inches each.

The number of turns per layer in the end sections would then be $1.92 \div .087=22$; but in order to wind the 240 turns economically, 24 turns per layer and 10 layers will be better, necessitating $24 \times .087=2.09$ inches space for insulated copper.

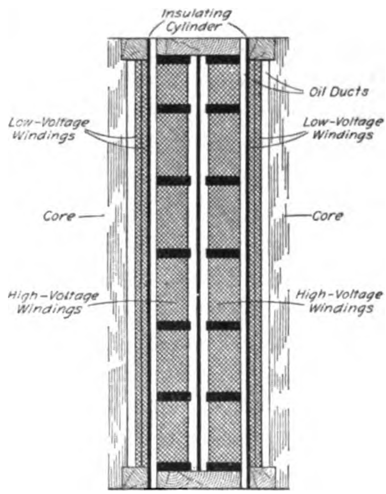


FIG. 13

The ten layers in the end section will wind to approximately the same depth as eleven layers of the .080-inch insulated wire in the remaining sections, and better results are obtained by having all the sections nearly the same depth. The insulation between layers will add .030 inch to their thickness and a collar at the ends of each section will project .25 inch above the section. The end section will build up to $(.087+.030)\times 10+.25=1.42$ inches and the other sections to $(.080+.030)\times 11+.25=1.46$ inches. Each of the five main sections will contain ten layers of forty-three turns each and one layer of two turns. The insulated copper in each of the first ten layers will fill a space of $.080\times 43=3.44$ inches and the space required for the insulated copper in the five sections will be 17.2 inches.

44. The height and width of the window of the core structure may now be determined as follows:

HEIGHT OF WINDOW	INCHES
Creepage distance from windings to yoke at each end for a test voltage of 44,000 (creepage curve, Fig. 10), 1.5 inches, two ends	3.00
Length of layer of heavily insulated wire in end section (24×.087)	2.09
Total length of lower layers in other five sections (43 ×.080) 5	17.20
Insulation between sections, 5 barriers, .4 inch each	2.00
Total	24.29

WIDTH OF WINDOW	INCHES
Radius of core (Fig. 12)	4.610
Oil duct between core and low-voltage winding	.500
Paper cylinder on which inner low-voltage coil is wound	.040
Thickness of inner, low-voltage, coil	.240
Mean radius of low-voltage winding	5.390
Layer insulation	.080
Thickness of outer, low-voltage, coil	.240
Distance from low-voltage to high-voltage winding puncture curve, Fig. 10, 44,000 volts, approximate allowance .94 inch	.940
High-voltage winding	1.460
Distance between high-voltage winding and center of winding space (one-half allowable space between high-voltage coils on the two legs of the core $\frac{.940}{2}$)	.470
	8.580
	2
Distance between centers of core legs	17.160
Less width of core leg (Fig. 12)	8.250
Width of window	8.910

Copper space factor (Art. 14), $\frac{.187\times 96+.00385\times 4{,}800}{8.91\times 24.29}=.168$

45. The yokes connecting the core ends should have the same cross-sectional area as the cores, 55 square inches. The

width of the yoke will be the same as that of the core, 8.25 inches, and the thickness will be $55 \div 8.25 = 6.67$ inches.

The mean length of the magnetic path through the cores and yokes can be found by adding to the periphery of the window opening the circumference of a circle of which the diameter is 6.67 inches (the thickness of the yoke), and also a length equal to four times the difference between one-half of the thickness of the core and one-half of the thickness of the yoke. The latter correction is made to bring the curve tangent to the center lines of the cores and yokes. The length of the path will be

$$2(24.29+8.91)+(6.67\times 3.1416)+4\left(\frac{8.25}{2}-\frac{6.67}{2}\right)=90.51 \text{ inches}$$

The weight of steel will be taken as .275 pound per cubic inch, and .9 of the gross cubic inches as the net cubic inches of steel. The total weight of the core will be $.275\times .9\times 90.51\times 55 = 1,230$ pounds, approximately. From the upper curve of Fig. 3, a core loss of very nearly .8 watt per pound is indicated for a density of 83.5 kilolines. The total core loss will be $1,230 \times .8 = 984$ watts.

The per cent. of core loss based on full-load output will be $\frac{984}{100,000} = .984$ per cent.

46. The magnetizing ampere-turns per inch length of the mean magnetic circuit (Fig. 4), at a density of 83.5 kilolines per square inch, will be 12.4. If 400 ampere-turns are allowed for the four joints in the core, the total magnetizing ampere-turns will be $12.4\times 90.51+400=1,522$. The per cent. of magnetizing ampere-turns based on the ampere-turns of the primary at full load will be

$$\frac{1,522}{4,800\times 4.545}=7 \text{ per cent.}$$

47. The mean radius of the low-voltage turns is 5.39 inches (Art. **44**), and the length of this winding, in feet, will be $\frac{2\times 5.39\times 3.1416\times 96}{12} = 270$, approximately.

The mean radius of the high-voltage coil will be $5.39+.08+.24+.94+\frac{1.46}{2}=7.38$ inches. The total length of the high-voltage conductor will be $\frac{2\times 7.38\times 3.1416\times 4{,}800}{12}=18{,}550$ feet, approximately.

The resistance of the low-voltage conductor at 75° C. will be $\frac{9.94\times 270}{10^6\times .187}=.01435$ ohm; of the high-voltage conductor, $\frac{9.94\times 18{,}550}{10^6\times .00385}=47.89$ ohms.

The fractional voltage drop in the windings due to their resistance will be $\frac{.01435\times 227.27}{440}=.00741$, or .741 per cent. in the low-voltage winding, and $\frac{47.89\times 4.545}{22{,}000}=.00989$, or .989 per cent. in the high-voltage winding. The per cent. drop in both windings will be $.741+.989=1.73$, and the combined copper and steel loss in the transformer will be $1.73+.984=2.714$ per cent. The efficiency of the transformer will therefore be $\frac{100}{100+2.714}=.974$, or 97.4 per cent.

The loss in watts in each of the windings is equal to the product of the total output in kilovolt-amperes (100,000) and the fractional voltage drop expressed decimally, or 741 and 989, respectively, the total loss being 1,730 watts, which is also the product of .0173 and 100,000. The mathematical reason for this is as follows: $E_p\times\%\times I_p$ = watts loss in primary, where % is the fractional voltage drop expressed decimally. $E_p\times\%\times\frac{W}{E_p}$ = watts loss in primary. Canceling E_p, $\%\times W$ = watts lost in primary.

48. The reactance of this transformer, measured on the primary side, may be determined by the use of the formula of Art. **17.** In this equation, the value of p is 2, and of N is $\frac{4{,}800}{2}$

=2,400 turns, as there are two leakage paths and two legs of the core on which the windings are mounted. The average value of l for both the primary and secondary windings is

$$\frac{2[(7.38\times3.1416)+(5.39\times3.1416)]}{2\times12}=3.34 \text{ feet}$$

The value of t_1 is $1.46-.25$ (collar allowance) $=1.21$ inches.

The value of t_2 is $.24+.24+.08=.56$ inch.

The values of t, f, and l_1 are given directly in the data of the problem. The reactance will be

$$X=275\times\frac{25\times2\times2{,}400^2\times3.34}{10^8\times22}\times\left(\frac{1.21+.56}{3}+.94\right)$$
$$=184 \text{ ohms, nearly}$$

The reactance drop will be $4.545\times184=836.28$ volts $=\frac{836.28}{22{,}000}\times100=3.8$ per cent. of primary voltage.

The impedance drop in percentage of the primary voltage will be $\sqrt{1.73^2+3.8^2}=4.18$ per cent.

The regulation found by the formula of Art. **35** will be $1.73+\frac{3.8^2}{200}=1.8$ per cent.

49. The cylindrical radiating surface of the low-voltage winding will be 2×5.39 (mean radius) $\times3.1416\times22\times4=2{,}980$ square inches; of the high-voltage winding, 2×7.38 (mean radius) $\times3.1416\times22\times4=4{,}081$ square inches.

The watts loss per square inch of surface of the low-voltage winding will be $\frac{741}{2{,}980}=.25$ watt; of the high-voltage winding, $\frac{989}{4{,}081}=.24$ watt.

A tank constant, Fig. 5, of approximately .14 watt per square inch of effective tank surface would be suitable if the watts per square inch of coil surface were .24 and the temperature rise 40° C. The total effective tank surface required will be $\frac{1{,}730 \text{ (copper loss)}}{.14}=12{,}360$ square inches, approximately.

The radiating surface on the legs of the core will be 2[2×4.61 (radius)×3.1416×24.29 (height of window)] = 1,400 square inches, approximately; on the four side faces of the yoke, 2[6.67 (width)×2 (17.16+8.25) (length of one yoke)] = 680 square inches, approximately. The total core radiating surface will be 1,400+680 = 2,080 square inches. The watts core loss per square inch will be $\frac{984}{2,080} = .473$ watt per square inch. Only a portion of the whole surface of the yoke is considered in calculating the radiating surface, because part of the yoke is so covered that the oil does not wash the surface.

DESIGN OF A 1,000-KILOVOLT-AMPERE, THREE-PHASE, SHELL-TYPE TRANSFORMER

50. Let it be required to design a 1,000-kilovolt-ampere, 60-cycle, oil-and-water-cooled, three-phase, shell-type trans-

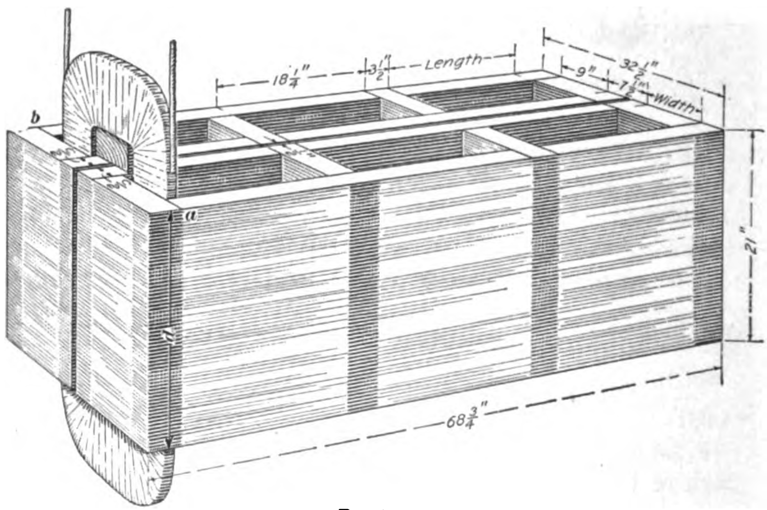


Fig. 14

former; the primary coils to be designed for 38,100 volts when delta-connected and for 66,000 volts when star-connected, the secondary coils to be delta-connected for 2,300 volts.

The core and windings will be submerged in oil in a tank made of boiler plate, the oil being cooled by running water in pipes. The temperature rise of the windings is not to exceed 40° C. above the temperature of the cooling water when the transformer is operating continuously under full load.

51. The core will be made in the form shown in Fig. 14, that is, in two parts separated by a vertical oil duct, each part having three rectangular windows through which pass the sides of thin pancake coils, as indicated in Fig. 14 and also in Fig. 15, which is a vertical section made on the line *a b*, Fig. 14. The planes of the coils will be vertical, and when the core and coils are submerged in oil the oil can rise freely through the oil ducts as it becomes heated. In Fig. 16 is shown a plan of two of the six windows indicating the relative positions of the sides of the high-voltage and low-voltage coils that make up a transformer for one phase.

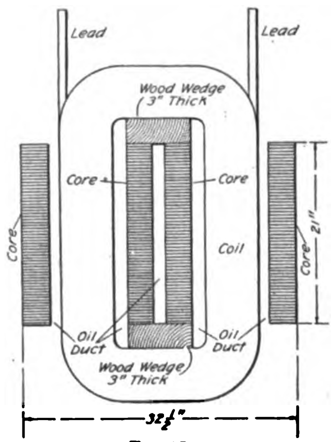


FIG. 15

52. The design problem of the coils is worked for one set of the three groups of windings on the complete transformer. The output per phase will be $\frac{1{,}000}{3} = 334$ kilovolt-amperes. By formula **2**, Art. **2**, $\Phi_m N = \frac{22.5 \times 2{,}300}{60} = 862.5$ for the low-voltage winding. By the 60-cycle shell-type curve, Fig. 1, the flux Φ_m in a 334-kilovolt-ampere transformer should be approximately 10.3 megalines, making the number of low-voltage turns per phase in this case $\frac{862.5}{10.3} = 83.7$, approximately. Any convenient number of turns near this value can be used and the flux changed accordingly so as to keep the product $\Phi_m N$

$=862.5$. As will appear later, 80 turns will be convenient for the secondary, and the number of primary turns will then be $80 \times \frac{38,100}{2,300} = 1,325.2$. It will be better to use the nearest even number, 1,326, for the primary turns.

The primary current will be $\frac{334 \times 1,000}{38,100} = 8.77$ amperes; the secondary current will be $\frac{334 \times 1,000}{2,300} = 145.22$ amperes.

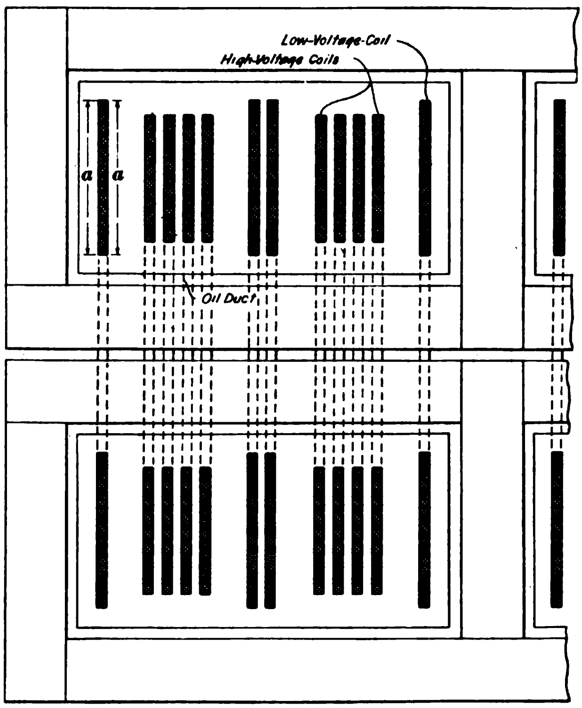


Fig. 16

53. Each winding must be divided into a number of coils, and this number is influenced by the shape of the window, by the value of reactance desired, by the allowable volts per coil, and by some other considerations. The number of coils in

each winding can be determined only by assuming different values and calculating the results; several assumptions are often necessary before the best results can be obtained. Good practice for transformers of this size is to make the window length about twice its width. In very large transformers the ratio of the length to the width of the window is sometimes much larger than 2 to 1.

For the present problem, eight high-voltage and four low-voltage coils per phase will be found to give good results when arranged as indicated in Fig. 16. The number of turns in each low-voltage coil will be $80 \div 4 = 20$, and the average number in each high-voltage coil will be $1{,}326 \div 8 = 165\frac{3}{4}$.

54. In water-cooled transformers it is desirable to operate at current densities as high as 2,500 amperes per square inch of conductor cross-section, and at moderate voltages, that is, under 50,000 volts, with surface radiation of .41 watt per square inch. In coils for high voltage, that is, over 50,000 volts, the watts per square inch of coil surface should be less than this, on account of the fact that a large portion of the surface is taken up by insulation, and a constant of .34 watt is a good average value.

55. The length of the cooled surface of the high-voltage winding when pancake coils are employed is measured along the height of the built-up flat surface. The oil is in contact with both faces *a*, Fig. 16, making the length of the cooled surface per coil 2 *a*. The sum of the lengths of the cooled surfaces of all the eight high-voltage coils is the length of the total cooled surface of the primary winding.

The length of the cooled surface of the high-voltage windings can be found by substituting $8.77 \times 1{,}326$ for $I\,N$, $\frac{1}{2{,}500}$ for $\frac{a}{I}$, and .34 for P_i in formula **3**, Art. **15**; $\frac{8.77 \times 1{,}326}{L} = \frac{1}{2{,}500} \times \frac{10^6 \times .34}{.828} = 71$ inches, approximately. The length of the surface on each of the eight high-voltage coils will therefore be

approximately 9 inches, and on each side of a coil, as *a*, Fig. 16, will be 4.5 inches.

56. For 2,500 amperes per square inch the primary copper must have a sectional area of $8.77 \div 2,500 = .0035$ square inch. If the average number of turns per coil, $165\frac{3}{4}$ (Art. **53**), were wound in the available space, 4.5 inches, the thickness of the insulated copper could not be over $4.5 \div 165\frac{3}{4} = .027$ inch, which is too thin to be economical, as the insulation would fill more than half of this thickness. Each coil will therefore be made in two sections side by side, separated only by an insulating collar.

The end coils of each winding must have heavier insulation, and, consequently, fewer turns than the other coils, the number in each case being enough to build up, when insulated, to about the same height. The insulation on the copper in the six inner coils will be double cotton, which will add .015 inch to each dimension. Between turns will be paper and mica .015 inch thick. The copper in the end coils will have four wrappings of cotton, adding .030 inch to each dimension, and the paper and mica between turns will be .030 inch thick.

For economical use of the space, the copper strip in the six inner coils should be thicker than the cotton insulation covering it. A convenient available thickness is .020 inch, which, when insulated with .015 inch of cotton and wound with a strip of .015-inch paper and mica, builds up to $.020 + .015 + .015 = .050$ inch per turn. The space available is approximately 4.5 inches, allowing 90 turns per section, 180 turns per coil, and 1,080 turns in six coils. This leaves $1,326 - 1,080 = 246$ turns for the two end coils, or 123 turns per coil, and each coil will have two sections of 61 and 62 turns, respectively. Each end turn builds up to $.020 + .030 + .030 = .080$ inch, and 62 turns would build up to 4.96 inches, which is higher than the six inner coils. By winding 92 turns per section, 184 turns per coil, in the inner coils and 111 turns in 55- and 56-turn sections in each end coil, the turns in the inner coils will build up to $92 \times .050 = 4.6$ inches, and in the end coils to $56 \times .080 = 4.48$ inches, which are near enough alike.

The width of the copper strip for .0035 square inch sectional area must be $.0035 \div .020 = .175$ inch, but the nearest available standard width is .170 inch and this will answer; the final size for the primary conductor will then be .0034 square inch. The current density will then be $\frac{8.77}{.020 \times .170} = 2{,}580$ amperes per square inch.

The inside turns of the two sections of a coil will be connected together and the dielectric strength of the insulating pressboard collar separating the sections must allow a good factor of safety over the maximum voltage between the outer turns of the two sections; .125-inch board will do this. The collar will be made to extend 1 inch beyond the outer turns so as to prevent creepage around it, and the space at each side of this projection will be filled with wood 1 inch thick. The whole coil assembly will then be wrapped with tape, adding .062 inch to the build of the coil.

According to the creepage curve, Fig. 10, approximately 2.25 inches for a 76.2-kilovolt test $\left(2 \times \frac{38{,}100}{1{,}000}, \text{Art. } \mathbf{20}\right)$ should be allowed between the copper of the coil and the steel of the core. The greatest danger of breakdown is from the coils to the core, and here the 1-inch layer of wood affords some protection. An allowance of 1.625 inches between the finished coil and the core, both outside and inside the coil, will be safe.

The width of window space required for primary winding, all dimensions being in inches and previously stated, will be as follows:

Width of Window	Inches
92 turns insulated copper with separators	4.600
Wood filling	1.000
Coil tape	.062
Height of insulated coil	5.662
Space allowance, 1.625 inch to steel outside and also inside of coil	3.250
Total	8.912

The portion of the length of the window space required for the high-voltage coils, neglecting allowances for spacing, will be as follows:

Length of Window	Inches
Two strips insulated copper, each .185 inch	.370
Insulating collar between sections	.125
Coil tape	.062
Width of each coil	.557
Width of eight coils	4.456

Note.—The .030 inch additional width of insulation in the end coils can be disregarded here.

57. The width of the wire space being 8.912 inches, the maximum allowable build of the low-voltage coil can theoretically be obtained by subtracting from 8.912 inches the very small insulation space required for a $2{,}300 \times 2 = 4{,}600$-volt test. There are, however, several reasons why it is advisable to employ coils of lesser build and to have a greater space allowance between core and coil than the indicated theoretical value. If the low-voltage coils have a greater build than the high-voltage coils, eddy currents in the copper of the high-voltage coils are increased. Mechanical considerations require more space allowance. Oil ducts must be provided between coil and core to cool the core. There is danger of high voltages and high frequencies being induced in low-voltage coils, and also of high voltage at normal frequency being electrostatically transferred to the low-voltage coils.

58. If 1.25 inches are allowed between the core and the secondary coil, both outside and inside, the build of the coil will occupy a space of $8.912 - 2 \times 1.25 = 6.412$ inches, and each of the twenty turns in the coil will have a space allowance of $6.412 \div 20 = .3206$ inch for copper and insulation. A thick strip is hard to wind and is subject to eddy currents; three strands in multiple wound on each other will be better, the two outer strands double cotton covered to prevent eddy currents between strands. The cotton covering takes up a space of .030 inch. Paper and mica insulation to the thickness of .040 inch will be used between turns of the three-part conductor (not between the strands). This layer insulation must

be able to withstand the crushing effect due to the process of winding the coil.

The total insulation space per turn will be .030+.040 =.070 inch. Subtracting this value from .3206 gives .2506 inch available for copper, or $\frac{.2506}{3} = .0835$ inch for copper per strand. The thickness of copper selected will be .080 inch.

59. Assuming 71 inches as the length of cooled surface of low-voltage coils, which was the value taken for the high-voltage coils, Art. **55**, the length per coil will be $\frac{71}{4} = 17.75$ inches, which corresponds to a coil height of 8.88 inches. The space available will be only 6.412 inches, which indicates that the length of the cooling surface of the low-voltage coils must be less than that required for the coil constants originally assumed. The total length of the cooled surface may be increased by increasing the number of low-voltage coils from four to six, but this is objectionable on account of the increased expense. The length of cooled surface that will be required will be reduced by decreasing the current density in the copper by using a conductor of larger cross-section. The watts per square inch of coil radiating surface corresponding to a coil build of 6.412 inches and a current density of $\frac{I}{a} = 2{,}500$ amperes per square inch, Art. **54**, will be (formula **2**, Art. **15**),

$$P_i = \frac{.828}{10^6} \times \frac{N\,I}{L} \times \frac{I}{a} = \frac{.828}{10^6} \times \frac{80 \times 145.22}{8 \times 6.412} \times 2{,}500 = .469$$

To reduce this value to the more suitable one of .4 watt per square inch, the current density would be reduced to $\frac{.4 \times 2{,}500}{.469} = 2{,}130$ amperes per square inch, which corresponds to a cross-section of copper of $\frac{145.22}{2{,}130} = .0682$ square inch, and the width of the copper becomes $\frac{.0682}{3 \times .080} = .284$ inch. A copper strip .280 inch wide and .080 inch thick will be selected. The

cross-section of the conductor will be $.280 \times .080 \times 3 = .0672$ square inch and the corresponding current density will be $\frac{145.22}{.0672} = 2,160$ amperes per square inch.

INCHES OF SPACE REQUIRED FOR SECONDARY COIL BUILD IN DIRECTION OF WIDTH OF WINDOW

	Inches
Thickness of three copper strips (3×.080)	.240
Double cotton covering on two strands	.030
Paper and mica insulation between turns	.040
Height of each insulated turn	.310
Height of twenty turns (.310×20)	6.200
Coil taping	.062
Height of complete coil	6.262
Space allowance to steel outside of coil	1.325
Space allowance to steel inside of coil	1.325
Height of wire space	8.912

INCHES OF SPACE REQUIRED FOR SECONDARY COIL BUILD IN DIRECTION OF LENGTH OF WINDOW

	Inches
Width of copper strip	.280
Double cotton covering (two sides of a strip)	.015
Width of strip with double cotton covering	.295
Coil taping	.062
Width of coil	.357
Total width of space occupied by four secondary coils (.357×4)	1.428

60. The length of the window in the core may now be determined.

LENGTH OF WINDOW	Inches
Space occupied by secondary coils	1.428
Space occupied by primary coils	4.456
Total space between groups of primary coils and adjacent secondary coils, puncture curve, Fig. 10, 132,000 volts, approximate allowance 1.75 inches (1.75×4)	7.000

Total space for six oil ducts between primary coils (.375×6)	2.250
Space for one oil duct between two central secondary coils	.375
Total space between end secondary coils and core (1.325×2)	2.650
Total	18.159

The window dimensions will be, therefore, 18¼ inches by 9 inches, as indicated in Fig. 14. The ratio of the window dimensions will be approximately as 2 to 1, which is satisfactory.

61. The product of the flux in megalines, and the low-voltage turns, Art. **52**, was calculated as 862.5 and the number of turns selected was 80, therefore the total flux will be $\frac{862.5}{80}$ $=10.78$ megalines, or 10,780,000 lines. Assuming a flux density of 80,000 lines per square inch, the cross-section of steel will be $\frac{10{,}780{,}000}{80{,}000}=134.8$ square inches, which corresponds to a gross cross-section of $\frac{134.8}{.9}=150$ square inches, approximately.

In Fig. 14 the steel portion of the two-part core within the coils has a total width of c inches; each part being $\frac{c}{2}$ inches. The depth of the core is d inches. In good design, the ratio d to c varies from 2.5 to 1, to 3 to 1. The cross-sectional area $c\,d$ should equal 150 square inches, approximately. The rectangular punchings selected will be 3.5 inches in width, therefore $\frac{c}{2}=3.5$ inches and $c=7$. The depth d selected will be 21 inches; and $c\,d=147$ square inches, gross, or $147\times.9$ $=132.3$ square inches, net. The flux density will be $\frac{10{,}780{,}000}{132.3}$ $=81{,}500$ lines, or 81.5 kilolines per square inch. The core loss at this density (Fig. 3) will be 1.44 watts per pound.

62. To build the core, the rectangular steel strips are arranged as indicated in Fig. 14. An oil duct $\frac{1}{2}$ inch wide will be provided between the halves of the core within the center space of the coils. In assembling these laminations, each layer must be laid on the one preceding in such a manner that the joints do not come in the same place. This interleaving of the ends of the laminations strengthens the core and reduces the reluctance of the magnetic circuit.

63. The net contents of the whole core for the three sets of windings can be calculated from the dimensions indicated in Fig. 14 and the factor .9; thus: $.9 \times 21 \times (68.75 \times 3.5 \times 4 + 9 \times 3.5 \times 8) = 22{,}954$ cubic inches. At .275 pound per cubic inch, the core will weigh $22{,}954 \times .275 = 6{,}310$ pounds, approximately. The total core loss will be $6{,}310 \times 1.44 = 9{,}090$ watts, which is .91 per cent. of the 1,000,000-watt rating of the transformer.

64. To find the mean length of the turns in a coil, the dimensions of the rectangular opening inside the coil, Fig. 15, and the build of the coil must be known. The core inside the coil is 21 inches by $7\frac{1}{2}$ inches (including the $\frac{1}{2}$-inch oil duct); the wooden wedges are 3 inches by $7\frac{1}{2}$ inches; the clearance between the inner core and the inside of the primary and the secondary coils is approximately 1.625 inches and 1.325 inches, respectively. The opening inside the primary coil is, therefore, $21+6=27$ inches high, and $7.5+2\times 1.625=10.75$ inches wide; in the secondary coil these dimensions are 27 inches high and $7.5+2\times 1.325=10.15$ inches wide. The build of the insulated primary turns is 4.6 inches, and of the insulated secondary turns is 6.2 inches. The mean length of turns in a primary coil is $2(27+10.75+4.6\,\pi)=104.4$ inches; in a secondary coil $2(27+10.15+6.2\,\pi)=113.3$ inches.

The length of the high-voltage conductor will be $\frac{104.4\times 1{,}326}{12}$ $=11{,}536$ feet per phase; and of the low-voltage conductor, $\frac{113.3\times 80}{12}=755$ feet per phase.

65. The resistance of the high-voltage conductor will be $\frac{9.94 \times 11{,}536}{10^6 \times .0034} = 33.73$ ohms per phase; of the low-voltage conductor, $\frac{9.94 \times 755}{10^6 \times .0672} = .112$ ohm per phase.

The resistance drop in per cent. of normal voltage for the high-voltage conductor will be $\frac{33.73 \times 8.77}{38{,}100} \times 100 = .776$ per cent.; for the low-voltage conductor, $\frac{.112 \times 145.22}{2{,}300} \times 100 = .707$ per cent. The total resistance drop in percentage, also the per cent. copper loss, will be $.776 + .707 = 1.483$ per cent.

The total copper and steel loss, expressed in percentage, will be $1.483 + .91 = 2.393$ per cent., and the efficiency of the transformer will be $\frac{100}{100 + 2.393} \times 100 = 97.6$.

The copper space factor will be $\frac{1{,}326 \times .0034 + 80 \times .0672}{18.25 \times 9} = .06$.

66. The reactance of the primary circuit can be calculated by the formula of Art. **17**, in which the letters will have values as follows: $f = 60$ cycles; $p = 4$, there being four 1.75-inch spaces between primary and secondary coils of each phase; N = approximately one-fourth of the total number of primary turns, or 332, this number lying adjacent to each secondary coil; $l = \frac{104.4 + 113.3}{2 \times 12} = 9.1$ feet; $l_1 = 9$ inches, the width of the wire space, Fig. 14; t_1 = width of two primary coils and the oil duct between them, or $.557 \times 2 + .375 = 1.489$ inches, $t_2 = .357$ inch; and $t = 1.75$ inches.

$$X = 275 \times \frac{60 \times 4 \times 332^2 \times 9.1}{10^8 \times 9} \times \left(\frac{1.489 + .357}{3} + 1.75\right) = 174 \text{ ohms}$$

The reactance drop, in percentage of primary voltage, will be $\frac{174 \times 8.77}{38{,}100} \times 100 = 4$ per cent.

The impedance drop, in percentage, will be $\sqrt{1.483^2 + 4^2} = 4.27$ per cent.

The regulation, in percentage (Art. **35**), will be $1.483+\frac{4.27^2}{200}$ $=1.574$ per cent.

67. The transformer will be placed in a rectangular boiler-plate tank made to fit the core, Fig. 14, with allowance for copper clearance. The distance between the core and the side of the tank will be 4 inches. The ends of the tank will be curved and will extend 15 inches beyond the ends of the core.

The dimensions of the tank will be: width, $32\frac{1}{2}+8 = 40\frac{1}{2}$ inches; length, $68\frac{3}{4}+30=98\frac{3}{4}$ inches. The height of the tank will be 10 feet, which allows space for the core, the windings, the connection board, and the cooling coils. The effective cooling surface of the tank will be 29,000 square inches, approximately, and this will be capable of dissipating heat at the rate of .25 watt $\times$ 29,000 square inches = 7,250 watts. Subtracting this value from the total number of watts transformed into heat in the transformer, $1{,}000{,}000\times.02393=23{,}930$ watts, gives $23{,}930-7{,}250=16{,}700$ watts, approximately, to be dissipated by the cooling coils. A wrought-iron cooling coil with an inside diameter of $1\frac{1}{2}$ inches will be used. This coil can absorb heat at the rate of 1 watt per square inch of inside surface; therefore, 16,700 square inches of cooling surface will be provided. The total length of the cooling coil will be $\frac{16{,}700}{1.5\times3.1416\times12}=295$ feet.

With a permissible rise in temperature of the cooling water of 10° C., 1 gallon of water will absorb heat dissipated at the rate of 2,640 watts per minute. The total rate of flow of water will be $\frac{16{,}700}{2{,}640}=6.3$ gallons per minute.

DESIGN OF ALTERNATING-CURRENT MACHINES
(PART 1)

GENERAL FEATURES OF STATIONARY ARMATURES

WINDING DISTRIBUTION AND PITCH

WINDING DISTRIBUTION

1. A thorough understanding of the design of direct-current machines is an aid in learning to design alternating-current machines, the magnetic circuits and many of the mechanical details of both classes of machines being calculated in much the same way. Differences in design of machines of both classes are due to the experience and taste of different designers and the duty required of the machines. In general, the design that gives the best results at the lowest cost is the best.

2. Concentrated Winding.—When an alternating magnetic flux Φ is made to vary through a number of turns T connected in series, at the rate of f cycles per second, the effective electromotive force generated is

$$E = \frac{4.44\,\Phi\,T\,f}{10^8}$$

This general formula assumes that the generated electromotive force follows a sine wave and that all turns are active at the same instant; that is, they are all cut simultaneously by

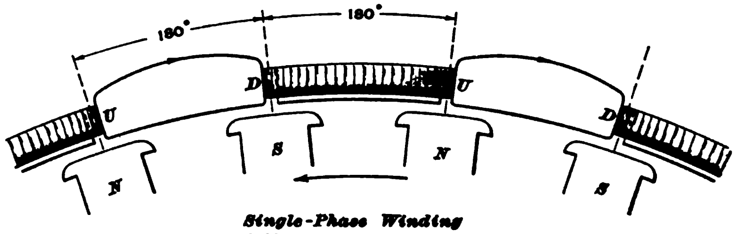


FIG. 1

the flux. For example, Fig. 1 shows a portion of a single-phase armature winding with all the turns of each coil concentrated in one slot per pole. These slots are 180 electrical degrees apart, so that all the turns of each coil are active at the same instant. In this case the armature is assumed to be stationary and the poles NS to move counter-clockwise. When the centers of the poles are adjacent to sides of coils as shown, the direction of generated electromotive force is upwards, that is, toward the reader, in conductors U adjacent to N poles and downwards in conductors D adjacent to S poles. The armature coils are connected in series and the total electromotive force generated in them is as given by the foregoing formula.

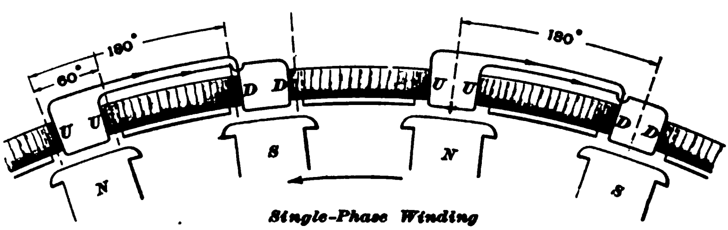


FIG. 2

3. Single-Phase Distributed Winding.—If the winding is subdivided or distributed, the electromotive forces in all turns of a given coil do not reach their maximum value at the

same instant, and, for a given number of turns, the electromotive force is less than is obtained with a concentrated winding.

Fig. 2 shows part of a single-phase winding with two slots per pole, the centers of the slots being 60 electrical degrees, or one-third pole pitch, apart. The electromotive forces induced in the conductors of the two coils under a given pole will therefore differ in phase by 60°. If alternate coils were connected in series so as to form two sets of coils and if each coil contains one-half as many turns as a coil of Fig. 1, then

$$E_1 = E_2 = \frac{4.44\ \Phi\ T f}{10^8} \times \frac{1}{2}$$

in which $E_1 = E_2$ = electromotive force induced in each of the two sets of coils, Fig. 2;

T = number of turns in both sets of coils, which is the same as the number of turns in the winding shown in Fig. 1.

If all the turns shown in Fig. 2 are connected in series, the total electromotive force E will be the resultant found by

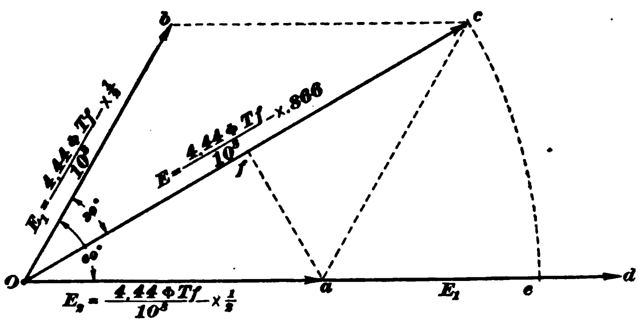


FIG. 3

combining the electromotive forces of each set, as shown in Fig. 3, where $O\,a$ represents the electromotive force E_2 in one set and $O\,b = E_1$ in the other, the two differing in phase by an angle of 60°.

The resultant of the two forces is $O\,c$, which represents the total electromotive force E. The value of this resultant in

terms of the component $O\,a$ can be found by drawing the line $a\,f$ perpendicular to $O\,c$; then,

$$O\,f = \tfrac{1}{2}\,O\,c = O\,a\cos 30° = \left(\frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{2}\right)\cos 30°$$

$$E = 2\left(\frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{2}\right)\cos 30° = \frac{4.44\,\Phi\,T\,f}{10^8}\times .866$$

If the winding were not distributed, the total electromotive force would be $O\,d = O\,a + a\,d$, $a\,d$ being made equal to $O\,b$. The resultant $O\,c = O\,e$; hence, the reduction in voltage by distributing the winding is $e\,d = 1 - .866 = .134$, or 13.4 per cent.

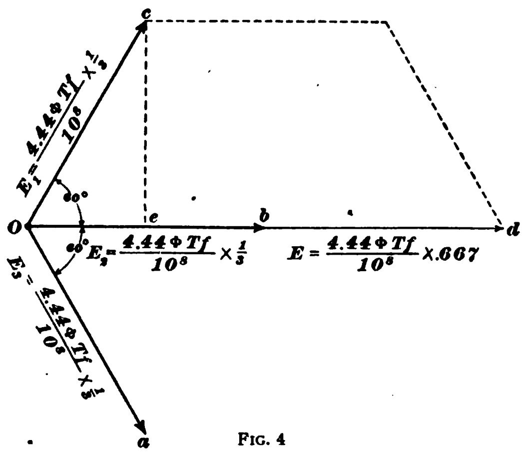


FIG. 4

If the winding were subdivided into three slots per pole, and the total number of turns kept the same as before, the resultant electromotive force would be found as shown in Fig. 4. The three electromotive forces E_1, E_2 and E_3 differ 60° from one another in phase, and each equals $\frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{3}$. The vector representing the resultant electromotive force $O\,d$ can be found by drawing $C\,e$ perpendicular to $O\,b$, thus bisecting $O\,b$. $O\,e = \left(\frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{3}\right)\cos 60°$ and $O\,d = O\,b + 2\,O\,e$. Therefore,

$$E = \frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{3} + 2\left(\frac{4.44\,\Phi\,T\,f}{10^8}\times\frac{1}{3}\right)\cos 60° = \frac{4.44\,\Phi\,T\,f}{10^8}\times .667$$

In this case the reduction caused by distributing the winding is 33.3 per cent. (100−66.7).

4. Polyphase Distributed Windings.—In polyphase windings, the distribution of the coils in two or more slots per phase always reduces the electromotive force for a given number of turns, but since the distance between slots is not large, the reduction in voltage is not very great. For example, in a three-phase winding with two slots per pole per phase there will be six slots per pole and the slots will be 30 electrical degrees apart. The electromotive forces E_1 and E_2 of each group of coils will therefore be combined as shown in Fig. 5,

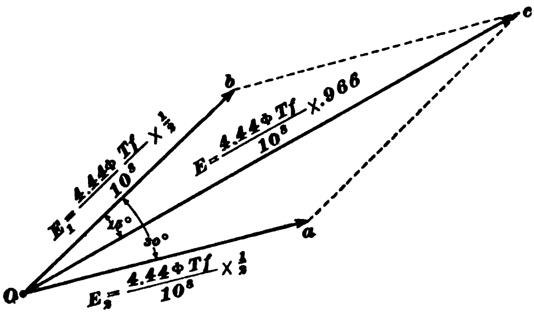


FIG. 5

and if there are T turns per phase, the resultant, obtained as explained in connection with Fig. 3, will be

$$O\,c=2\left(\frac{4.44\ \Phi\ T\,f}{10^8}\times\frac{1}{2}\right)\cos 15°=\frac{4.44\ \Phi\ T\,f}{10^8}\times.966$$

The arrangement of the winding in two slots per pole per phase has therefore reduced the voltage, for a given total number of turns, only 3.4 per cent. (100−96.6).

5. In order to take account of the winding distribution in any given case, the electromotive force given by the formula of Art. **2** for a single slot or concentrated winding must be multiplied by a number, called a *winding-distribution factor*, that corrects for the distribution For any winding, the general formula is therefore written as follows:

$$E_p = \frac{4.44\ \Phi\ T_p f}{10^8} k_w$$

in which E_p = effective volts generated per phase;
Φ = flux per pole;
T_p = turns in series per phase;
f = frequency, in cycles per second;
k_w = winding-distribution factor.

Table I gives the value of k_w for a few of the more common types of winding. These values are obtained, for any given winding, as previously explained. When there is only one slot per pole per phase, the value of k_w is unity.

TABLE I

VALUES OF WINDING-DISTRIBUTION FACTOR k_w

Style of Winding	Value of k_w
Single-phase, with two slots per pole (slots 60° apart)	.866
Single-phase, with three slots per pole (slots 60° apart)	.667
Two-phase, two slots per pole per phase	.924
Two-phase, three slots per pole per phase	.911
Two-phase, four slots per pole per phase	.906
Three-phase, two slots per pole per phase	.966
Three-phase, three slots per pole per phase	.960
Three-phase, four slots per pole per phase	.958
Three-phase, five slots per pole per phase	.957

6. Distributing a winding in more than four slots per pole per phase causes very little reduction in the value of k_w. If a single-phase winding is made by taking two-phase armature punchings and using only half the slots, that is, using the slots of only one phase, the value of k_w is the same as given for two-phase windings. In the same way, if the slots for only one phase of a three-phase armature are used and two-thirds of the slots left empty, the value of k_w is the same as for three-phase windings. If, however, two-thirds of the slots for a

three-phase winding are filled to form a single-phase winding of T_p turns, the corresponding three-phase winding would have $\frac{1}{2}$ T_p turns per phase and would generate electromotive forces differing in phase by 120°. The electromotive force of each phase of the three-phase winding would be

$$E_1=\frac{4.44\ \Phi\ \frac{1}{2}\ T_p f}{10^8}k_w$$

and the resultant of these two phases connected in series would be

$$E=E_1\sqrt{3}=\frac{4.44\ \Phi\ \frac{1}{2}\ T_p f}{10^8}k_w\times\sqrt{3}$$

Therefore, the single-phase voltage

$$E=\frac{4.44\ \Phi\ T_p f}{10^8}k_w\times\frac{\sqrt{3}}{2}=\frac{4.44\ \Phi\ T_p f}{10^8}k_w\times .866$$

Hence, if a single-phase winding is made by using two-thirds of the slots of a three-phase winding, the values of k_w as given for the three-phase winding must be multiplied by $\frac{\sqrt{3}}{2}=.866$. For example, the single-phase winding shown in Fig. 2 is in two slots per pole, the slots being 60° apart. If this armature had another slot per pole, it would answer for a three-phase winding with one slot per pole per phase, so that the arrangement of coils is the same as if a three-phase armature were utilized and one-third of the slots left empty. The three-phase armature would have one slot per pole per phase, and k_w would be 1. A single-phase winding using two-thirds of these slots would therefore have a winding factor of .866, or the same as shown in Fig. 3. Single-phase machines are frequently made by using existing two- or three-phase armature punchings, and the foregoing remarks regarding the winding factor in such cases are important.

WINDING PITCH

7. In short-pitch, or fractional-pitch, windings, the electromotive forces induced in the two sides of a coil are not in phase, since the sides of the coil are less than a pole pitch, or

180 electrical degrees, apart. The resultant electromotive force is therefore less for a short-pitch winding than for a similar full-pitch winding, and the electromotive force given by the general formula of Art. 2 must be multiplied by a *winding-pitch factor* to obtain the voltage generated in fractional-pitch windings. This pitch factor is the cosine of one-half the difference between the full pitch and the winding pitch and may be represented by k_o; that is, $k_o = \cos \frac{1}{2}(180°$ minus the coil pitch).

The general formula for determining the electromotive force per phase, taking into consideration both distribution and pitch, is, therefore,

$$E_p = \frac{4.44\ \Phi\ T_p f}{10^8} k_w k_o$$

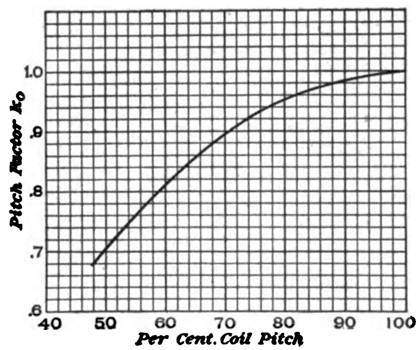


FIG. 6

Values of k_o are given by the curve, Fig. 6. Coil pitch is usually stated in per cent. of full pitch, and the curve is plotted accordingly. A coil has full pitch when its sides span the full number of slots per pole; thus, if there are eight slots per pole, full slot pitch is from slot 1 to slot 9. Coils lying in slots 1 and 8 therefore include seven slots and have $\frac{7}{8}$, or $87\frac{1}{2}$ per cent., of full pitch.

EXAMPLE.—A 60-cycle 3-phase alternator has nine slots per pole and a flux per pole of 2,000,000 lines. The coil pitch is from slot 1 to slot 8. If there are 180 turns per phase, what is the electromotive force generated per phase?

SOLUTION.—There are nine slots per pole, or three slots per pole per phase; hence, $k_w = .96$ (Table I). In a full-pitch coil the slot pitch is from slot 1 to slot 10 and spans nine slots, whereas in this winding the slot pitch is from 1 to 8 and spans seven slots. Hence, the per cent. slot pitch is $\frac{7}{9} = 77.8$ per cent., and from Fig. 6, $k_o = .94$. $\Phi = 2{,}000{,}000$; $T_p = 180$, and $f = 60$. Therefore,

$$E_p = \frac{4.44 \times 2{,}000{,}000 \times 180 \times 60}{10^8} \times .96 \times .94 = 865 \text{ volts}$$

WINDING CONNECTIONS

8. The common methods of **interconnecting the phases of a three-phase winding** are shown in Fig. 7; (*a*) (*b*), and (*c*)

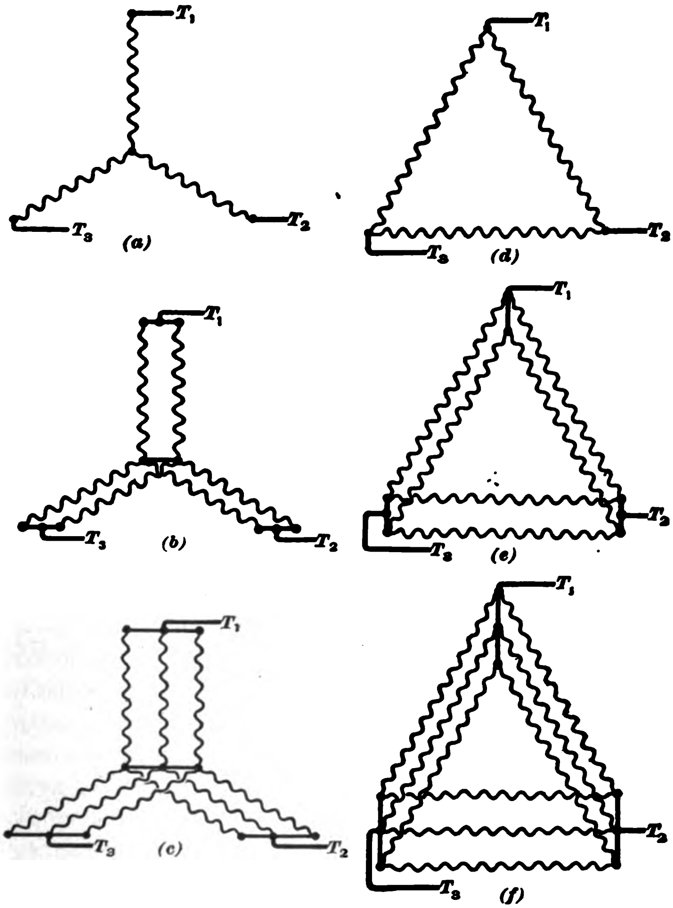


FIG. 7

are Y connections, and (*d*), (*e*), and (*f*) are Δ connections. Connections (*a*) and (*d*) are most commonly used, but cases sometimes arise where it is necessary to use the others,

particularly with low-voltage machines having large current output. In (*a*), the coils of each phase are connected in series and the three phases are connected to form a single-circuit Y winding; (*d*) is a single-circuit Δ winding. As a general rule, when the conditions permit its use, the Y method of connection is preferable to the Δ method. For a given electromotive force between line terminals T_1, T_2, T_3, the Y-connected armature requires fewer turns per phase than the Δ-connected armature; hence, there is less slot space occupied by insulation. Again, with the Δ connection, the phases form a closed circuit within the machine, and if the electromotive forces are unbalanced, a local current may circulate in the armature, thus causing undue heating of the conductors.

9. The method of connection to be used in any given case is decided largely by the voltage and the current capacity required. Generally there is little difficulty in using the single-circuit Y connection for fairly high-voltage armatures; but an alternator may have so many poles and slots that the voltage will be too high when all the conductors of a phase are connected in series, even with only one conductor per slot. In such a case, the Δ connection, Fig. 7 (*d*), or the two-circuit Y connection, view (*b*), should be used in preference to the single-circuit Y connection.

In the two-circuit Y connection, the coils of each phase are divided into two groups, each consisting of an equal number of coils connected in series, and the two groups are connected in parallel, thus halving the effective number of turns per phase but doubling the current-carrying capacity. In a similar manner, the coils can be connected in three parallel groups, as in (*c*), forming a three-circuit Y winding, provided the number of coils per phase is divisible by 3. The corresponding two-circuit and three-circuit Δ connections are shown in (*e*) and (*f*). Only those coils in which the electromotive forces are in phase can be connected in parallel on an armature; that is, the connected coils must occupy corresponding slots, and corresponding terminals must be connected together. Three-circuit windings are seldom required, but the two-circuit connections are often

very useful, especially where an existing frame with a fixed number of slots must be wound for a voltage considerably lower than that for which the machine was originally designed.

ARMATURE REACTION

10. Armature reaction in an alternator may distort the direction of magnetic flux and weaken the field so as to cause a drop in voltage and hence affect the regulation or maintenance of voltage with change of load. In order to estimate the regulation, it is necessary to know the demagnetizing effect that the armature exerts on the field.

Let N, S, Fig. 8, represent two poles of a revolving field and c, c' two groups of conductors forming a coil in the armature slots. With no current in the armature and with the fields excited, the magnetic field in the air gap will be nearly uniform over the pole faces, as indicated in (*a*) by the lines with arrowheads. The flux then passes from a north pole N into the armature and from the armature into a south pole S. In (*b*), the fields are unexcited, but electricity flows through the armature from some outside source, the direction of the current being downwards from the reader through conductors c and upwards through conductors c'. These armature currents set up magnetic lines of force, as indicated by the circular dotted lines.

11. With the fields excited as in (*a*) and rotated so as to set up current in c, c' in the same direction, as in (*b*), the fields shown separately in (*a*) and (*b*) will be superimposed, with the result that the flux is made more dense at the left-hand side of the poles and thinned out at the right-hand side, as shown in (*c*), because the directions of the two fluxes agree at the left-hand sides of the poles and are opposed to each other at the right-hand sides. In (*c*), assuming that the alternating current is in phase with the induced electromotive force, the current in the conductors is at the maximum value the instant the poles are in the position shown, because the conductors are then opposite the pole centers. Under these conditions the armature

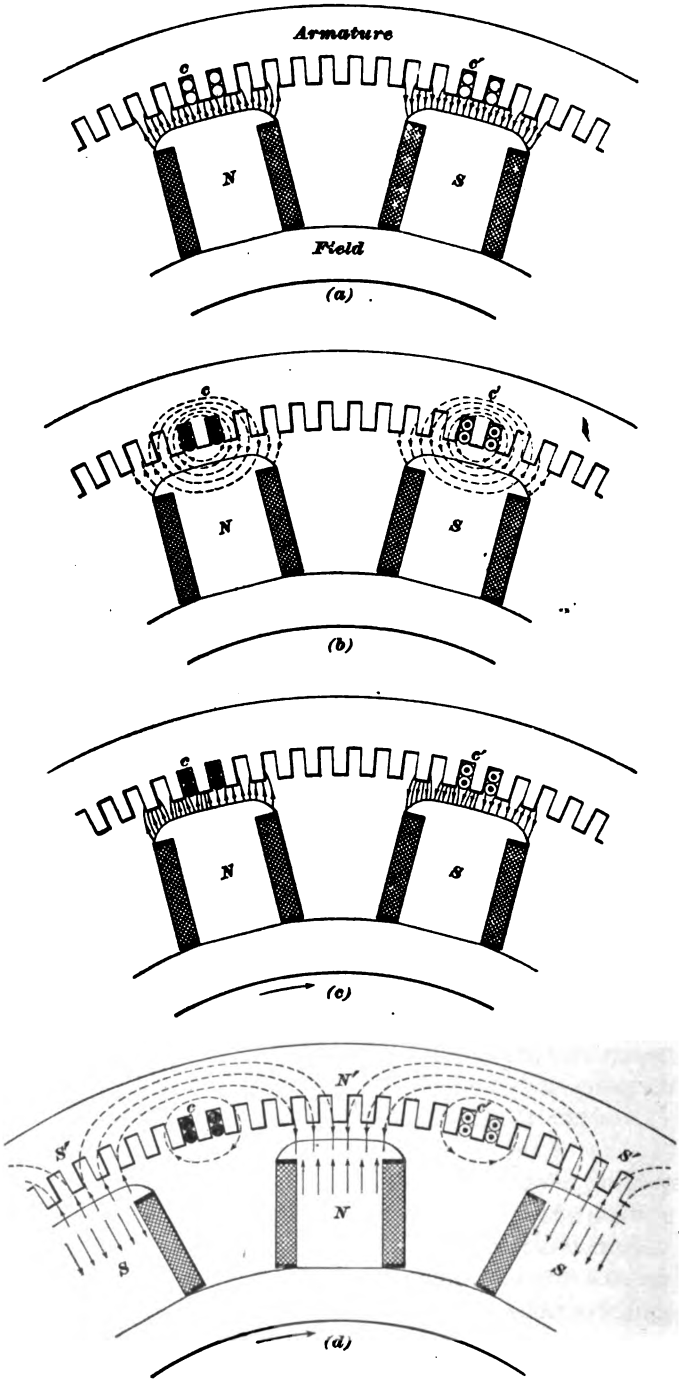


FIG. 8

reaction simply distorts the direction of the flux, but does not produce any appreciable weakening of it unless, as is not usual, the crowding of the lines causes such a high degree of saturation in the teeth as to choke back some of the lines.

If, however, owing to self-induction, the current in the armature lags behind the induced electromotive force, the magnetic effect of the armature currents tends to weaken the field. In (*d*), is indicated the extreme case where the armature current lags 90° behind the electromotive force; the current in conductors c, c' does not reach its maximum until pole N has passed around to a position midway between the two conductors, as shown. The induced electromotive force in c, c' at this instant is zero, but, on account of the 90° lag, the armature current tends to form poles N', S' that are squarely opposed to those on the field, and the armature thus exerts a powerful demagnetizing action, which, if not offset by a corresponding increase in the field excitation, will cause a decided weakening of the field and a drop in voltage.

On the other hand, if the armature current could be made to lead the electromotive force it would strengthen the magnetic field, but such a condition rarely occurs in practice. The maximum strengthening would occur with the current leading the electromotive force by 90°, in which case the direction of the flux set up by the armature current would agree with that set up by the field current.

12. The **total armature ampere-turns per pole** on an armature for a polyphase alternator can be calculated by means of the formula

$$I\,T_p = .707\,m\,T_{pp}\,I_p$$

in which $I\,T_p$ = armature ampere-turns per pole:
m = number of phases;
T_{pp} = turns per pole per phase;
I_p = current in armature turns.

13. The **demagnetizing armature ampere-turns per pole** can be determined approximately by multiplying the total armature ampere-turns per pole by the sine of the angle by

which the current lags behind the electromotive force. When this angle is 0, there is no demagnetizing action. At zero power factor, the angle of lag is 90°, its sine is 1, and all the armature ampere-turns oppose those of the field. This method is useful for making preliminary calculations but is not exact, as the demagnetizing effect of the armature current depends also on the distribution of the winding, on the winding pitch, and on the ratio of the *pole arc*, or circumferential width of pole face, to the *pole pitch*, or circumferential distance between pole centers. The following formula can be used to obtain more exact values after the style of winding and the shape of the poles have been determined:

$$D\,I\,T_p = .9\,m\,T_{pp}\,I_p\,k_w\,k_o\,k_p \sin a$$

in which $D\,I\,T_p$ = demagnetizing ampere-turns per pole;
a = angle of lag of the current;
k_p = a factor depending on the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$.

The other letters in the formula have the meanings previously given. Values of k_w are given in Table I, and of k_o in Fig. 6. Values of k_p for different ratios of pole arc to pole pitch are as follows:

When ratio is	.5	.6	.7	.8
k_p is	.9	.855	.81	.75

EXAMPLE 1.—A three-phase twelve-pole alternator, when fully loaded, delivers 50 amperes per line. The armature is Y-connected and has 108 slots, each containing 20 conductors. (*a*) How many ampere-turns per pole are supplied by the armature at full load? (*b*) If the angle of lag between the current and the induced electromotive force is approximately 30°, how many ampere-turns on the armature are opposed to the field?

SOLUTION.—(*a*) There are $\frac{108 \times 20}{12} = 180$ conductors per pole, or 60 per pole per phase. Then, $T_{pp} = 60 \div 2 = 30$; $I_p = 50$; and $m = 3$. Hence, by the formula of Art. 12,

$$I\,T_p = .707 \times 3 \times 30 \times 50 = 3{,}182, \text{ nearly. Ans.}$$

(*b*) The angle of lag $a = 30°$ and $\sin a = .5$; then, the approximate number of demagnetizing ampere-turns per pole on the armature is 3,182 $\times .5 = 1{,}591$. Ans.

EXAMPLE 2.—In the machine referred to in example 1, assume that it has been decided to make the ratio $\frac{\text{pole arc}}{\text{pole pitch}} = .6$, and the coil pitch from slot 1 to slot 8. Find by the more exact method the number of demagnetizing armature ampere-turns per pole.

SOLUTION.—The number of slots per pole is $108 \div 12 = 9$, and the number per pole per phase is $9 \div 3 = 3$. For a three-phase machine with three slots per pole per phase, $k_w = .96$ (Table I). The coil pitch is $\frac{7}{9}$, or 77.8 per cent. of full pitch, and by Fig. 6, $k_o = .94$; $k_p = .855$. The other quantities in the formula of this article have the values previously given; hence,

$$D\,I\,T_p = .9 \times 3 \times 30 \times 50 \times .96 \times .94 \times .855 \times .5 = 1{,}562. \quad \text{Ans.}$$

NOTE.—In this case the exact value, 1,562, is only 29 less than the approximate value.

14. In turbo-alternators with cylindrical rotors, the ratio of pole arc to pole pitch is 1, and if the winding is concentrated, as on a salient-pole machine, k_p may be taken as .6, making

$$D\,I\,T_p = .54\, m\, T_{pp}\, I_p\, k_w\, k_o \sin a \qquad (1)$$

If the field winding is distributed as will be explained later, this value must be multiplied by the ratio of the maximum number of field ampere-turns to the average number. This ratio may be taken as 1.5; hence, for a cylindrical rotor with a distributed winding

$$D\,I\,T_p = .81\, m\, T_{pp}\, I_p\, k_w\, k_o \sin a \qquad (2)$$

ARMATURE SELF-INDUCTION

15. All alternator armatures have self-induction, the amount in each case depending on the design of the armature. The self-induction of an alternator armature can be measured easily after the armature is completed, but to calculate it in advance is difficult. Self-induction increases the impedance of an armature, thus increasing the voltage drop, and also causes the armature current to lag, thus increasing the demagnetizing effect, or reaction.

In general, armatures wound with a few heavy coils embedded in large slots have a high self-induction, because the armature current is able to set up a large number of lines of force around the coils. Machines with this style of armature winding

usually give an electromotive-force curve that is more or less peaked and irregular. Such windings are easily applied to the armature and necessitate few crossings of the coils where they project at the ends. They are therefore used largely for high-voltage machines, because they admit of high insulation, and the percentage of slot space occupied by insulation is less than in a machine having a large number of coils.

16. The inductance of the slot portions of an armature coil is usually larger than the inductance of the end connections, unless the slot portions are short and the end connections unusually long. The self-induction of an armature can sometimes be reduced by increasing the number of slots and using

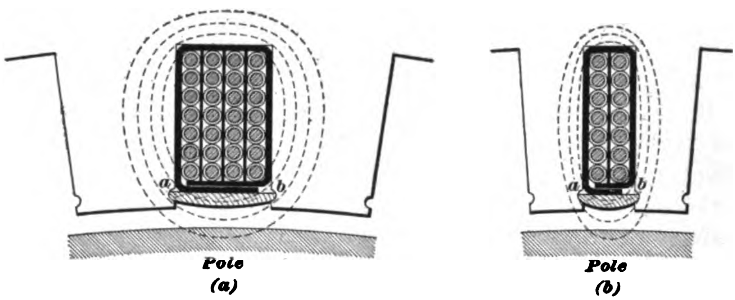


Fig. 9

fewer conductors in each slot, thereby reducing the inductance of the slot portions.

Fig. 9 (*a*) shows a cross-section of a slot containing a group of conductors forming one side of a coil of twenty-eight turns. Current in the coil sets up around it magnetic flux, the value of which depends on the strength of the current and on the reluctance of the path indicated by the dotted lines around the sides of the coil. The self-induced electromotive force in the slot portion of the coil depends on this flux and on the square of the number of turns per coil or conductors per slot.

The inductance can, therefore, be decreased by splitting the coil into two or more sections and placing them in separate slots, thus reducing the number of conductors per slot. For example, an armature with ten twenty-eight-turn coils, each

having an inductance of .01 henry, will have a total inductance of $10 \times .01 = .1$ henry. However, if the winding is split up into twenty coils of fourteen turns each, the shape and general arrangement of the coils being kept the same, the reluctance of the local magnetic paths the same, and the total number of turns the same, each coil will have only half as many turns per coil or half as many conductors per slot. Hence, the inductance of each coil will be one-fourth of what it was before, or $\frac{1}{4} \times .01 = .0025$ henry, and the total inductance will be $.0025 \times 20 = .05$ henry, or one-half that in the former case.

17. In the example just given, it was assumed that the reluctance of the path around the coil is the same for the heavy coils as for the light one. This, however, is not the case in practice, and the reduction of inductance by subdividing the winding is not as great as the example indicates. In Fig. 9 (*a*), the greater part of the reluctance occurs at the air gaps around the openings of the slots, as between *a* and *b*, and with a wide, shallow slot, the reluctance between the sides is considerably larger than with a deep, narrow one. When the coils are subdivided, it is necessary to use rather deep, narrow slots, as shown in (*b*), and the reluctance between *a* and *b* is much less than with the wider slot. The result is that the decrease in the number of conductors per slot may be largely offset by the decreased reluctance. Therefore, splitting up the winding reduces the total inductance somewhat less than in direct proportion to the reduction in the number of turns per coil, but is advisable for machines in which low armature inductance and close regulation are desirable. Another important reason for distributing the windings is that the wave shape is thereby made more nearly a smooth sine wave than can be obtained with a concentrated winding.

18. The inductance of the end connections of the coils is reduced by the use of short-pitch coils, because the lengths of these parts are thereby reduced. The flux around the free coil ends is reduced in proportion to the length of the coil ends, and these, in turn, decrease in proportion to the decrease in percentage of the coil pitch.

19. Short-Circuit Current.—If an alternator is run at normal speed with its armature terminals short-circuited, the field excitation necessary to establish full-load current in the armature will be only a small part of that required for normal voltage. Under such conditions, the electromotive force induced in the winding divided by the short-circuit current will give the **synchronous impedance**, that is, the apparent impedance of the armature. The current in the armature, when short-circuited in this way, lags greatly behind the induced electromotive force; the power factor is practically zero, and the armature ampere-turns are directly opposed to those on the field. The ampere-turns on the field must overcome the armature demagnetizing ampere-turns and also set up enough flux to induce an electromotive force sufficient to establish full-load current in the armature against its resistance and self-induction. Since the field required to do this is very small, the iron parts are unsaturated; and, within the current limit at which it is safe to operate the machine without overheating, the graph is a straight line showing the relation between short-circuit current and field ampere-turns, as shown at *a a*, Fig. 10. This line is known as the **short-circuit line**, or the **short-circuit current characteristic.**

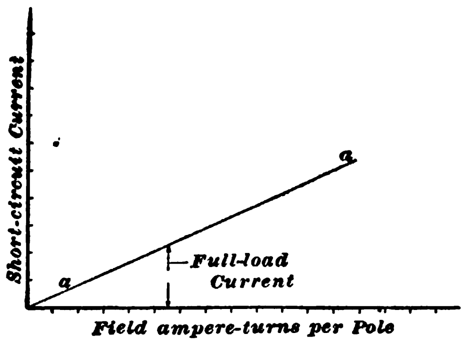


FIG. 10

20. A knowledge of the short-circuit line is useful in calculating the regulation. In most cases, this line can be drawn with a fair degree of accuracy by calculating one point according to the formula

$$I_p = \frac{I_f\, T_f}{k_c\, m\, T_{pp}}$$

in which I_p = short-circuit armature current per phase;
$I_f T_f$ = ampere-turns per pole on the field;
m = number of phases;
T_{pp} = turns per pole per phase on the armature;
k_c = a factor depending on the type of machine = .75 to .9 for ordinary two- and three-phase revolving-field alternators. This factor is readily determined by tests for any class of machines, after which it can be used in calculating others of the same class.

EXAMPLE.—A three-phase alternator has sixteen poles and ninety-six slots, with six conductors per slot. What short-circuit current will be established in the armature when the excitation per pole is 800 ampere-turns?

SOLUTION.—There are six slots per pole and $6 \times 6 = 36$ conductors, or 18 turns, per pole, and $T_{pp} = 6$; $I_f T_f = 800$; $m = 3$; and k_c can be taken as .9; then, according to the formula,

$$I_p = \frac{800}{.9 \times 3 \times 6} = 49.4 \text{ amperes, approximately. Ans.}$$

VOLTAGE REGULATION

REGULATION DIAGRAMS

21. Non-Inductive Load.—The regulation of an alternator is usually expressed in per cent. increase of terminal voltage as the current is decreased from full-load value to zero, full-load voltage being taken as the base. Fig. 11 serves to explain the effect of self-induction on the terminal voltage when the load is non-inductive, as is the case when the current is used in incandescent lamps. Let $O\,b$ represent the terminal voltage E, and $O\,a$ the current I in phase with it. The voltage drop in the armature, due to resistance, is $I\,R_a = O\,c$, which is also in phase with the current and electromotive force. The vector $O\,d$ represents the electromotive force required to overcome the self-induction of the armature. This vector is drawn 90° ahead of the current vector $O\,I$ because the electromotive force of self-induction is always 90° behind the current.

The resultant of $O\,c$ and $O\,d$ is $O\,e$, which represents the electromotive force required to overcome the impedance of the armature. Vector E_1 representing the induced electromotive force is the resultant of vectors $O\,b$ and $O\,e$. Though the load is non-inductive and the current is in phase with the terminal electromotive force, the self-induction of the armature causes the current to lag behind the induced electromotive force E_1 by the angle α, and consequently the armature current exerts a demagnetizing effect on the field.

22. The induced electromotive force in an armature coil is proportional to the rate of change in the number of lines of

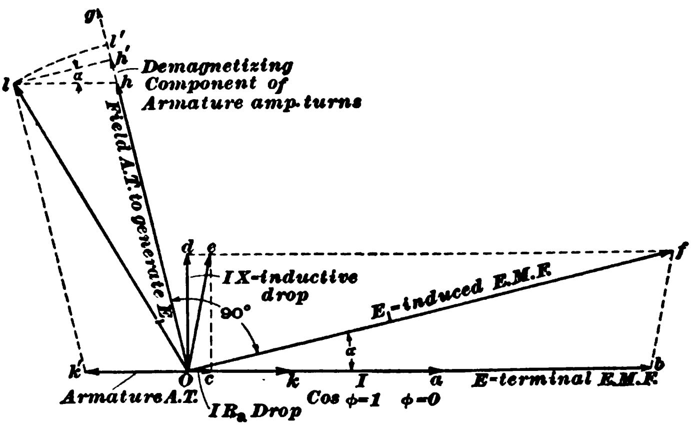


FIG. 11

force passing through the coil, and not to the total magnetism linked by the coil. For example, in Fig. 8 (*a*), the effective number of lines of force passing through the coil $c\,c'$ at the instant represented is zero, because an equal number is passing in the opposite direction, but as the poles move on the rate of change in the number of lines is at its greatest; hence, the electromotive force induced in the coil at this instant is maximum. At the position shown in (*d*), a maximum number of lines is passing through the coil, but the rate of change in this number is least; hence, the electromotive force is minimum. In short, when the resultant field flux through the coil is least, the induced electromotive force is greatest; and when the

resultant flux is greatest, the induced electromotive force is least—that is, the field flux is 90° ahead of the induced electromotive force, or in the direction $O\,g$, Fig. 11.

The field ampere-turns may be assumed proportional to the flux and hence can be laid off along $O\,g$ to any convenient scale. Let $O\,h$ represent the field ampere-turns necessary to induce electromotive force E_1 when the armature current is zero; then, when current is established, the total ampere-turns required on the field are $O\,h$ plus the number required to offset the demagnetizing effect of this current. The ampere-turns on the armature will be in phase with the armature current, and

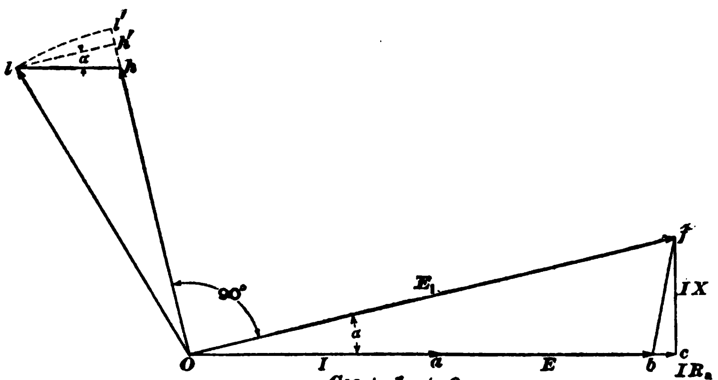


Fig. 12

can be represented to scale by $O\,k$ in phase with $O\,b$. The ampere-turns required to overcome the armature reaction will be $O\,k'$, equal and opposite to $O\,k$. The total ampere-turns required on the field to induce the electromotive force E_1 and to overcome the armature reaction $O\,k$ is represented by vector $O\,l$, found by completing the parallelogram $O\,h\,l\,k'$ and drawing its diagonal. The component of the armature reaction parallel with the field is $h\,h'$, found by drawing $l\,h'$ perpendicular to $O\,g$; $l\,h = O\,k'$ = armature ampere-turns, and it can be shown that angle $h'\,l\,h = a$. The demagnetizing component of the armature ampere-turns is therefore

$$h\,h' = l\,h \sin a = \text{armature ampere-turns} \times \sin a$$

23. The excitation $O\,h$ alone is not sufficient to obtain an induced electromotive force E_1 with current in the armature, but if the field ampere-turns are increased to $O\,l' = O\,l$, the added excitation will be more than sufficient to overcome the demagnetizing component $h\,h'$.

If the excitation is adjusted to give E volts at the machine terminals when it is loaded and the load is then thrown off, the voltage at the terminals will at once rise to that corresponding to an excitation $O\,l$, with the armature on open circuit.

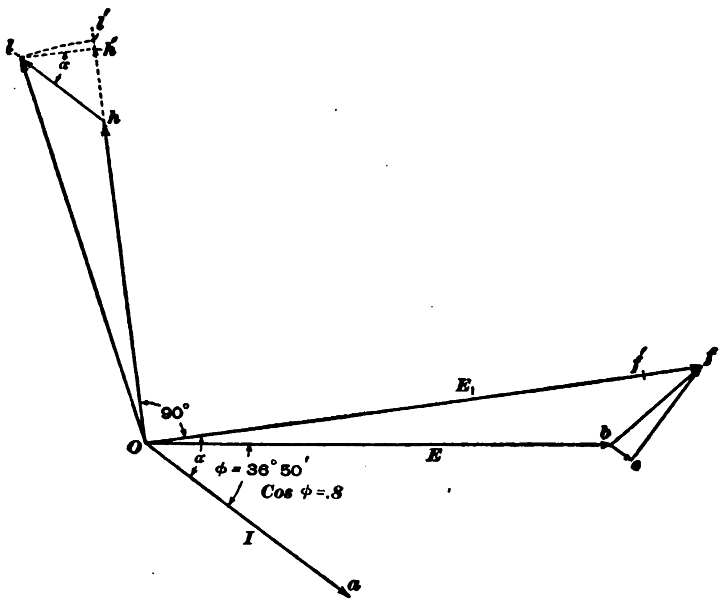


Fig. 13

Fig. 12 is the same as Fig. 11 except that a number of the lines have been omitted and triangles used instead of parallelograms in order to simplify the figure. The armature-drop triangle $O\,e\,c$, Fig. 11, has been transferred to position $b\,f\,c$, Fig. 12, but the lengths of the various lines in the two figures are the same.

24. Inductive Load.—In Fig. 13, the diagram is for an inductive load with a power factor of .8. The current and the

terminal voltage have the same values as before but the current now lags 36° 50′ (cos 36° 50′ = .8, approx.) behind the terminal voltage. Vector $O\,a$ is therefore drawn 36° 50′ behind vector $O\,b$. The resistance drop is always in phase with the current; therefore, $b\,c$ must be drawn parallel with the current vector $O\,a$, while $c\,f$, representing the electromotive force to

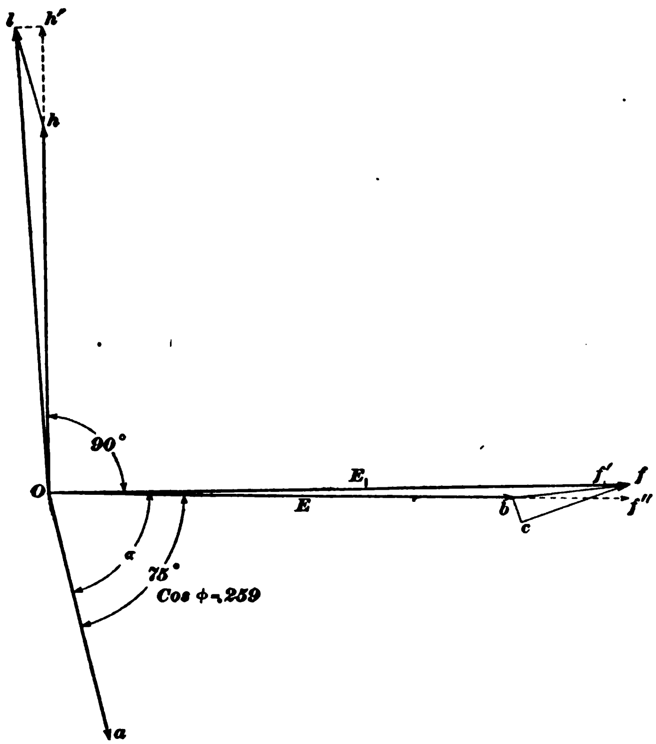


FIG. 14

overcome the drop due to the inductance of the armature, is at right angles to the current and hence to $b\,c$. The triangle $b\,f\,c$ is therefore swung around to the position shown, and the total induced electromotive force E_1 is represented in value and in phase relation by vector $O\,f$. Vector $O\,f'$ is equal to $O\,f$, Fig. 12; that is, reducing the power factor from 1 to .8 necessitates increasing the induced electromotive force by

an amount represented by $f' f$, Fig. 13, in order to maintain the same terminal voltage E. The excitation at no load is represented by vector $O h$ perpendicular to $O f$, and the armature ampere-turns by vector $h l$ parallel to the current vector $O a$, because the armature ampere-turns are in phase with the current. The demagnetizing component, represented by $h h'$, is much larger than with the non-inductive load, and the field excitation vector is therefore increased to $O l' = O l$; if the load is thrown off, the voltage at the terminals rises to that corresponding to an excitation $O l$ with the armature on open circuit.

25. Fig. 14 is a diagram showing the regulation of an alternator when carrying an inductive load with a power factor of .259; that is, the angle of lag is 75°. Vector $O f$ represents the induced electromotive force E_1, and $O f'$ is equal to $O f$, Fig. 13, so that the decrease in power factor from .8 to .259 has necessitated an increase in E_1 only by the amount $f' f$, Fig. 14, for the same terminal voltage E. The decrease from power factor 1 to power factor .8 made the difference $f' f$ in Fig. 13. When the power factor becomes low, say below .25, a further reduction makes very little difference in the induced electromotive force required to maintain a given terminal voltage. Moreover, with a very low power factor, the armature ampere-turns become so nearly in direct opposition to the field ampere-turns that the excitation must be increased by very nearly the full number of armature ampere-turns; that is, vector $h h'$ is very nearly equal to $h l$, and $O h'$ to $O l$. Likewise, the induced electromotive force E_1 is so nearly in phase with the terminal voltage E that vector $O f$ can be drawn in position $O f''$ with small error, $b f''$ being equal to $b f$. Because of these facts the most convenient load for which to estimate the regulation is that having a very low power factor, or approximately a zero power factor. After the regulation for this load has been determined, the approximate regulation for loads of other power factors may be estimated, as will be explained.

WAVE FORM OF ELECTROMOTIVE FORCE AND FLUX DISTRIBUTION

26. A sine-wave form of the electromotive-force curve has been assumed in all the preceding calculations. In order to obtain an electromotive force of such form the rate of cutting lines of force by each armature conductor must vary according to a sine curve, and this may be brought about by making the flux most dense at the pole centers, and decreasing the density toward the pole tips In other words, the flux distribution must approach a sine-wave form. Such distribution can be obtained by chamfering the pole faces, that is, by shaping them so that the length of the air gap is least at the pole center, increasing gradually toward either edge.

27. In order to obtain a smooth form of voltage curve, the stator slots must be closed. Open stator slots cause the lines of force to enter and leave the armature in tufts, but the influence of these tufts can be minimized by making the number of stator slots such that the number of teeth opposite a pole is always the same whether the pole center is opposite a tooth or a slot. With such proportions the pole is said to have **uniform reluctance.**

28. Fig. 15 shows three flux distribution curves plotted from test data on an alternator to show the influence of shaping the pole faces. Curve *a* was taken when the machine was first tested with the pole faces concentric with the armature bore. The machine was noisy, and the pole tips were trimmed off to some extent, resulting in improved flux distribution, as shown by curve *b*, and reduced noise. The poles were then chamfered, giving the flux distribution shown in curve *c* and eliminating all noise.

29. If an armature winding is concentrated in one slot per pole per phase, its voltage wave will have the same shape as the flux distribution curve. The curves in Fig. 16 show voltage waves of a stator winding with one slot per pole per phase;

a shows the voltage and the flux distribution at no load; *b* with a non-inductive load, and *c* with an inductive load.

Curve *a* deviates considerably from a sine curve and shows that the pole face is not properly chamfered; *b* and *c* show the distortion of the voltage wave shape, due to armature reaction. Such distortion always occurs with load when the winding is concentrated in one slot per pole per phase.

30. If the armature winding is distributed in several slots per pole per phase, the voltage curve will approach a sine wave irrespective of the flux distribution curve. This is clearly

FIG. 15

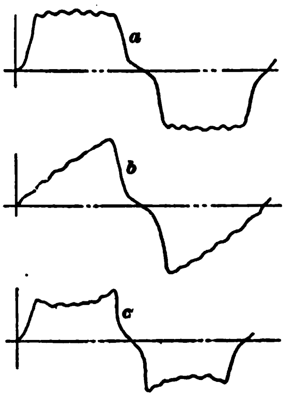


FIG. 16

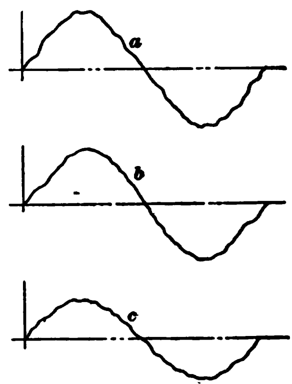


FIG. 17

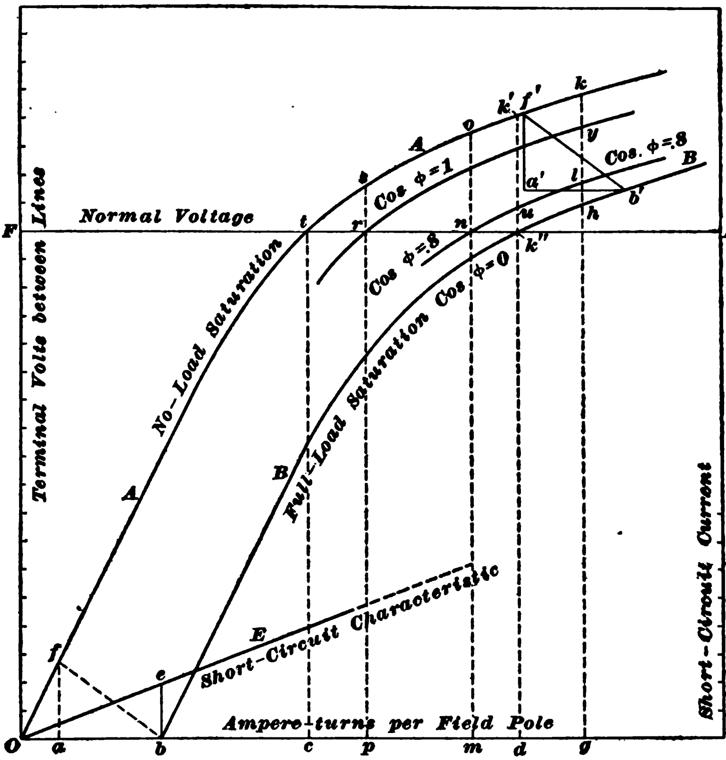
shown by the curves in Fig. 17, which were taken with an armature winding arranged in three slots per pole per phase, all other conditions being the same as when obtaining the curves in Fig. 16. With a distributed winding the voltage wave is thus shown to approach a sine curve under all load conditions.

31. The winding pitch also influences the voltage wave shape, a short-pitch winding giving more nearly a sine wave form than does a full-pitch winding. A sine-shaped voltage wave is desirable, because with it an alternator operates better and more efficiently and the insulation of the stator coils is subjected to lower maximum stress. A sine-shaped flux distribution insures low core loss and noiseless operation.

SATURATION CURVES

NO-LOAD CURVES

32. No-load saturation curves of salient-pole alternators are calculated in the same manner as those of direct-current machines. The curve for a turbo-alternator must be



ab=Armature Demagnetizing Ampere-turns Cos. φ=0
be=Full-Load Short-Circuit Current

FIG. 18

based partly on test data from similar machines, because of the difficulty of calculating accurately the number of ampere-turns required to set up the flux in the cylindrical rotor. The air-gap

ampere-turns for a turbo-alternator are calculated in the usual manner, and to the number thus found is added the number of ampere-turns shown by tests to be necessary for the stator and rotor steel. Curve *O A A*, Fig. 18, is the no-load saturation curve of an alternator, and from this curve can be found the full-load saturation curve for any power factor.

FULL-LOAD CURVES

33. The **full-load saturation curve** of an alternator can be drawn after calculating the no-load curve and the ampere-turns in the field coil necessary to set up full-load current in the armature when short-circuited. The latter calculation is made as explained in Art. **20**, and it determines the point *b*, Fig. 18, at which the full-load saturation curve for any power factor must begin. Incidentally, the calculation of the excitation *O b* necessary for full-load current on short circuit and the terminal voltage *b e* necessary to set up this current against the impedance of the armature winding determines a point on the short-circuit characteristic, which can then be drawn through this point and the origin.

34. At zero power factor the armature demagnetizing ampere-turns *a b*, calculated according to Art. **13**, can be subtracted directly from the field ampere-turns as explained in Art. **25**, leaving *O a* effective ampere-turns, which on open circuit induce electromotive force *a f* in the armature, *f* being a point on the no-load saturation curve. Line *a f* then represents the armature drop at full load. If the armature current remains constant and the power factor remains zero while the terminal voltage is increased, the armature drop *a f* and the armature reaction *a b* remain constant, and points on the curve *b B B* can be located by moving the triangle *f a b*, keeping the point *f* on the curve *O A A* and the line *a f* vertical. One position is shown at $f' a' b'$ obtained by making $f' a' = f a$ and $a' b' = a b$; b' is one point on the required curve. The curve *b B B*, drawn through the points thus found, represents the full-load saturation curve for zero power factor.

35. With zero power factor, field excitation $O\,d$ sets up normal voltage $d\,k''$ at full load, and if the load is removed with this excitation the terminal volts will rise to $d\,k'$; that is, the terminal volts will increase $k''\,k'$. On open circuit, excitation $O\,c$ is necessary to obtain normal voltage, and if the fields were not designed to admit the increase $c\,d$ with the available exciter voltage, the normal terminal voltage could not be maintained at full load with zero power factor. Operation at zero power factor is rarely necessary, but if an alternator

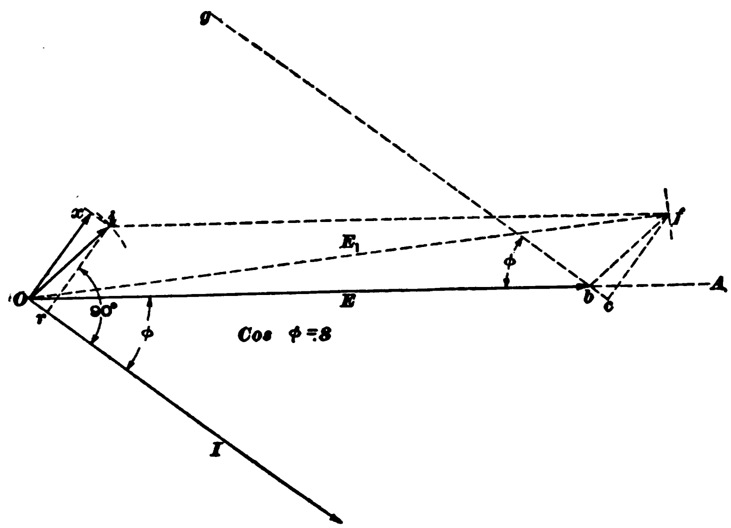


Fig. 19

is designed for such operation it can easily be made to maintain full voltage and good regulation at any power factor.

36. **A curve for any power factor** can now be found by the use of Fig. 19. For example, a point on the curve $n\,l$, Fig. 18, for a power factor of .8 can be found as follows: Any convenient excitation, say $O\,g$, would maintain open-circuit voltage $g\,k$ and full-load voltage $g\,h$ on zero power factor; the full-load voltage drop on zero power factor is $h\,k$. The problem is to determine the full-load voltage on power factor .8. The line $O\,A$, Fig. 19, is drawn to represent the direction of the

terminal voltage to be found, and $O I$ to represent the direction of the current, cos ϕ being .8. The electromotive force $O i$ ($=h k$, Fig. 18) required to overcome the synchronous impedance is the resultant of the known resistance drop $O r$ in phase with $O I$, Fig. 19, and the unknown reactance drop $O x$ at right angles to $O I$; hence, by laying off $O r$ equal to the resistance drop and drawing a line from r perpendicular to $O I$, the point i can be found by striking an arc from O as a center with a radius equal to $h k$, Fig. 18. The reactance drop, represented by $O x$ equals $r i$, Fig. 19. The total induced

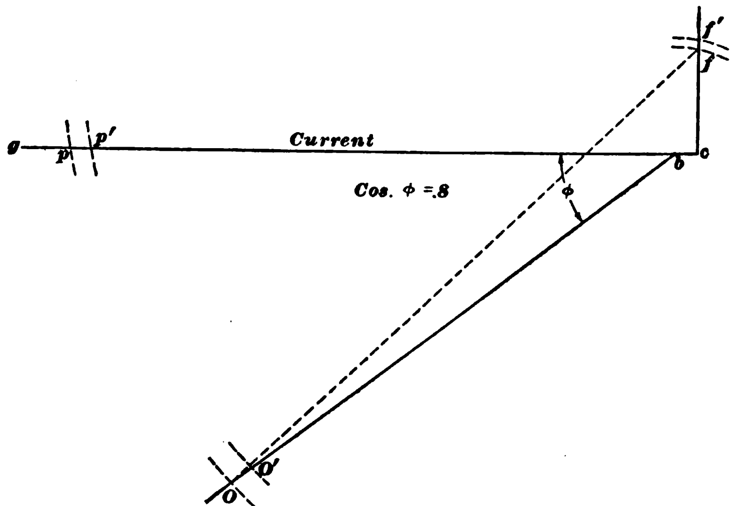


FIG. 20

electromotive force $g k$, Fig. 18, is the resultant of $O i$ volts, Fig. 19, drop in the armature and the unknown terminal volts. From the point i a line is drawn parallel to $O A$ and this line is intersected at f by an arc drawn with O as a center and with a radius equal to $g k$, Fig. 18. The line $f b$, Fig. 19, is then drawn parallel to $O i$, locating the point b and determining the length of the vector $O b$ representing the terminal voltage E at full load with a power factor .8. The length of this vector is laid off at $g l$, Fig. 18, giving one point l on the curve. With a power factor of .8, excitation $O g$ will therefore give $g k$ volts at no load and $g l$ volts at full load with a power factor of .8.

37. Fig. 20, which is a modification of Fig. 19, affords a more ready means of locating several points on saturation curves after the no-load curve has been calculated. At one end c of a horizontal line $g\,c$ a vertical line is drawn, and a distance $b\,c$ is laid off from the same end to represent the resistance drop in the armature. From the point b a line $b\,y$ of indefinite length is drawn making an angle ϕ with the horizontal line $g\,c$, such that $\cos\phi = .8$. With b as a center and with a radius equal to the total armature drop $h\,k$, Fig. 18, an arc is drawn cutting the vertical line $c\,f$, Fig. 20. Then with f as a center and with a radius equal to the open-circuit voltage $g\,k$, Fig. 18, an arc is drawn to cut the line $b\,y$, Fig. 20, at O; the distance $g\,l$, Fig. 18, is made equal to $b\,O$, thus locating the point l on the curve $n\,l$.

The polygons of electromotive forces $O\,f\,c\,b\,O$ are exactly the same in both Figs. 19 and 20. This fact will be more evident if the dotted line $c\,g$ parallel with $O\,I$, Fig. 19, is taken as the direction of the current and the whole figure conceived to be swung around until line $c\,g$ is horizontal.

A point on the curve $n\,l$, Fig. 18, corresponding to excitation $O\,d$, can be found by drawing an arc with a radius $k''\,k'$ and a center b, Fig. 20, to locate the point f'. With this point as a center and with a radius equal to $d\,k'$, Fig. 18, an arc is drawn cutting the line $b\,y$, Fig. 20, at O'. The point u, Fig. 18, is located so that the ordinate $d\,u$ is equal to the distance $b\,O'$, Fig. 20. Other points can be located in a similar manner and the curve $n\,l$ drawn through them. Only the useful part of the curve need be drawn; if continued it would terminate in the point b.

38. In locating points on the curve for unity power factor, the angle ϕ, Fig. 20, becomes zero, and the intercepts, such as p, p', on the horizontal determine the values of terminal electromotive forces, the arcs being struck from centers located, as before, on the perpendicular through c. A part of the curve for unity power factor and full-load current is shown at $r\,y$, Fig. 18. This curve, if fully plotted, would end at b.

39. The regulation can be estimated from the saturation curves. At unity power factor, field excitation $O\,p$, Fig. 18, is

required to maintain normal voltage. When the load is thrown off, the voltage rises to $p\,s$; hence, the regulation is $\frac{r\,s}{p\,r}$, or $\frac{r\,s}{p\,r}$ $\times 100$ if it is to be expressed as a percentage. With a power factor of .8, the regulation is $\frac{o\,n}{m\,n}$, and the variation in voltage when the load is thrown off is very much greater than on a non-inductive load. With zero power factor, the regulation is $\frac{k'\,k''}{d\,k''}$. The curves plainly show the great influence the character of the load has on the regulation, and why it is necessary to state the power factor of the load in every case where regulation is specified.

40. The foregoing method of estimating regulation is not exactly correct, because it is difficult to separate the effects of armature reaction and self-induction; also, there are a number of points that are not taken into account in the method. For example, with current in the armature, the field ampere-turns must be increased in order to keep up the voltage, and this increases the magnetic leakage between poles; whereas, in Fig. 18, the open-circuit saturation curve is used in determining the regulation, and the leakage is therefore taken as the same with and without load. Again, there is liable to be more or less inaccuracy in the calculations of the short-circuit line. It is possible to make corrections that add to the accuracy of the calculations, but the foregoing method illustrates the principles involved, and is therefore very useful in forming an approximate estimate of the behavior of a machine when operating on loads of various power factors.

FIELD LEAKAGE

41. The ability of an alternator to hold up its voltage on loads of low power factor depends very largely on the field leakage. The greater part of the flux set up through the poles passes through the armature core and is useful in generating electromotive force; but some lines of force leak across the space

between the poles without entering the armature core, and this leakage flux produces no electromotive force. The *field leakage coefficient* is the ratio

$$\frac{\text{total flux}}{\text{useful flux}} = \frac{\text{useful flux} + \text{leakage flux}}{\text{useful flux}}$$

In ordinary alternators, the value of the leakage coefficient usually lies between 1.1 and 1.5.

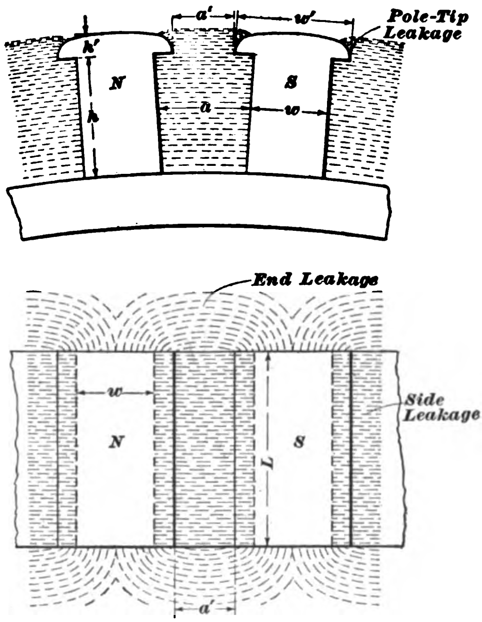


Fig. 21

42. Leakage flux can be calculated when the dimensions shown in Fig. 21 are known; N and S represent two adjacent poles of a revolving field, and the dotted lines represent the leakage flux between poles. For convenience, the flux may be divided into four parts as follows: (1) Leakage between the pole-core sides of height h and length L; (2) leakage between pole tips of height h' and length L; (3) end leakage between ends of pole cores of height h and width w; and (4) end leakage between ends of pole shoes of height h' and width w'.

The density of the leakage flux is greatest at the upper part of the poles and between the pole shoes, because at these places all the ampere-turns are effective, and also because the leakage path between the pole shoes is shorter than elsewhere; the flux density decreases as the yoke is approached. Since the leakage paths are in parallel with the path of the useful flux, the

ampere-turns effective in setting up leakage are the same as those effective in setting up useful flux through the air gap, armature core, and teeth; for all practical purposes, these ampere-turns may be taken as the same as the ampere-turns for the teeth and the air gap, because the number required for the core is very small. If X represents the number of ampere-turns per pole required for the teeth and for the air gap, and the dimensions shown in Fig. 21 are in inches, the leakage flux

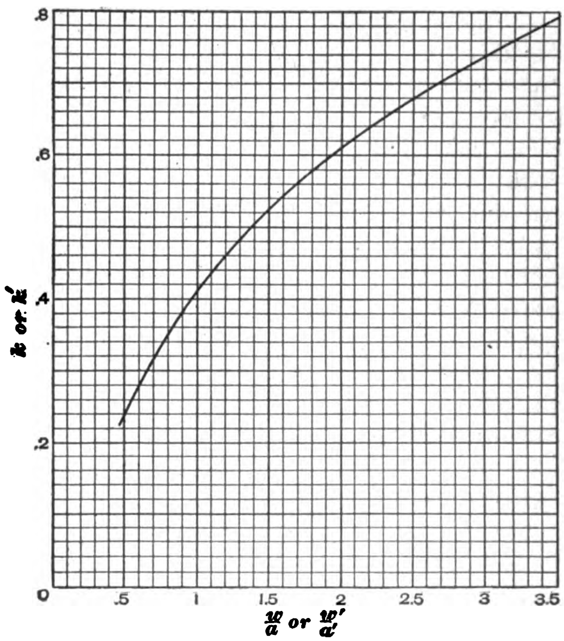


FIG. 22

per pole Φ_l can be calculated approximately by the following formula:

$$\Phi_l = 3.19 \times \left[\left(\frac{4\,h'}{a'} + \frac{2\,h}{a}\right) L + 3.2\,h\,k + 6.42\,h'\,k'\right]$$

in which k and k' are coefficients depending for their values on the ratios $\frac{w}{a}$ and $\frac{w'}{a'}$, respectively. For known values of $\frac{w}{a}$ or $\frac{w'}{a'}$

the values of k and k' can be obtained from the curve given in Fig. 22.

EXAMPLE.—An alternator has poles of the following dimensions, in inches: $h'=.5$; $h=6$; $w=3$; $w'=5$; $a=4$; $a'=2.4$; and $L=8$. The ampere-turns per pole for the teeth and the air gap are 3,000, and the useful flux per pole is 2,500,000 lines. (*a*) Calculate the leakage flux Φ_l. (*b*) What is the leakage coefficient of the field?

SOLUTION.—(*a*) First find the values of k and k' from Fig. 22. $\frac{w}{a}=\frac{3}{4}$ $=.75$; hence, $k=.335$; $\frac{w'}{a'}=\frac{5}{2.4}=2.08$; and $k'=.625$. Substituting the other values in the formula,

$$\Phi_l=3.19\times 3{,}000\left[\left(\frac{4\times .5}{2.4}+\frac{2\times 6}{4}\right)8+3.2\times 6\times .335+6.42\times .5\times .625\right]$$
$$=3.19\times 3{,}000\ (30.6+6.4+2)$$
$$=3.19\times 3{,}000\times 39=373{,}230 \text{ lines, approximately. Ans.}$$

(*b*) The useful flux is 2,500,000 lines, and the total flux is 2,500,000 +373,230=2,873,230 lines; hence,

$$\text{leakage coefficient is } \frac{2{,}873{,}230}{2{,}500{,}000}=1.15, \text{ nearly. Ans.}$$

As indicated in the formula, the leakage flux is the sum of the products of the magnetomotive force 3.19 X and four separate terms as follows: $\frac{4\,h'}{a'}\times L$; $\frac{2\,h}{a}\times L$; $3.2\,h\,k$; and $6.42\,h'\,k'$. The first of these gives the leakage flux between the pole tips facing each other; the second, the flux between the pole sides; the third, the leakage between the ends of the pole cores; and the fourth, that between the ends of the pole shoes.

43. In order to avoid excessive field leakage, the distances a' and a, Fig. 21, must be made as large as practicable; this means that the pole pitch, or distance between centers of poles, must be liberal. Machines built with small diameters and having crowded field poles invariably have large magnetic leakage. In other words, machines of large diameter and with fairly short poles, measured parallel with the shaft, and with liberal space between the poles, have much less leakage than those of smaller diameter with longer poles crowded closely together.

RATIO OF FIELD AND ARMATURE AMPERE-TURNS

44. In order to minimize the effect of armature reaction on the voltage regulation, the ratio of the number of field ampere-turns per pole required to set up the flux in the air gap to the number of armature ampere-turns per pole in salient-pole alternators should be from 1.4 to 2.5 for 60-cycle machines and from 1.25 to 2 for 25-cycle machines. In turbo-alternators these ratios should be from 1.5 to 2.5 for 60-cycle machines, and from 1.35 to 2.25 for 25-cycle machines.

45. The number of gap ampere-turns per pole is calculated by the formula

$$I\,T_g = .313\,B_g\,l_g \qquad (1)$$

in which B_g = flux density in air gap, in lines per square inch;
l_g = radial length of air gap, in inches.

The flux density B_g must be kept within practicable limits, which will be specified later, and the air gap must be made long enough to give the required number of gap ampere-turns $I\,T_g$. In some cases, therefore, a longer air gap than necessary for mechanical clearance between the stator and the rotor is required The number of gap ampere-turns can be ascertained, after the number of armature ampere-turns per pole is decided, as explained in Art. **44.** The required length of air gap can then be calculated by transforming formula **1** as follows:

$$l_g = \frac{I\,T_g}{.313\,B_g} \qquad (2)$$

SUMMARY OF REQUIREMENTS FOR GOOD REGULATION

46. The following is a summary of the main points that should be observed in order to obtain good regulation:

1. Make the armature of sufficiently large diameter so that the poles will not be crowded together and cause excessive magnetic leakage.

2. Design the armature to have as low self-induction as is practicable, so as to keep down the inductive drop and reduce the angle by which the current lags behind the induced electromotive force. The slots should neither be too deep nor too narrow, and the winding should be subdivided if possible. This statement applies more especially to alternators of slow and moderate speeds.

3. Make the field magnetically strong as compared with the armature; that is, make the air gap of such length that the ampere-turns per pole on the field necessary to set up the required air-gap density will be much greater than the armature ampere-turns per pole.

4. Increase the density of the magnetic circuit so that the machine will be worked well up on the bend of the saturation curve. Considerable variation in the effective ampere-turns per pole will not then affect the flux and voltage so much as if the magnetic circuit were unsaturated.

47. Low armature reactance is objectionable, however, in high-speed alternators, because in case of accidental short circuit it permits excessive transient current and correspondingly heavy stresses on the long coil ends projecting beyond the stator slots. On turbo-alternators either these ends must be securely braced or the self-induction must be made comparatively high, resulting in relatively poor regulation. This reasoning does not apply to slow-speed alternators with many field poles, small coil pitch, and short coil ends; such machines can be made with lower self-induction and better regulation than is practicable with turbo-alternators.

48. Moreover, good regulation necessitates heavy and expensive construction. Low armature reaction requires large magnetic flux and heavy magnetic circuits, and long air gaps require many ampere-turns and heavy field coils. On the other hand, poor regulation permits higher armature reaction, smaller flux, and lighter magnetic circuits; also, short air gaps require fewer ampere-turns and lighter field coils.

In many cases an alternator with poor regulation and an automatic voltage regulator can be purchased cheaper than an

alternator for the same output with regulation approximating that obtained with the combined alternator and voltage regulator. The tendency in modern alternator design is, therefore, to disregard regulation and observe only the limiting temperature. The regulation thus obtained ranges from 12 to 18 per cent. at unity power factor and from 26 to 40 per cent. at 80 per cent. power factor.

ELECTRICAL CALCULATIONS

MAGNETIC DENSITIES

49. The allowable magnetic densities in the teeth and cores of alternator armatures depend somewhat on the frequency, being highest for low-frequency machines. In any case these densities are somewhat lower than in the corresponding parts of direct-current machines. In alternators that are to operate at temperature rises not exceeding 40° C. with full rated load, the following densities are common in well-proportioned machines. The lesser of the two values in each case is for short machines and the higher values for longer machines provided with fans.

Frequency Cycles	Tooth Density Lines per Square Inch	Core Density Lines per Square Inch
60	92,000 to 110,000	65,000 to 70,000
25	115,000 to 120,000	75,000 to 90,000

50. The **air-gap densities** generally range from 40,000 to 58,000 lines per square inch for 60-cycle alternators, 48,000 being a fair average. In 25-cycle alternators the maximum density may be as high as 65,000 lines per square inch. These densities are limited by the allowable tooth density. With subdivided armature windings the teeth are frequently very little wider than the slots, in which case the air-gap density must be approximately one-half the tooth density. With very wide teeth and narrow slots the air-gap densities can be made somewhat higher than the foregoing numbers without making the tooth densities excessive.

51. Field-core densities in laminated-steel or cast-steel poles can be made from 80,000 to 100,000 lines per square inch. High field-core densities are usually economical, as the cross-sections and peripheries of the cores are then small, making the mean length of field turn relatively short and the weight of field copper small.

52. The **densities in the spider rims** of revolving-field alternators are usually low enough if the rims are designed for mechanical strength. The density should not be over 40,000 lines per square inch in cast-iron rims and not over 60,000 lines per square inch in cast-steel rims, but mechanical considerations generally make it advisable to have the densities lower than these numbers indicate.

AIR-GAP AMPERE-TURNS

53. In an alternator, the armature teeth are usually rather coarse and the slots wide, so that the magnetic flux from the pole is in *tufts*, somewhat as shown in Fig. 23. The actual density in the gap is greater than the value obtained by dividing the total flux per pole by the area of the pole face, and the

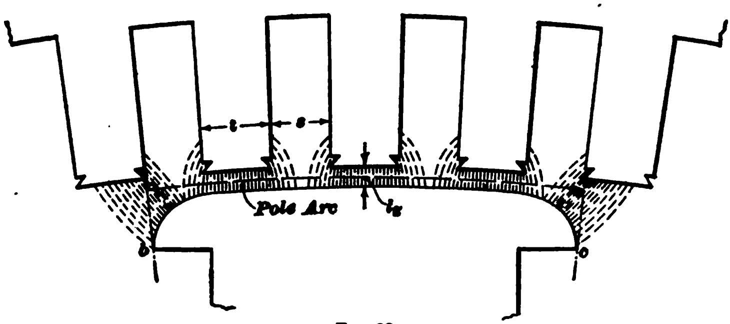


FIG. 23

actual ampere-turns required for the air gap are therefore greater than those obtained from calculation based on the assumption that the lines are uniformly distributed. To allow for this bunching of the lines, the ampere-turns can be calculated for an effective air gap slightly longer than the actual

gap l_g, the increased length depending on the dimensions of the slots and teeth and the length of the gap.

54. The **effective air gap** can be calculated by the following formula, in which all dimensions are in inches:

$$l'_g = \left(\frac{l + \frac{s}{t}}{l + k' \frac{s}{t}} \right) l_g$$

in which l'_g = effective air gap;
s = width of slot opening;
t = width of tooth at air gap;
k' = coefficient obtained from Fig. 24;
l_g = actual air gap.

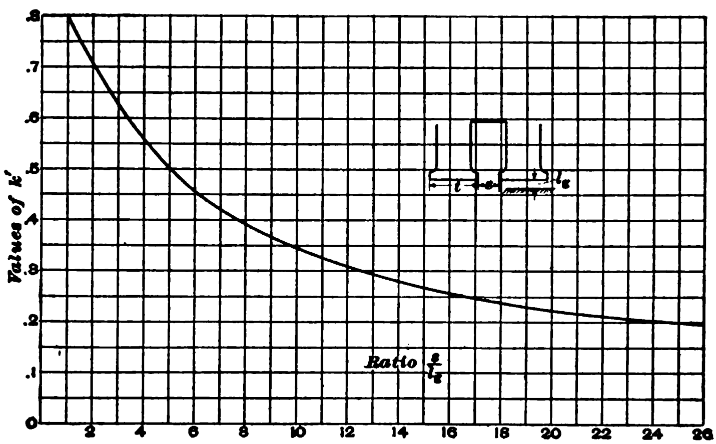


FIG. 24

55. Salient-Pole Rotors.—The effective polar arc in an alternator with salient, or projecting, poles is greater than the actual pole arc, Fig. 23, owing to the fringing of the flux at the pole tips. The effective arc can be calculated by the formula

$$\text{Effective arc} = \text{actual arc} + k\ l_g$$

in which k is equal to the coefficient obtained from Fig. 25.

56. The effective pole arc multiplied by the pole-face dimension parallel to the shaft gives the effective area of the

pole face. The total flux divided by this effective area, in square inches, gives the average air-gap density B_g in lines per square inch. The formula for air gap ampere-turns can then be written

$$I\,T_g = .313\,B_g\,l'_g$$

EXAMPLE.—The pole pitch of an alternator is 8 inches, and the open slots have a width of .75 inch. The width of the teeth at the air gap is 1 inch; the length of the single air gap is .2 inch; the polar arc is 5 inches; and the length of the pole parallel with the shaft is 10 inches. Assuming that the total flux per pole is 2,500,000 lines (2.5 megalines), calculate the ampere-turns for the air gap, making allowance for pole and tooth fringing.

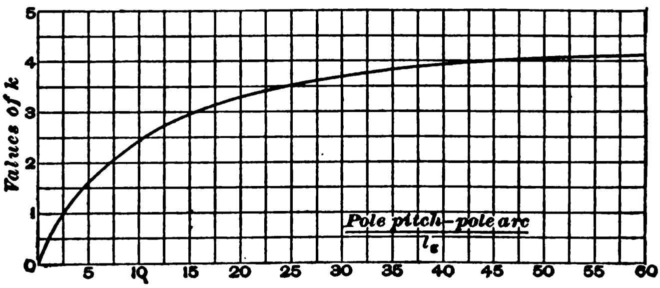


FIG. 25

SOLUTION.—To obtain the effective air gap, use $s = .75$; $t = 1$; and $\frac{s}{l_g} = \frac{.75}{.2} = 3.75$. From Fig. 24, the corresponding value of k' is .58. When these values are substituted in the formula of Art. **54**.

$$l'_g = \left(\frac{1+\frac{.75}{1}}{1+.58\times\frac{.75}{1}}\right)\times .2 = \frac{1.75}{1.435}\times .2 = .244,\text{ nearly}$$

To find the effective pole arc, $\frac{\text{pole pitch}-\text{pole arc}}{l_g} = \frac{8-5}{.2} = 15$, and from Fig. 25, $k = 3$, nearly. Then, by the formula of Art. **55**, the effective arc is $5+3\times .2 = 5.6$. The effective pole area is $5.6\times 10 = 56$, and the average air-gap density is $2,500,000 \div 56 = 44,600$ lines per sq. in., approximately. By the formula of this article,

$$I\,T_g = .313\times 44,600\times .244 = 3,400,\text{ approximately. Ans.}$$

57. Cylindrical Rotors.—Fig. 26 is a cross-section of a four-pole rotor with a winding consisting of three coils per pole, spaced, respectively, in slots *1–1*, *2–2*, and *3–3*, these numbers

being used here for identification purposes only and for only one pole. The coils for each of the other field poles are placed in the same relative manner. The three field coils of each pole are effective in establishing flux through the center pole and the arc *a* in the adjacent air gap; the two coils in slots *1–1* and *2–2* cause flux through the arc *a* and arcs *b* and only the coil in slots *1–1* is effective in producing flux through the arcs *c*.

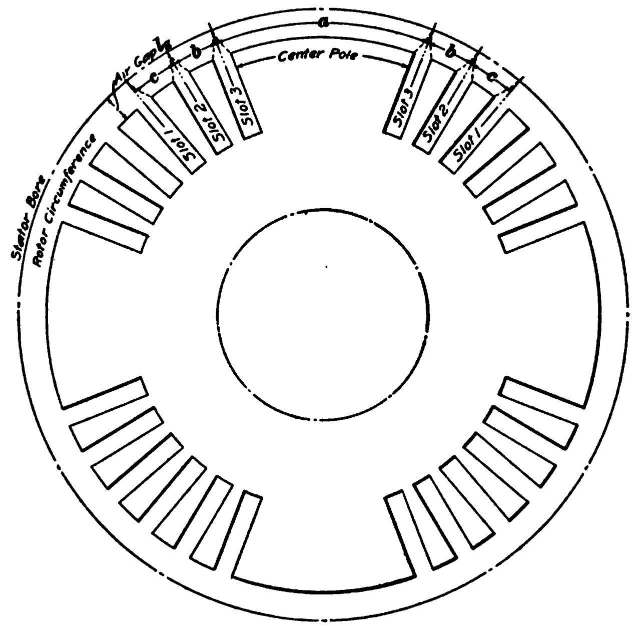


FIG. 26

58. Fig. 27 shows a development of one pole and the flux distribution curve in the air gap at arcs *a*, *b*, and *c*, Fig. 26. The maximum flux density B_a occurs over the center pole, and this density decreases over each adjoining rotor tooth to zero over the teeth between coils belonging to adjacent poles. The arcs are measured at the center of the air gap, in order to allow for the fringing of the lines of force at the rotor slots. Arcs *b* and *c* are usually of equal length.

The base of the distribution curve, Fig. 27, is equal to the span of the outer coil on the pole. The ordinates represent the

fluxes from the teeth and the pole center, and the ordinate over each tooth B_1, B_2 is therefore proportional to the ampere-turns surrounding that tooth. The ordinate B_m of the curve over the pole center represents the maximum flux caused by all the ampere-turns on the pole. If the area enclosed by the curve is divided by the length of its base, the quotient B_a will be the average flux as represented by the broken line. In laying out a design, the maximum flux can be assumed to be 1.5 times the average flux until dimensions of the rotor core and slots have been found, and then the ratio can be checked by laying out a curve and determining the exact ratio as just explained.

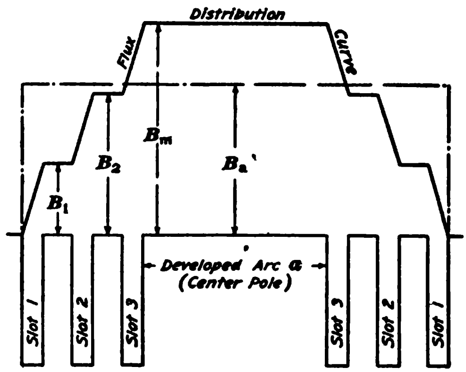


FIG. 27

59. In the following formulas

$I T_g$ = gap ampere-turns per pole;

Φ = useful flux per pole;

l_g = air gap, in inches;

l'_g = effective length of air gap, in inches, determined as in Art. 54;

l_r = axial length of rotor core, in inches;

l = axial length of net stator iron, in inches;

a' = developed length, in inches, of arc a, Fig. 26;

b' = developed length, in inches, of arc b, Fig. 26;

S_p = number of rotor slots per pole;

d = diameter of stator bore, in inches;

t_o = stator slot pitch, in inches $= \dfrac{\pi d}{\text{number of stator slots}}$;

t_m = mean width of stator teeth, in inches;

B_{tm} = density in stator teeth opposite pole arc a.

Then,
$$I\,T_g = \frac{.313\,l'_g\,\Phi}{l_r\left[a' + \left(\frac{S_p}{2} - 1\right)b'\right]} \qquad (1)$$

$$B_{tm} = \frac{\Phi\,(d - l_g)\,t_o}{\left[a' + \left(\frac{S_p}{2} - 1\right)b'\right]d\,t_m\,l} \qquad (2)$$

The density in the air gap is limited in the same manner as in a salient-pole machine. In machines designed for 40° C. rise, allowable tooth densities B_{tm} are from 90,000 to 115,000 lines per square inch for 60-cycle machines, and from 115,000 to 120,000 lines per square inch for 25-cycle machines. The corresponding gap densities are from 40,000 to 53,000 lines per square inch for 60-cycle machines, and from 50,000 to 65,000 lines per square inch for 25-cycle machines.

INSULATION

60. Alternator windings must be insulated for voltages much higher than the rated voltages at which the machines are to operate; that is, the insulation must be designed with a large factor of safety. The **standard dielectric test** agreed upon by the American Institute of Electrical Engineers for the insulation of alternator armature windings is a sine-wave alternating voltage of two times the rated voltage of the alternator plus 1,000 volts, this test to be continued for 60 seconds. The insulation of the field windings should be tested by a voltage of the same character ten times the exciter voltage but never less than 1,500 volts.

According to these specifications, the armature insulation of a 2,200-volt alternator should be tested with $2 \times 2{,}200 + 1{,}000 = 5{,}400$ volts alternating, and if the exciting voltage is 125 the field coil insulation should be tested with 1,500 volts, because $10 \times 125 = 1{,}250$, which is less than the minimum specified.

61. Some of the insulating materials most commonly used in alternators are micanite, varnished cloth (one variety of which is called empire cloth), oiled cambric or linen, paper and

fuller's board, cotton tape, horn fiber, fish paper, and similar tough, fibrous materials.

Micanite, which is formed of thin sheets of mica cemented together with insulating varnish or shellac, is used in alternators both in sheets from .015 to .02 inch thick and as tape from .005 to .01 inch thick. Varnished cloth and cotton tape are usually from .005 to .01 inch thick and .75 to 1 inch wide.

62. In **applying insulating materials** to armature conductors, single conductors, sometimes in several strands, may be insulated with fine cotton thread spun on in layers or with half-lapped cotton tape, as in Fig. 28. This taping is satisfactory for comparatively low voltages, but if the voltage between adjacent conductors in a coil is high, additional insulation, such as half-lapped micanite tape or varnished cloth, is applied. If high internal coil temperatures are expected, micanite tape is added to the cotton taping or strips of micanite are placed between the conductors.

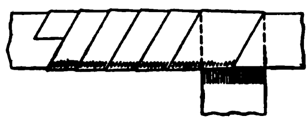


FIG. 28

After the conductors have been properly insulated, formed into coils, and tied together, the coils are insulated preparatory to placing them in the armature slots. In machines for voltages not exceeding 440, the slot insulation generally is fish paper, together with varnished cloth and cotton taping. In machines for more than 440 volts, varnished cloth, together with micanite, is used. The micanite is applied generally next to the conductor, and then the proper number of layers of half-lapped varnished cloth is applied over the micanite. In order to eliminate all air pockets, which invariably form between the different layers of coil insulation, the *vacuum impregnating process* is used. The insulated coils are placed in an air-tight tank or vacuum oven, from which the air is then exhausted. When the air is almost wholly removed from the insulation, hot insulating compound or insulation varnish is forced into the tank under pressure and fills the openings in the insulation from which the air was removed On cooling, this compound solidifies, making the coil a solid unit. The

straight portion is then covered with a layer of fish paper, after which the coil is heated and pressed to exact size.

63. In calculating the dimensions of finished armature coils, allowance must be made for the space occupied by insulation. Each layer of half-lapped cotton tape or varnished cloth adds four times its thickness to each dimension of a conductor section. This insulation swells when impregnated, so that .006 inch must be allowed for each thickness of .005-inch tape. Micanite does not swell appreciably when impregnated. The outer covering, or armor, of the coil, usually of micanite or fish paper, as indicated in Fig. 29, adds twice its thickness to the width of the coil and is lapped above so as to add three times its thickness to the depth.

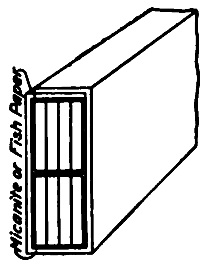


FIG. 29

64. In calculating the dielectric strength, only one thickness of tape can be counted for each layer half lapped and only one thickness of the micanite or fish paper applied as in Fig. 29. The dielectric strengths in kilovolts (thousands of volts) of some of these materials are shown by the curves in Fig. 30. A single layer of half-lapped cotton tape has a dielectric strength of 1,000 volts when impregnated with insulating compound. A double layer of fine cotton threads spun over a conductor to a thickness of only .0034 inch will withstand 600 volts when impregnated, but only 160 volts

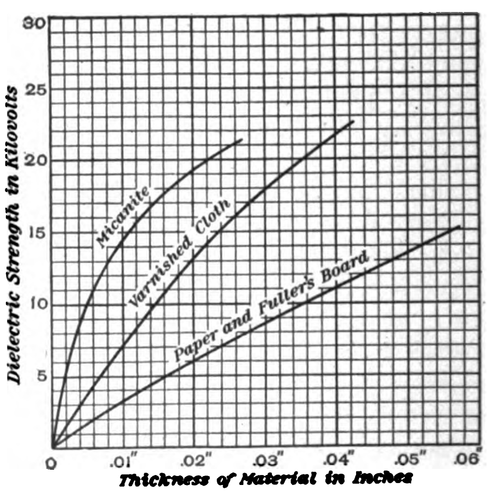


FIG. 30

without impregnation. Calculations based on the dielectric strengths of insulating materials serve only as rough guides in designing the insulation of machines, and large factors of safety must be allowed. Insulating materials are not uniform in quality and, moreover, they may be injured while being applied, thus greatly reducing their dielectric strengths.

EXAMPLE.—An armature coil consists of one turn of $\frac{1}{4}$-in. × $\frac{1}{2}$-in. conductor insulated with one layer of half-lapped .005-inch cotton tape. The coil insulation consists of one turn of .02-inch micanite lapped on top and bound on with three layers of half-lapped .005-inch varnished cloth, outside which is one turn of .013-inch fish paper lapped on top. The insulated coil is then impregnated. Calculate the dimensions of the coil section and the dielectric strength of its insulation.

SOLUTION.—The allowance for each thickness of impregnated cotton tape and varnished cloth must be .006 inch, and each layer adds four times its thickness to each dimension of the coil section. The dielectric strength of one layer of cotton tape has been given as 1,000 volts; Fig. 30 shows that .02-inch micanite has a dielectric strength of 19,000 volts, each layer of .006-inch varnished cloth about 4,500 volts, and .013-inch paper 4,000 volts. The calculations can be tabulated as follows:

Insulation Material	Width of Space Inches	Depth of Space Inches	Dielectric Strength Volts
Tape on conductor	4×.006=.024	4×.006=.024	1,000
Micanite on coil	2×.02 =.040	3×.02 =.06	19,000
Varnished cloth	3×4×.006=.072	3×4×.006=.072	13,500
Fish paper	2×.013=.026	3×.013=.039	4,000
Total	.162	.195	37,500

The width of the insulated coil, therefore, is .25+.162=.412 inch, and the depth is .5+.195=.695 in. Ans.

65. The **slot space required** is somewhat greater than indicated by the calculated coil dimensions, because of inaccuracies in thicknesses of insulating material and in the assembly of core laminations. Table II gives safe allowances for insulation on a coil section; that is, both the width and the depth of the slot space required for a coil may be found by adding the given allowance to the dimensions of the conductors. If the width of the conductor forming one side of a coil for a 220-volt armature is .3 inch and the depth .5 inch, the slot

should be from .08 to .10 inch wider, or approximately .4 inch, and the depth allowed should be approximately .6 inch.

TABLE II

SPACE ALLOWANCE FOR ALTERNATOR SLOT INSULATION

Generated Voltage	Total Allowance for Insulation Inches	Generated Voltage	Total Allowance for Insulation Inches
110–220	.08 to .10	6,600	.28 to .38
440–600	.10 to .14	11,000	.38 to .42
1,100–2,200	.16 to .20	13,200	.42 to .50
4,400	.24 to .28		

66. Insulation of alternator field coils is comparatively easy, because exciter voltages are generally low. The conductor for salient-pole field coils is usually strip copper wound on edge, as indicated in Fig. 31. The strips are sepa-

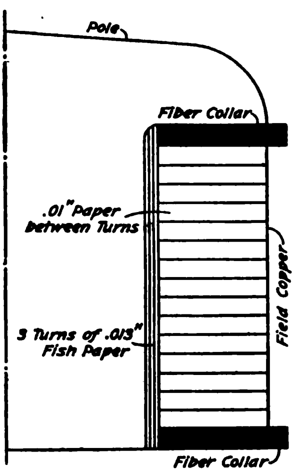


FIG. 31

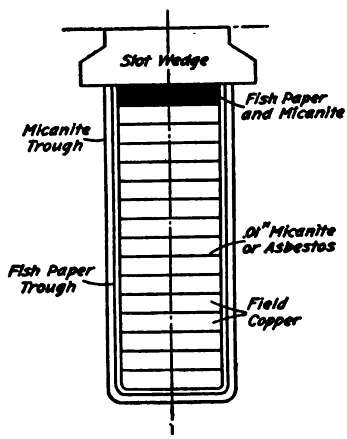


FIG. 32

rated by layers of paper from .01 inch to .013 inch thick; if the coils must withstand very high temperatures, asbestos paper is used. The inside of the coil is separated from the pole by

wrappings of fish paper, and the coil ends are protected by fiber collars. Salient-pole field coils are also sometimes made of double-cotton or asbestos-covered wire, in which case no paper is used between turns.

The field coils of turbo-generators are made of copper strip with fireproof insulation, such as mica or asbestos paper. The strip is wound flat in slots, as indicated in Fig 32, the slots being previously insulated with fish paper and micanite. The coil ends extending beyond the rotor body are insulated with half-lapped asbestos tape.

HEATING

67. Heating is the most common limitation to the output of alternators. The copper, or I^2R, losses, and the iron, or hysteresis, and the eddy-current losses are converted into heat, which raises the temperature of the machine. As this temperature increases, more heat is dissipated to the surrounding air, and when the machine temperature has risen enough above the air temperature so that heat is dissipated as fast as developed in the machine, the machine temperature remains constant, or flat. The maximum capacity of the machine, so far as heating is concerned, is the load it can carry without reaching a temperature so high as to injure the insulation.

68. The highest safe temperatures for some of the more common insulating materials are as follows:

Dry or untreated fibrous materials.........	90° C.
Treated or oil-immersed fibrous materials...	100° C.
Mica, asbestos, etc., in combination........	150° C.
Pure mica..............................	200° C.

The maximum temperatures in a machine are not indicated by thermometers applied to exposed parts; the heated parts are inside the coils where thermometers cannot be placed. The indicated temperatures should therefore be less than the maximum by an amount depending on the thickness of the coil

insulation. As this thickness varies with the voltage for which the machine is designed, the maximum indicated temperatures should be less than in the foregoing list by approximately 10° C., if the rated voltage is not over 4,000; by 20° C, if the voltage is between 4,000 and 13,000; and by 25° C., if the voltage is over 13,000.

HEATING OF ARMATURE COILS

69. The temperature of a stator winding is influenced by the temperature of the surrounding air and core steel, the axial length of the machine, the rate at which heat is developed in the coil, and the thickness of the insulation wall surrounding the conductors.

70. The design of an electrical machine should be based on the maximum surrounding air temperature that is likely to arise, and this temperature is usually understood to be 40° C., if not otherwise specified. The temperature of the core steel surrounding the armature coils depends largely on the iron losses, which are influenced by the flux densities in the armature core, by the frequency of the current, and by the provisions for ventilating the core.

71. The portion of the stator coil that is surrounded by the heated core necessarily has a higher temperature than the coil ends, since the ends come in contact with the air blast from the rotor. In a narrow machine, a considerable amount of the heat from the central portion of the coil flows to the coil ends, and the temperature difference between the hottest and coolest parts of the coil is slight; but as the width of a machine is increased, this difference is increased also. In long machines, such as turbo-alternators and high-speed motors and generators of large output, a considerable temperature difference exists between the hottest part of the coil, which is located in the central part of the core, and the coolest part, or the coil ends.

72. In order to find the rate at which heat will be developed in the armature winding, the ampere conductors per inch of

inside periphery must be known. This can be calculated as follows:

Let C = number of conductors per slot;
K = ampere-conductors per inch;
m = number of phases;
I_p = amperes per phase;
T_p = turns per phase;
d = inside armature diameter.

Then, $$K=\frac{2\,m\,I_p\,T_p}{\pi\,d} \qquad (1)$$

If the number of conductors per slot C, the amperes in each conductor I_p, and the slot pitch in inches t_o, Fig. 33, are known, the number of ampere-conductors per inch can be found by the formula

$$K=\frac{C\,I_p}{t_o} \qquad (2)$$

The number of ampere-conductors per inch influences both heating and regulation. This number is chosen with reference to regulation, and then the size of the conductor is chosen to get the carrying capacity required to keep within safe heating limits. For alternators to have regulation at unity power factor of from 7 to 10 per cent., K is made from 500 to 750; if the regulation is to be from 12 to 18 per cent., K can be chosen from 750 to 1,300. In order to prevent eddy currents, which help to heat large solid conductors, the armature conductors in a 60-cycle alternator are usually laminated if they exceed $\frac{5}{8}$ inch in depth and those in a 25-cycle alternator if they exceed $\frac{3}{4}$ inch in depth.

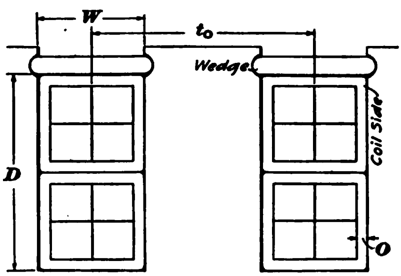


FIG. 33

73. A formula can now be derived for calculating the rate at which power is converted into heat in the armature windings.

Let P_s = watts dissipated per square inch of armature-coil surface;
k_a = kiloampere-conductors per inch of inside armature circumference $(= K \div 1{,}000)$;
I_k = kiloamperes per square inch of conductor cross-section;
M = sectional area of conductor, in square inches;
N = sectional area of conductor, in square mils $(= 1{,}000{,}000\, M)$.

W, D, and t_o have the meanings shown in Fig. 33 and are in inches; C and I_p have the meanings given in Art. **72.**

The resistance of 1 circular-mil-foot of copper is 10.8 ohms cold, and for design purposes may be considered as 12 ohms hot. The hot resistance of 1 circular-mil-inch is, therefore, 1 ohm, and of 1 square-mil-inch .7854 ohm, or, for practical purposes, .79 ohm. Then, the resistance in ohms per inch of one conductor is $\frac{.79}{N}$, and of all the conductors in 1 inch of slot space considered in series $\frac{.79\, C}{N}$, or $\frac{.79\, C}{1{,}000{,}000\, M}$. The loss in watts in 1 inch of slot space is $\frac{.79\, C\, I_p^2}{1{,}000{,}000\, M} = \frac{.79\, C\, I_p}{1{,}000{,}000} \times \frac{I_p}{M}$. But $C\, I_p = 1{,}000\, k_a\, t_o$ and $\frac{I_p}{M} = 1{,}000\, I_k$; therefore, the loss in 1 inch of slot space is $\frac{.79 \times 1{,}000\, k_a\, t_o \times 1{,}000\, I_k}{1{,}000{,}000} = .79\, k_a\, t_o\, I_k$.

The coil surface per inch of slot space is $2\,(W+D)$ square inches; therefore,

$$P_s = \frac{.79\, k_a\, t_o\, I_k}{2\,(W+D)} = \frac{.395\, k_a\, t_o\, I_k}{W+D}$$

Example.—The inside diameter of a stator is 100 inches and the number of slots is 360. The slots are .47 inch wide and 1.6 inches deep under the slot sticks. Each slot contains two conductors, and each conductor consists of four strips .15 in. × .24 in. The current per conductor is 300 amperes. Find the watts loss per square inch of coil surface.

SOLUTION.—

$$k_a = \frac{360 \times 2 \times 300}{100 \times 3.1416 \times 1{,}000} = .688$$

$$t_o = \frac{100 \times 3.1416}{360} = .873$$

$$I_k = \frac{300}{4 \times .15 \times .24 \times 1{,}000} = 2.08$$

$$W = .47;\ D = 1.6$$

$$P_s = \frac{.395 \times .688 \times .873 \times 2.08}{.47 + 1.6} = .238. \quad \text{Ans.}$$

74. The thickness of insulation covering the armature coils and its ability to conduct heat affects the rate at which heat escapes and thus affects the final temperature inside the insulation. As this inside temperature must not exceed the limiting safe value for the insulation, the effect of the insulating wall must be considered. Experience has shown that reasonably accurate results with well-impregnated insulation can be obtained by the formula

$$T = \frac{P_s O}{.003}$$

in which T = temperature difference, in centigrade, between the inside and the outside of the insulation walls;
P_s = watts loss per square inch of coil surface;
O = thickness of the insulation wall, in inches. (See Fig. 33.)

The external temperature of the coils and the core can be measured by thermometers. By adding the calculated temperature difference T, the temperature inside the insulation can be determined.

EXAMPLE.—Find the maximum internal coil temperature of an alternator carrying a load giving $P_s = .3$ watt per square inch of coil surface. The insulation around the coils is .2 inch thick, and the highest temperature of the machine is 65° C., measured by thermometer on the stator core touching the stator coils in the middle of the machine.

SOLUTION.—As the outside of the insulation wall has the same temperature as the adjacent core steel, the maximum internal coil temperature is 65° C. + T. In the formula, $P_s = .3$, $O = .2$, and $T = \frac{P_s O}{.003} = \frac{.3 \times .2}{.003} = 20°$ C. Then, the maximum internal coil temperature is 65 + 20 = 85° C. Ans.

75. The power loss in watts per square inch of armature coil surface in low-voltage alternators designed for 40° C. maximum temperature rise may vary from .25 in machines with peripheral velocities below 3,500 feet per minute to .35 with velocities of 6,000 feet per minute. The better ventilation obtained with the higher velocities causes more rapid dissipation of heat. With forced ventilation the value of P_s may be as high as .4 without causing excessive temperature. In high-voltage machines the effect of the insulation on the internal temperature must be considered, and P_s must be low enough to keep the inside temperature within safe limits.

EXAMPLE.—A 13,200-volt alternator is specified not to exceed 55° C. temperature rise, measured by thermometer, above atmospheric temperature taken at 25° C. The insulation is mica and varnished cloth of which the maximum temperature should not exceed 100° C. (Art. **68**). Find the maximum safe watts per square inch of coil surface.

SOLUTION.—The hottest part of the machine will be the armature conductors, and their maximum temperature must not be over 55° C. +25° C. = 80° C. As indicated in the problem, the temperature of the coils inside the insulation should not exceed 100° C., and the allowable difference when the coil temperature is at its highest point is 100 − 80 = 20° C. The thickness of insulation on armature coils for a 13,200-volt machine may be taken as one-half the maximum allowance given in Table II, or .25 inch. Then in the formula of Art. **74**, $T = 20$, $O = .25$, and $20 = \frac{P_s \times .25}{.003}$; $P_s = \frac{20 \times .003}{.25} = .24$. Ans.

ROTOR HEATING

76. Heat is readily dissipated from salient-pole field coils made of strip copper wound on edge, especially if the speed is high. Experience has shown that the results indicated by the curves in Fig. 34 may be expected with such coils. The coil surface referred to is the outside exposed surface only, or $2c(a+b)$, and does not include the ends or the surface inside next the pole core. The curve for 40° C. temperature rise shows that at a peripheral velocity of 9,000 feet per minute, power can be dissipated from the field coils at the rate of 3.8 watts per square inch of surface thus calculated. If the entire surface were considered, as is done with stator coils, this number would

be approximately 1.55, whereas the maximum for stator coils, as given in Art. **75**, is .4, thus showing that with the better ventilation possible with this form of field coils the rate of heat dissipation is increased nearly four times.

77. In a rotating field turbo-generator, the field winding is usually distributed in several slots per pole. The main portion of the coils is embedded in the rotor core, as indicated in Fig. 35, and only the coil ends are exposed directly to the air. Hence, only part of the heat due to rotor losses is dissipated from the coil ends, whereas the remainder must work its way through the rotor core to the surrounding air. In designing a turbo-rotor, the simplest method is to assume that all the heat

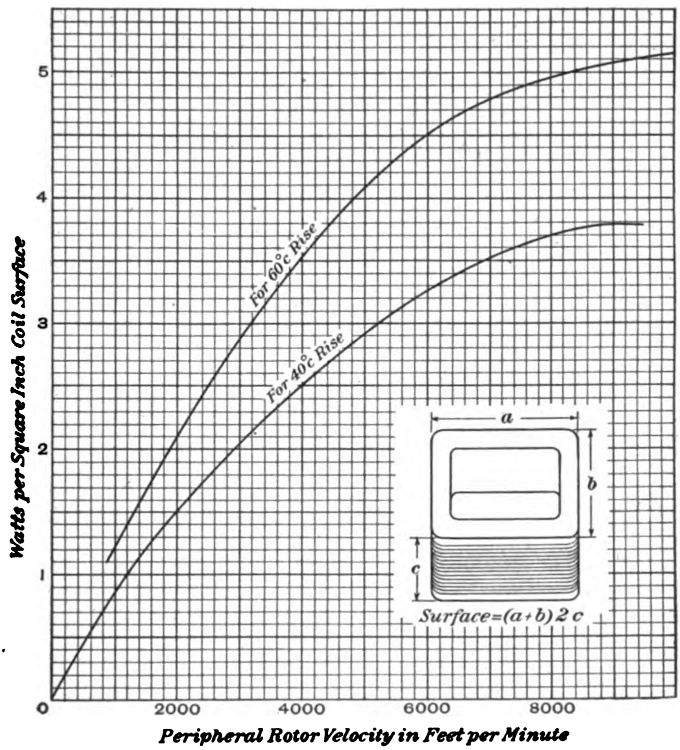


Fig. 34

due to rotor losses is dissipated from the cylindrical rotor surface. The increase in temperature varies considerably according to rotor length and speed. However, for each watt

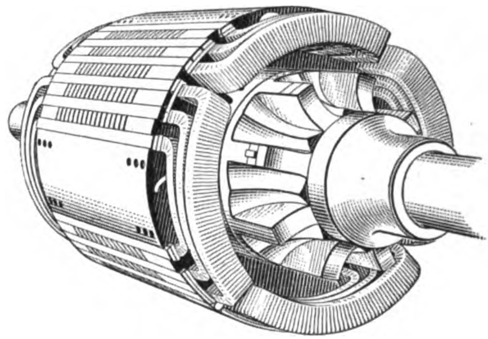


FIG. 35

dissipated per square inch cylindrical surface, a temperature rise of 12° C. may be taken as a fair average.

Let T_r = temperature rise;
d = rotor diameter, in inches (Fig. 35);
L = length of rotor core, in inches (Fig. 35);
P_r = total watts loss in rotor winding.

Then, $$T_r = \frac{12\ P_r}{\pi\ d\ L}$$

EXAMPLE.—A turbo-rotor is 20 inches in diameter and 36 inches long. At what rate can power be dissipated as heat from the cylindrical surface without exceeding a temperature rise of 40° C.?

SOLUTION.—Here $T_r = 40$, $d = 20$ in., $L = 36$ in., and P_r is to be found. By the formula, $40 = \frac{12 \times P_r}{3.1416 \times 20 \times 36}$;

$$P_r = \frac{40 \times 3.1416 \times 20 \times 36}{12} = 7{,}540 \text{ watts. Ans.}$$

FORCED VENTILATION

78. Forced ventilation is common in turbo-alternators. The high speed and correspondingly large output per unit weight of such machines result in the liberation of large

quantities of heat in small spaces, and much of this heat is carried off by air-currents forced through passages in the heated

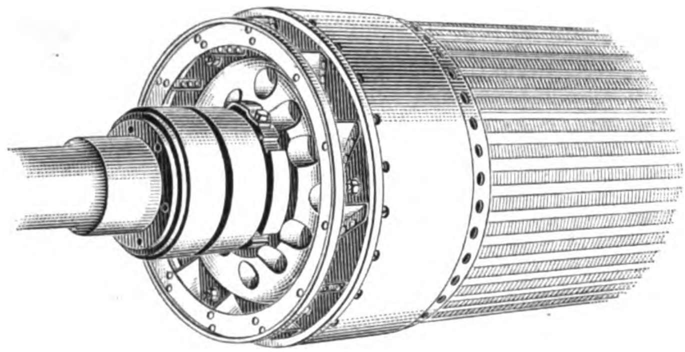


FIG. 36

parts. In order to control the direction of the air-currents, as well as to reduce noise, the machines are usually enclosed.

79. The air movement is caused in some cases by ventilating fans mounted on the rotors, as in Fig. 36, and in other cases by separate blower systems. The fan blades shown in Fig. 36 cause air-currents as indicated in the sectional view, Fig. 37. Air enters a chamber at the bottom of the machine, passes up in two parallel paths, through the stator core, and escapes through the top of the yoke. This method is preferable for the small alternators.

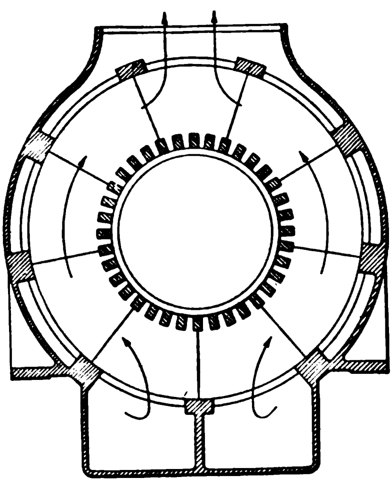


FIG. 37

Fig. 38 shows another method used in larger turbo-alternators. Fans as shown in Fig. 36 or separate blowers cause air-currents, as indicated by arrows, through a large number of ventilating ducts in the stator core, as well as over the surface of the rotor

and coil ends. The sections A and B form an inner annular belt just back of the stator core. The sections A open directly to the incoming air and the sections B directly into a space C beneath the machine and into an outer annular belt D; the heated air escapes at the top of the machines. The stator vent segments are provided with curved ribs a to guide the air. Air for cooling the rotor surface passes in through the air gap at each end of the rotor and finds its way out through the ducts in the core.

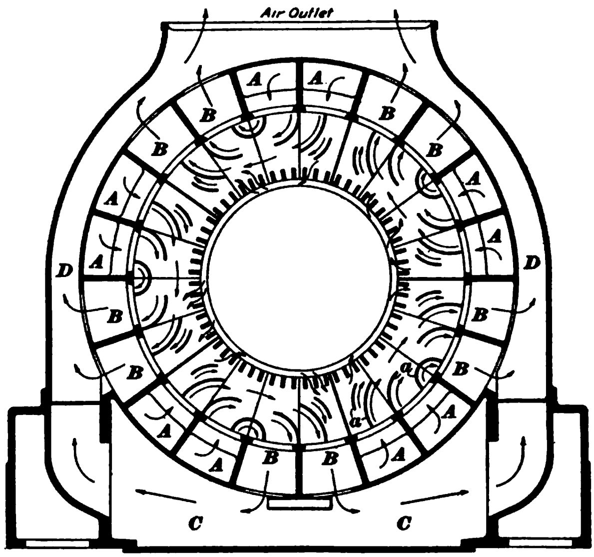


FIG. 38

80. From 100 to 150 cubic feet of air per minute for each kilovolt-ampere loss in a turbo-alternator must be provided for good ventilation. The passages must be large enough to keep the velocity of this air below 6,000 feet per minute. The heat carried away depends on the volume of air and on the difference between its temperatures on entering and on leaving the machine.

Let t_1 = temperature of entering air, in degrees centigrade;
t_2 = temperature of escaping air, in degrees centigrade;
P = power in watts to be dissipated as heat;
V = volume of ventilating air, in cubic feet per minute.

Then, $$P = .536\ V\ (t_2 - t_1)$$

EXAMPLE.—If the temperature rise in the ventilating air escaping from a turbo-alternator is 18.6° C., what volume is necessary for each kilowatt loss?

SOLUTION.—Here $P=1{,}000$ and $t_2-t_1=18.6$. Then $1{,}000=.536\,V\times18.6$;

$$V=\frac{1{,}000}{.536\times18.6}=100 \text{ cu. ft. per min.} \quad \text{Ans.}$$

EFFICIENCY

81. The efficiency of a machine affects its heating, because the efficiency is the ratio of the output to the input, and the output is the input minus the losses. The lower the efficiency the greater are the losses and consequently the heating. The losses in an alternator are of the same classes as those in direct-

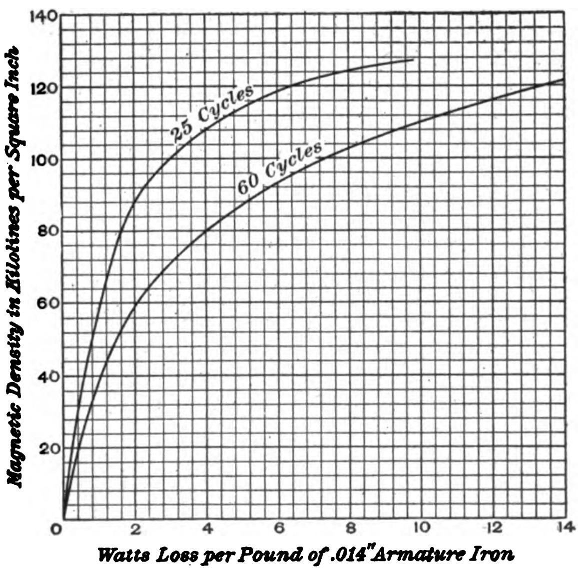


FIG. 39

current machines, namely: core loss, friction and windage, and copper losses in the windings; alternators have additional stray losses also.

82. The iron losses at 25 and 60 cycles can be calculated by the aid of the curves in Fig. 39. These losses must be

calculated separately for each part of the cores in which the magnetic density differs materially, as in the teeth and the core body. For example, the weight of metal and the magnetic density in the teeth are calculated and the weight is multiplied by the loss per pound at the calculated density, this loss being obtained from one of the curves. Similar calculations are performed to obtain the loss in the core, and the sum of the tooth and core losses is the total iron loss in the armature.

83. Friction and windage losses cannot be calculated accurately, the usual method being to estimate them from tests on similar machines. These losses are usually considered as belonging to the engine or other prime mover, rather than to the alternator, when the latter is direct driven.

84. The copper losses are simply the I^2R losses found by calculating the resistance of each winding or each phase, and multiplying this resistance by the square of the current in the winding.

Let i = amperes in each phase of the armature winding;
r = resistance per phase, in ohms.

Then, $3\,i^2\,r$ is the copper loss in watts in a three-phase armature, and $2\,i^2\,r$ that in a two-phase armature.

The field loss is calculated by the same method, I^2R being the loss when I represents the field current and R the field resistance. Some contracts specify that rheostat losses must be included, in which case the field loss is $I\,e$, e representing exciter voltage.

85. The losses caused by the magnetizing action of the armature current, usually called *stray losses*, include eddy-current losses in the conductors and in the steel and hysteresis loss in the steel. These losses occur only when the machine is loaded and are therefore not included in the losses measured when the machine is open-circuited. The stray losses are not important in slow-speed salient-pole machines, but are from two to five times the copper losses in a large high-speed alternator. Stray losses cannot be calculated accurately, but must

be estimated from tests on machines similar to the one for which a design is being calculated.

86. The curve in Fig. 40 shows average efficiencies of alternators ranging in capacities from 500 to 11,000 kilovolt-

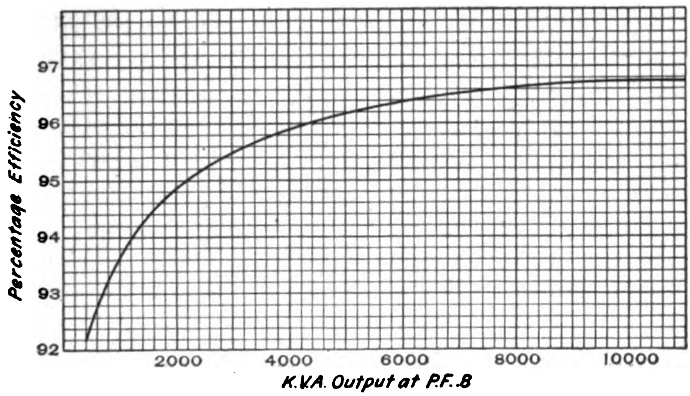


FIG. 40

amperes at .8 power factor This curve gives a fairly good idea of what may be expected from well-designed machines. The efficiency is based on losses in the machine alone, no bearing losses being included.

MECHANICAL DESIGN

STRESSES IN ROTOR STRUCTURES

87. The rotating parts of alternators should usually have little weight beyond that necessary for the required strength. The shaft as well as the stator yoke must be as stiff as is practicable. The yoke must not sag appreciably under the weight of the armature core and windings; it must, also, be strong enough to withstand the stresses occasioned by handling during manufacture and assembly. These stresses must also be considered in connection with the rotor spider; in fact, slow-speed rotors in service are seldom subjected to stresses greater than those met with during shop handling.

Machines direct-connected to high-speed water turbines, however, must be designed to withstand the excessive stresses that will be caused if the governor fails to work properly and allows the turbine to race. Such accidents are not common, but they sometimes happen when the load is suddenly removed from the alternator and the governor fails to shut off the water promptly.

88. The relation of centrifugal force to rotative speed is illustrated by the fact that a 1-pound weight revolving at 1,800 revolutions per minute with a velocity of 20,000 feet per minute is acted on by a centrifugal force of 2,000 pounds. The centrifugal force acting in any revolving body depends on the weight of the body, the radius of revolution, and the square of the number of revolutions per minute; that is,

$$F = .00034\,W\,R\,n^2$$

in which F = centrifugal force, in pounds;
W = weight, in pounds;
R = radius of revolution, in feet;
n = number of revolutions per minute.

EXAMPLE.—If the field structure of an alternator rotates at 300 revolutions per minute and its poles, weighing 65 pounds each, have centers of gravity 20 inches from the center of revolution, calculate the centrifugal force acting on each pole.

SOLUTION.—In the formula, $W = 65$, $R = 20 \div 12 = \frac{5}{3}$, and $n = 300$; $F = .00034 \times 65 \times \frac{5}{3} \times 300^2 = 3{,}315$ lb. Ans.

89. To calculate the centrifugal force acting on the parts of a machine, the weight of each part and the distance of its center of gravity from the center of revolution are determined, and the formula then applied for the desired speed. When designing an alternator, the dimensions of the parts are first tentatively determined and sketches of the outlines are made. The volume of each part in cubic inches is then calculated and multiplied by the weight per cubic inch of the material to obtain the weight of the part. If the sketches are accurately laid out to scale, the center of each part can be located with reasonable accuracy and its distance from the center of revolution scaled off.

SALIENT-POLE ROTORS

90. Figs. 41, 42, and 43 are sketches of those parts of a salient-pole rotor on which centrifugal force acts most strongly. The reference letters appearing in more than one of these figures have the same signification in each. The construction is typical of many alternators, namely, laminated poles dove-

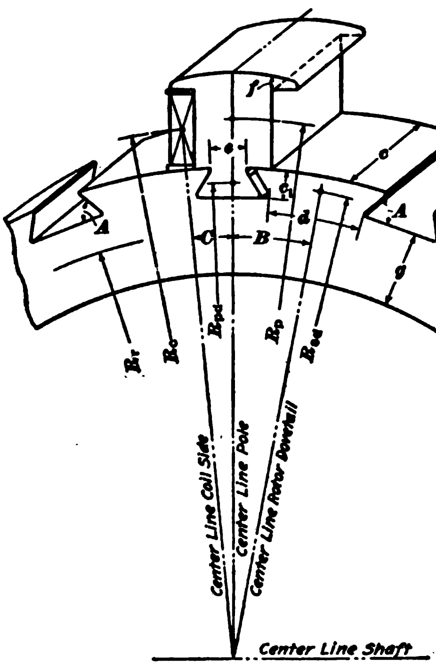


FIG. 41

tailed into a cast-steel spider and held by wedges. The approximate centers of gravity are indicated by black dots near the center of each part, and the radii of revolution are indicated by R_p for the pole only and also for the pole with field coil and coil wedges, R_c for one side of the coil, R_{pd} for the pole dovetail with its holding wedge, R_{sd} for the spider dovetail; and by R_r for the spider rim. When the dimensions indicated in

the sketches are known, the centrifugal forces can be calculated for any number of revolutions per minute.

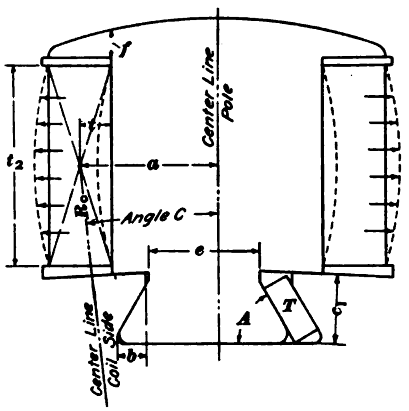


FIG. 42

91. The chief stresses that will act on the structure indicated in Fig. 41 will be those tending to separate the pole from the rim, to break the corners off the pole tip, and to distort the shape of the coil. To ascertain what these stresses will be, the following centrifugal forces must first be calculated, after which the stresses tending to break or distort the parts can be determined:

F_p acting on the pole only, without dovetail or field coil;
F_{pd} acting on the pole dovetail and holding wedge;
F_c acting on the field coil and the coil washers combined;
F_1 acting per inch mean length of field turns;
F_{sd} acting on the spider dovetail;
F_r acting on the spider rim;
F_t acting on the pole complete with dovetail and coil $=(F_p+F_{pd}+F_c)$;
F_s tending to separate the spider dovetails from the rim = number of poles (F_t+F_{sd}).

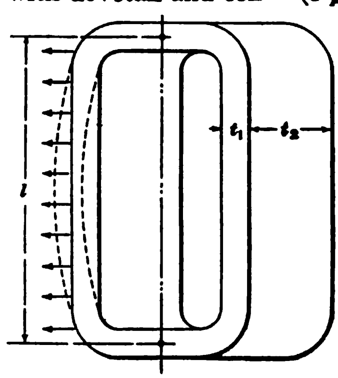


FIG. 43

92. Fig. 42 is an enlarged outline drawing of one pole with its coil, dovetail, and wedge; Fig. 43 shows the shape of a coil. Angle A, Fig. 41, at the spider dovetail is chosen according to the designer's experience; it is here shown as 60°, which is a conservative value. Angle B

between the center lines of a pole and the adjacent spider dovetail is always 360°÷2×the number of poles. Angle C between the center line of a pole and the center line of one side of its coil can be found in a table of natural sines when the perpendicular distance a, Fig. 42, from the center of one side of the coil to the center line of the pole, and the radial distance R_c are known, because $\sin C = a \div R_c$. Knowledge of these angles and of the forces listed in Art. **91** is essential in order to calculate the stresses. In the stress formulas to follow the dimensions referred to are in inches and the stresses are calculated

TABLE III

AVERAGE STRESSES IN METALS USED IN ALTERNATORS

Material	Stress in Pounds per Square Inch		
	Permissible Working	Elastic Limit	Ultimate Strength
Cast steel	12,000	35,000	80,000
Steel punchings	14,000	40,000	80,000
Steel forgings	14,000	40,000	80,000
Chrome vanadium steel	30,000	100,000	120,000
Bronze	10,000		50,000
Steel forgings, oil-tempered	18,000	55,000	90,000

in pounds per square inch. If the stresses thus found exceed the values given in Table III, changes must be made to make the design safe.

93. The parts most likely to break are as follows:

1. The corner may part from the spider dovetail so that the broken section will have the dimensions $c_1\,c$, Fig. 41. If b is a dimension indicated in Fig. 42, the stress, or bending moment, tending to break the corner off the spider dovetail is

$$S_{cc_1} = \frac{3\,\frac{F_t}{2}\,b}{c_1^2\,c} \qquad (1)$$

2. The whole spider dovetail may be broken off in a section $c\,d$, Fig. 41, the stress being

$$S_{cd}=\frac{F_{sd}+\dfrac{F_t \cos\,(A-B)}{\cos A}}{c\,d} \qquad (2)$$

3. The pole dovetail may break along its smallest section with dimensions $c\,e$. The pole is laminated and the ratio of the net iron parallel with the shaft to the gross iron varies from .9 to .95. If this ratio is represented by n, the stress tending to break the dovetail from the pole is

$$S_{ce}=\frac{F_p+F_c}{n\,c\,e} \qquad (3)$$

4. The pole tip may break off along the dimension f, the stress, or bending moment, here being

$$S_f=\frac{6\,F_1\,t\cos C}{f^2} \qquad (4)$$

5. The coil sides may arch outwards into the form indicated by dotted lines, Figs. 42 and 43. If the stress tending to cause this distortion exceeds 3,000 pounds per square inch, the coil sides are generally stiffened by retainers. With coil dimensions $l\,t_1\,t_2$, Fig. 43, and angle C, Fig. 42, this stress may be calculated by the formula

$$S_t=\frac{l\,F_1\sin C}{2\,t_1^2\,t_2} \qquad (5)$$

6. The rotor rim is subjected to stress tending to cause it to part in a section $c\,g$ at the root of a dovetail slot. This stress is

$$S_{cg}=\frac{F_s+F_r}{2\,\pi\,c\,g} \qquad (6)$$

STEAM TURBO-ROTORS

94. The rotors of steam turbo-alternators operate at very high normal speeds, and their mechanical design must receive careful attention in order that the internal stresses shall not

exceed safe limits. These stresses are kept low by making the diameters small and the rotors long in comparison with machines operating at slower speeds.

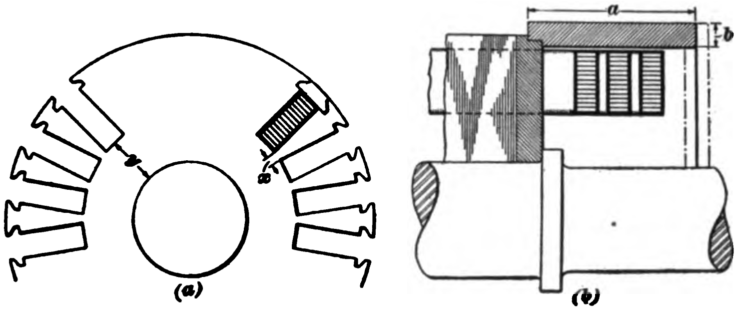


Fig. 44

In Fig. 44 (*a*) is shown part of a cross-section and in (*b*) part of a longitudinal section of a cylindrical rotor such as is commonly used in turbo-alternators. The windings are embedded in slots, except at the ends, where the projecting coils are held in places by rings. A section of one of these rings is shown in view (*b*) with dimensions *a b*. After determining the approximate dimensions and calculating the weights, the centrifugal forces can be calculated by the formula in Art. **88**, the letter C being here used instead of F.

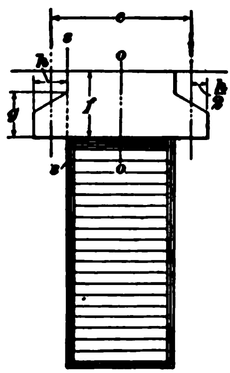


Fig. 45

C_t acting on 1 inch length of a rotor tooth;

C_s acting on 1 inch length of slot contents, including wedge;

C_c acting on 1 inch length of total rotor core, including teeth and slot contents;

C_f acting on all the windings projecting at one end of the core;

C_r acting on one of the rings holding the coil ends.

95. After determining these centrifugal forces, their effect in causing stresses tending to break or distort the structure can be calculated by substituting the forces in pounds, and the

dimensions referred to in the formulas are indicated in Figs. 44 and 45. The stresses thus calculated are in pounds per square inch, and they should not exceed the limiting values given in Table III.

1. The stress tending to burst the holding ring along a section $a\ b$ is

$$S_{ab}=\frac{C_f+C_r}{2\ \pi\ a\ b} \qquad (1)$$

2. The tensile stress S_c tending to break each rotor tooth is

$$S_c=\frac{C_t+C_s}{x} \qquad (2)$$

3. The stress S_d tending to burst the rotor core is

$$S_d=\frac{C_c}{2\ \pi\ y} \qquad (3)$$

4. The stress S_o tending to arch the wedge outward at its center is

$$S_o=\frac{.75\ C_s\ e}{f^2} \qquad (4)$$

5. The stress S_s tending to shear the corner off the wedge along the line $s\ s$ is

$$S_s=\frac{C_s}{2\ g} \qquad (5)$$

6. The bending stress S_b acting in the section $s\ s$ is

$$S_b=\frac{1.5\ C_s\ h}{g^2} \qquad (6)$$

7. The combined bending and shearing stress S_{os} acting in the section $s\ s$ is

$$S_{os}=.35\,S_b+.65\sqrt{S^2_b+2.3\,S^2_s} \qquad (7)$$

ROTOR BEARINGS AND SHAFTS

96. The length of a bearing is usually made from two and one-half to four times the diameter. The projected area of a bearing is the product of the diameter and length. The

dimensions should be selected so that the pressure per square inch of projected area will not exceed 70 pounds with rubbing velocities below 1,500 feet per minute, and proportionately less for higher velocities.

97. After the bearing sizes have been determined and the distance between centers of bearings is settled, the diameter of the shaft between the bearings can be estimated. This diameter is generally chosen so that the deflection of the shaft with the rotor will not exceed 8 per cent. of the air gap; a deflection of one-half this maximum is better practice.

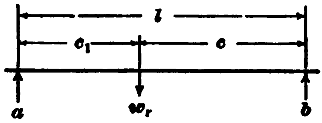


FIG. 46

In Fig. 46, a and b represent centers of bearings, c_1 and c the distances in inches from these centers to the central point of the rotor weighing w_r pounds, and l the total distance in inches between bearing centers $= c_1 + c$.

Let d = diameter of a steel shaft, in inches:

y = deflection of shaft, in decimal part of single air gap;

l_g = single air gap, in inches.

Then,
$$d = \sqrt[4]{\frac{w_r\, c^2_1\, c^2}{1{,}375{,}000\, y\, l_g\, l}} \qquad (1)$$

If the center of the rotor is midway between the bearing centers $c_1 = c = \frac{l}{2}$, the formula reduces to

$$d = \sqrt[4]{\frac{w_r\, l^3}{10^6 \times 22\, y\, l_g}} \qquad (2)$$

NOTE.—The fourth root of any quantity may be obtained by taking the square root of that quantity and then the square root of this result. For example, $\sqrt{256} = 16$; $\sqrt{16} = 4$; and $4 \times 4 \times 4 \times 4 = 16 \times 16 = 256$.

EXAMPLE.—The shaft of a horizontal turbo-alternator is carried in bearings with centers 6 feet 5 inches apart. The rotor weighs 3,050 pounds and is mounted so that its center is half way between the bearing centers. The air gap, or clearance, is $\frac{5}{8}$ inch. Calculate the diameter of a steel shaft for $3\frac{1}{2}$ per cent. deflection.

SOLUTION.—In the formula $w_r=3{,}050$, $y=.035$, $e=.625$, and $l=77$. Then,

$$d=\sqrt[4]{\frac{3{,}050\times77\times77\times77}{10^6\times22\times.035\times.625}}=7.35, \text{ nearly, or, say, } 7\tfrac{3}{8} \text{ in. Ans.}$$

98. Critical Speed.—Every shaft with its rotor has a certain natural rate of vibration; that is, if the shaft receives a blow it will vibrate like a string of a musical instrument at a certain definite rate, which is determined by the length and the stiffness of the shaft and the weight of the rotor. When operating, the weight of the rotor causes the shaft to bend to and fro for each revolution. If the speed of the rotor is changed, the rate at which the shaft is subjected to this bending is also changed; when the rate of this impressed bending is equal to the natural rate of vibration of the rotor, the effect will be cumulative and the vibrations may become violent enough to destroy the machine. The speed at which such phenomena occur is called the *critical speed* of the rotor.

The critical speeds of slow- and moderate-speed machines need not be considered in designing, for these speeds are nearly always far above the normal speeds. In designing turbo-alternators, however, care must be taken to select a shaft diameter of which the critical speed is not within 20 per cent. of the normal operating speed. The critical speed of a steel shaft for a turbo-alternator can be calculated by the formula

$$N_c=\frac{1{,}570{,}000\,d^2}{\sqrt{w_r\,l^3}}$$

in which $N_c=$ critical speed in revolutions per minute, and the other letters have the meanings given in Art. 97.

For example, a shaft 6 inches in diameter, 60 inches long, with a load $w_r=10{,}000$ pounds has a critical speed in revolutions per minute

$$N_c=\frac{1{,}570{,}000\times6\times6}{\sqrt{10{,}000\times60\times60\times60}}=1{,}215, \text{ approximately}$$

DESIGN OF ALTERNATING-CURRENT MACHINES

(PART 2)

GENERAL METHOD OF DETERMINING ARMATURE DIMENSIONS

1. In design of any kind careful preliminary consideration of several possibilities is required. A general comparison of the merits of these possibilities will usually indicate the most desirable to be selected for investigation in more detail. The design of an alternator is worked out most satisfactorily if approximate dimensions of several possible machines are first determined and their probable performances calculated. The most promising design can then be chosen and more accurate calculations made. The basis for determining the dimensions is the general formula

$$E_p = \frac{4.44\ \Phi\ T_p f\ k_w}{10^8} = \frac{2.22\ \Phi\ Z_p f\ k_w}{10^8}$$

in which E_p = effective volts per phase;
Φ = flux per pole;
T_p = turns per phase;
Z_p = conductors per phase = $T_p \times 2$;
f = frequency;
k_w = distribution factor.

2. In order to use this formula for the purpose indicated, it must be changed so that it contains quantities all of which, except the dimensions to be determined, are known or can be assumed from experimental data on similar machines.

Let P = watts output;
m = number of phases;
I_p = current per phase;
B_g = average number of lines of force per square inch in the air gap;
d = diameter of armature face, in inches;
l = number of inches of laminated steel core parallel to shaft;
$\%$ = fractional part of armature surface covered by pole faces, or ratio of pole arc to pole pitch;
p = number of poles;
K = ampere-conductors per inch of armature periphery;
S = revolutions per minute.

Then,
$$P = m E_p I_p \qquad (1)$$
$$\Phi = \frac{\% \pi d l B_g}{p} \qquad (2)$$
$$K = \frac{m Z_p I_p}{\pi d} \text{ and } m Z_p I_p = K \pi d \qquad (3)$$
$$f = \frac{p S}{120} \qquad (4)$$

These four formulas can be combined with the formula of Art. 1 as follows:

$$P = m I_p \times \frac{2.22 \frac{\% \pi d l B_g}{p} Z_p \frac{p S}{120} k_w}{10^8}$$
$$= \frac{2.22 \% \pi d l B_g m Z_p I_p S k_w}{10^8 \times 120}$$
$$= \frac{2.22 \% \pi d l B_g K \pi d S k_w}{10^8 \times 120} = \frac{\% d^2 l B_g K S k_w}{5.48 \times 10^8} \qquad (5)$$

3. Values of $\%$, B_g, and K can be assumed from experimental data, k_w is fixed by the type of winding chosen, and P and S are specified. The value of $d^2 l$, usually called the *cylindrical inches*, can then be calculated by transforming formula **5** of Art. **2**, thus:

$$d^2 l = \frac{5.48 \times 10^8 P}{\% B_g K S k_w}$$

If either d or l is known, both armature dimensions are now fixed. For example, if the armature is to be made of existing punchings d inches in diameter, the total length of the laminated core must be $d^2 l \div d^2$. The net length of steel is usually taken as from 88 to 90 per cent. of the total length.

If neither the diameter nor the length are fixed by the conditions, a diameter should be chosen so that the length l will be from .75 to 1.5 times the pole pitch. If the length is much greater than this, the poles become very long (parallel with the shaft) and narrow, thus making the periphery of the pole cores unnecessarily long and requiring a correspondingly increased weight of field copper. Long, narrow poles also have large field leakage. The diameters of turbo-alternator armatures are limited by the maximum peripheral speed permissible for the rotor, and if the output is large, long armatures with correspondingly long poles are unavoidable.

4. Output Coefficient.—A convenient expression in the discussion of alternator design is output coefficient, meaning *the number of cylindrical inches per kilowatt per 100 revolutions per minute.* The number of kilowatts per 100 revolutions per minute is $K.\,W. \div \frac{S}{100}$, or $\frac{100\,K.\,W.}{S}$, and the output coefficient is $d^2 l \div \frac{100\,K.\,W.}{S}$, or $\frac{d^2 l S}{100\,K.\,W.}$.

By the formula of Art. 3, $d^2 l S = \frac{5.48 \times 10^8 P}{\%\,B_g K k_w}$. Then, $\frac{100\,d^2 l S}{1{,}000} = \frac{5.48 \times 10^8 \times 100\,P}{\%\,B_g K k_w \times 1{,}000} = \frac{5.48 \times 10^8}{\%\,B_g K k_w} \times 100\,K.\,W.$, and the output coefficient

$$\frac{d^2 l S}{100\,K.\,W.} = \frac{5.48 \times 10^9}{\%\,B_g K k_w}$$

5. Possible values of $\%$, B_g, K, and k_w were given in *Design of Alternating-Current Machines*, Part 1. If $\% = .7$, $B_g = 48{,}000$, $K = 600$, and $k_w = .96$ (three-phase, three slots per pole per phase), the output coefficient will be

$$\frac{d^2 l S}{100\,K.\,W.} = \frac{5.48 \times 10^9}{.7 \times 48{,}000 \times 600 \times .96} = 283$$

If an alternator having 283 cylindrical inches per kilowatt per 100 revolutions is to be built for 500 kilowatts at 200 revolutions per minute and armature punchings having 90 inches internal diameter are to be used, the net length of the armature core parallel to the shaft can be calculated as follows:

$$\frac{90^2 \times l \times 200}{100 \times 500} = 283$$

$$l = \frac{283 \times 100 \times 500}{90 \times 90 \times 200} = 8.73, \text{ or, say, } 8\tfrac{3}{4} \text{ inches}$$

6. Values of Output Coefficient.—For a given output and speed, the cylindrical inches, $d^2 l$, which are a measure of the size of the armature, depend on the values of the quantities k_w, $\%_O$, B_g, and K. The value of k_w is nearly constant, since it depends merely on the arrangement of the armature coils; the pole arc cannot be made much over 75 per cent. of the pole pitch without causing a large amount of magnetic leakage and crowding of the field windings; also B_g cannot be increased beyond a certain amount for reasons previously given, but the number of ampere-conductors per inch K admits of some variation, and the greater this number the smaller will be the armature. However, if K is made too large the machine is liable to overheat, and it can be made high with safety only when the ventilation is good.

A machine with a large number of ampere-conductors per inch has a low output coefficient and may therefore have a small armature, but it must contain a relatively large amount of copper. In ordinary polyphase alternators, the output coefficient usually lies between 200 and 600, the lower limit being for machines operating at high peripheral speed, having usually good ventilation, and designed with a relatively large amount of copper. In single-phase machines, all the armature surface is not utilized, and the output coefficient usually lies between 500 and 900.

DESIGN OF 5,000-KILOVOLT-AMPERE SALIENT-POLE ALTERNATOR

SPECIFICATIONS

7. Let it be assumed that a three-phase alternator is to be designed to conform to the following specifications:

Maximum continuous capacity in kilovolt-amperes, 5,000.
Power factor, 80 per cent.
Speed in revolutions per minute, 300.
Phases, 3.
Normal terminal voltage, 6,600.
Amperes per terminal at full load and at 100 per cent. power factor, $\left(\frac{5,000,000}{\sqrt{3}\times 6,600}\right)=438$.
Frequency, in cycles per second, 60.
Alternator will be excited by a separately-driven exciter with a full-load voltage of 120.
The exciting current will not exceed 385 amperes, when generator carries its maximum rated load of 80 per cent. power factor.
Regulation: When 438 amperes per terminal at 80 per cent. power factor, is thrown off, the terminal voltage must not rise more than 30 per cent.
Temperature: With air temperature 30° C., the alternator must be able to carry continuously 438 amperes per terminal at normal terminal voltage and a power factor of 80 per cent. without developing in its hottest part a maximum temperature of over 90° C.
Field Winding: The field winding is to be made of copper strip wound on edge. The insulation of the field winding to ground will be subjected to an insulation test of 1,500 volts alternating current for 60 seconds.

Armature Coils: The armature coils must be form wound, interchangeable, and carefully insulated with the very best materials. The insulation of the armature winding from the core will be subjected to an insulation test of 13,200 volts alternating current for a period of 60 seconds.

Efficiency: The generator must have an efficiency at 80 per cent. power factor of not less than 95½ per cent. at a maximum-load current, at 100 per cent. power factor, of 438 amperes per terminal, 95 per cent. at three-quarter load, and 93½ per cent. at half load. These efficiencies are calculated from the core loss, armature and field copper losses, and the windage and friction losses.

8. General Construction.—The generator is to be of the revolving field type for direct connection to two horizontal waterwheels. The rotor must be designed for a runaway speed of 500 revolutions per minute.

The stator is to have open slots, and must have shields to protect the parts of stator coils that project beyond the stator core. The stator coils, under normal operating conditions, must be able to withstand a short circuit at terminals, without being deformed.

The design of the alternator is to include the stator, the rotor, and the rotor shaft. The base and bearings will be supplied by the turbine builder, who will determine the size of the bearings to support the rotor and the two waterwheels, one of which is to be mounted at each end of the shaft.

ARMATURE DESIGN

GENERAL DIMENSIONS

9. In order to determine the core dimensions, the cylindrical inches are first calculated by the formula of Art. **3.** The value of % for such a machine can be taken rather high, say .75; B_g should be fairly low, say 37,000; the ampere-turns per inch K can be taken at 1,140, and k_w for three slots per pole per phase will be .96. Then,

$$d^2 l = \frac{5.48 \times 10^8 \times 5{,}000{,}000}{.75 \times 37{,}000 \times 1{,}140 \times 300 \times .96} = 300{,}000, \text{ approximately}$$

The rotor spider will be made of a steel casting, and with such a rotor the peripheral speed can be between 8,000 and 9,000 feet per minute. The peripheral speed is calculated by multiplying the circumference in feet by the number of revolutions per minute. If d is taken at 105 inches, $\frac{\pi d}{12} \times 300 = 8{,}250$ feet per minute, which is satisfactory. Then, $l = \frac{300{,}000}{105 \times 105} = 27.2$ inches. The axial length of the core will, therefore, be taken as 27 inches. Between the core and each end plate will be a 1-inch air duct, making the distance between end plates 29 inches. The core will contain eight ½-inch ventilating ducts totaling 4 inches and leaving 23 inches gross steel and approximately $23 \times .88 = 20.2$ inches net steel.

10. The **number of poles** is determined by the specified speed and frequency, because $f = \frac{pS}{120}$; therefore, $p = \frac{120 \times 60}{300} = 24$. Three slots per pole per phase for a three-phase machine makes 72 slots per phase, and a total of 216 slots. As the armature coils must be interchangeable, a two-layer winding will be used.

11. Assumed Flux per Pole.—A trial value for the flux per pole is calculated by estimating the pole-face area and multiplying it by the density that has been found good practice. The length of the pole face can be a little less than that of the armature, or, say, 25.5 inches. If the outer diameter of the rotor is taken as 105 inches, the same as the inner diameter of the stator, the pole pitch will be $\frac{105 \times 3.1416}{24} = 13.74$ inches, and the pole arc will be $13.74 \times .75 = 10.3$, or, say, 10.25 inches, making $\%= .746$. The area of the pole face is, therefore, approximately $25.5 \times 10.25 = 261$ square inches. With an assumed flux density of 37,000 lines per square inch, the total flux per pole is approximately 9,670,000 lines.

12. Number of Conductors.—In the general formula of Art. 1 all the quantities are now known except E_p and Z_p. The value of E_p will depend on the connection of the phases, which in this case will be made *star*, making $E_p = 6{,}600 \div \sqrt{3} = 3{,}815$ volts. By transforming the general formula and substituting the values of known quantities,

$$Z_p = \frac{10^8 E_p}{2.22\, \Phi f k_w} = \frac{10^8 \times 3{,}815}{2.22 \times 9{,}670{,}000 \times 60 \times .96}$$
$$= 309, \text{ approximately}$$

But the number of conductors per phase must be an even multiple of the number of slots per phase, 72, and 288, the multiple nearest 309, will be used, making four conductors, or two turns per slot and 144 turns per phase. The actual number of ampere-conductors per inch inside periphery of the armature, $K = \frac{288 \times 438 \times 3}{3.1416 \times 105} = 1{,}147$.

13. Winding Pitch.—With 216 slots and 24 poles, full-winding pitch requires that each coil span $216 \div 24 = 9$ slots. In order to make it easier to install the coils as well as to reduce the length of coil ends and improve the wave shape of the electromotive force when the machine is loaded, the coils will be made to span 8 slots, making the winding pitch $\frac{8}{9}$, or 89 per cent., of full pitch. According to Fig. 6 of *Design of Alternating-Current Machines*, Part 1, the pitch factor $k_o = .982$, requiring still further increase in the flux first assumed.

14. Actual Flux per Pole.—When the factor k_o is introduced in the general formula and the formula is transposed to find the value of the flux per pole, it becomes

$$\Phi = \frac{10^8 E_p}{4.44\, T_p f k_w k_o} = \frac{10^8 \times 3{,}815}{4.44 \times 144 \times 60 \times .96 \times .982}$$
$$= 10{,}500{,}000, \text{ approximately}$$

ARMATURE SLOT DIMENSIONS

15. The **slot width** is determined by the fact that enough metal must be left in the teeth to keep the flux density within safe limits. This density may be taken as 95,000 lines per square inch. There are 9 slots (and teeth) per pole, 74.6 per cent. of which, or 6.7 teeth, are covered by the pole face. The total flux per pole divided by the product of the number of teeth under the pole face, the net length of steel in the teeth, 20.2 inches, and the allowable density gives the tooth width, or $\frac{10,500,000}{6.7\times 20.2\times 95,000}=.817$ inch. The slot pitch $t_o=\frac{105\times 3.1416}{216}=1.53$ inches; therefore, the slot width is $1.53-.817=.713$. As 95,000 lines per square inch is a comparatively low density, slot width can safely be increased to .75 inch, thus leaving $1.53-.75=.78$ inch width of tooth root and increasing the density to $\frac{.817}{.78}\times 95,000=99,500$ lines per square inch.

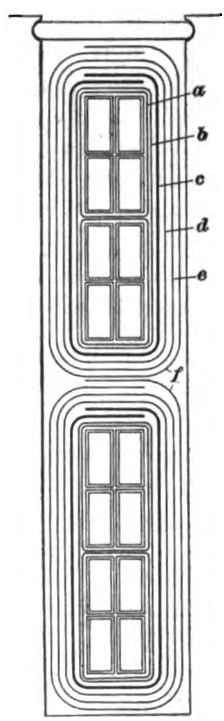


FIG. 1

16. The **slot depth** must be sufficient to hold conductors that will keep the temperature below the maximum limit, 90° C. The core temperature rise should not exceed 40° C., and this added to 30° C., the specified air temperature, gives 70° C. outside the coil insulation. The excess temperature inside the insulation cannot, therefore, be more than $90-70=20°$ C. The formula $T=\frac{P_s O}{.003}$ can be used to determine the allowable watts per square inch P_s for the temperature difference $T=20°$ C. by assuming the thickness of insulation $O=.19$ for 6,600 volts. Then, $P_s=\frac{.003\times 20}{.19}=.316$.

17. Each slot contains four conductors, which will be arranged four deep in the slot as shown in Fig. 1, each conductor consisting of four strands. The number of amperes per square inch of conductor cross-section may be assumed as 18,00 and the section must, then, be $438 \div 1,800 = .243$ square inch. The slot width is .75 inch, and about .38 inch of this will be occupied by insulation, leaving .37 inch for copper. The depth of copper must be approximately $.243 \div .37 = .657$ inch. The dimensions $.37 \times .657$ are unusual, and, moreover, the eddy-current loss in so large a conductor would be considerable. Each of the four strands of the conductor will be .18 inch $\times$ .35 inch, arranged as in Fig. 1.

The insulation above and below each pair of conductors forming a side of a coil will occupy about .21 inch, and the total depth required for the four insulated conductors will be $D = 4 \times 2 \times .35 + (4 \times .21) = 3.64$ inches. The kiloampere-conductors per inch of armature periphery, $k_a = 1.147$, the slot pitch $t_o = 1.53$, the kiloamperes per square inch of conductor section $I_k = \frac{438}{4 \times .18 \times .35 \times 1{,}000} = 1.738$, and the slot width $W = .75$. Then, the actual number of watts per square inch of conductor surface is $P_s = \frac{.395\, k_a\, t_o\, I_k}{W + D} = \frac{.395 \times 1.147 \times 1.53 \times 1.738}{.75 + 3.64} = .275$, and $T = \frac{P_s\, O}{.003} = \frac{.275 \times .19}{.003} = 17.4°$ C., which is only 2.6° C. below the maximum allowable temperature difference, showing that the assumptions are correctly made.

The slot stick will require a space of .21 inch, making the total slot depth $3.64 + .21 = 3.85$ inches.

STATOR-CONDUCTOR INSULATIONS

18. The insulation on the slot portions of the stator conductors is indicated in Fig. 1 and Table I. The complete insulation is as follows:

1. Double-cotton covering on each strand.
2. Each four-strand conductor is taped all around the coil with half-lapped varnished cloth *a*.

3. The two conductors in each layer are bound together with cotton tape *b*.

4. One turn of mica insulation *c* is placed around the slot portion of the coil.

5. One layer of half-lapped varnished cloth *d* is wrapped around the full length of the coil.

6. The coil is then impregnated by the vacuum process.

7. Six layers of half-lapped varnished cloth *e* are wrapped around the full length of the coil, and two extra layers are wrapped around the coil ends outside the slots.

8. The coil is again impregnated by the vacuum process.

9. One layer of half-lapped cotton tape is wrapped on the coil ends.

10. One turn of .013-inch fish paper *f* is placed around the slot portions of the coil.

11. The slot portions are hot-pressed to exact form and size.

TABLE I

THICKNESS OF INSULATION AND APPARENT DIELECTRIC STRENGTH

Insulating Materials on Each Layer of Winding	Thickness		Dielectric Voltage
	Width	Depth	
Cotton covering on strands	.034	.068	600
One layer ½-lap varnished cloth on conductors	.024	.024	4,500
One layer cotton tape	.012	.012	
One layer mica insulation	.040	.060	19,000
One layer ½-lap varnished cloth	.024	.024	4,500
Six layers ½-lap varnished cloth	.144	.144	20,000
One layer fish paper	.026	.039	4,200
Total	.304	.371	52,800

19. The dielectric strength, 52,800, divided by the specified puncture test, 13,200, gives the **factor of safety** of the armature insulation 4. The connectors between coils will be wrapped with eight layers of half-lapped varnished cloth and then one layer of cotton tape. After the stator is wound, two coats of insulating varnish will be applied to the coil ends.

20. The thickness of insulated coils is .36+.304=.664 inch, leaving an allowance of .75−.664=.086 inch. The depth of insulated copper in the slots is 4×.7+2×.371=3.542 inches, leaving an allowance of 3.64−3.542=.098 inch. These allowances are in accordance with good shop practice, for the coils can be placed in the slots without undue crowding.

ARMATURE CORE SECTION

21. The depth of steel in the core must be enough to carry the flux at a safe density, say 60,000 lines per square inch. The flux from each pole divides, so that the core section carries one-half of it each way, or 10,500,000÷2=5,250,000 lines of force. The sectional area must, therefore, be approximately 5,250,000÷60,000=87.5 square inches. The net axial length of the core is 20.2 inches, and the depth of steel under the slots must be 87.5÷20.2=4.33, or, say, 4.4 inches.

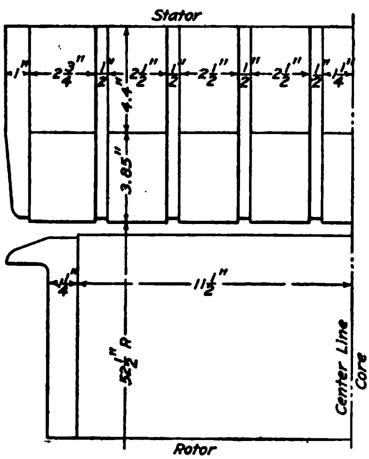


Fig. 2

22. Fig. 2 is an outline of part of the rotor and part of the stator. The slot depth has been found to be 3.85 inches, and the radial core depth under the slots 4.4 inches; the total radial core depth is 3.85+4.4=8.25 inches, and the outside diameter of the stator core is 105+2×8.25=121.5 inches. The laminations will be made in 18 segments having 12 slots each; they will be punched from .014-inch plates and will be varnished or japanned.

AIR GAP

23. The air gap must be long enough radially so that the number of field ampere-turns per pole required to establish the flux in it is at least 1.4 times the number of armature

ampere-turns per pole. In the machine considered, the constant 1.5 will be used. $I\,T_p = .707\,m\,T_{pp}\,I_p = .707 \times 3 \times 6 \times 438 = 5{,}574$, and the number of gap ampere-turns per pole must be $I\,T_g = 1.5 \times 5{,}574 = 8{,}361$. The ratio 1.5 is low, but it can be used here because of the low-regulation guarantee in the specifications. The length of air gap thus calculated is often greater than is required for mechanical clearance.

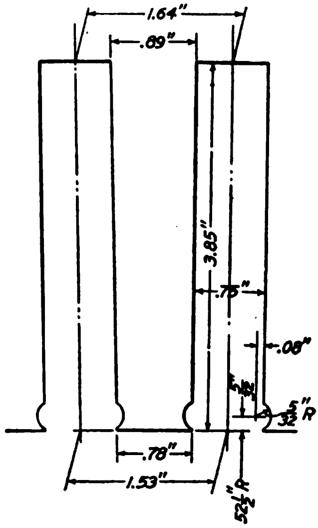


FIG. 3

The density in the air gap B_g has been assumed (Art. **9**) at 37,000 lines per square inch, and $l_g = \dfrac{I\,T_g}{.313\,B_g} = \dfrac{8{,}361}{.313 \times 37{,}000} = .722$ inch, the minimum length of the effective air gap necessary to secure the specified regulation. The pole face will be chamfered to improve the flux distribution, and the actual air gap will be made .625 inch at the center and .74 inch at the tip, the average being .68 inch.

24. The **effective air gap** is calculated by the formula $l'_g = \left(\dfrac{1+\frac{s}{t}}{1+k'\frac{s}{t}}\right) l_g$, in which the slot dimensions are as shown in Fig. 3; $s = .75$, $t = .78$, $\dfrac{s}{l_g} = \dfrac{.75}{.68} = 1.1$, $l_g = .68$, and by the curve given in connection with the discussion of effective air gap in *Design of Alternating-Current Machines*, Part 1, $k' = .8$. Then,

$$l'_g = \left(\frac{1+\frac{.75}{.78}}{1+.8\times\frac{.75}{.78}}\right) \times .68 = \frac{1.96}{1.77} \times .68 = .753 \text{ inch.}$$

MAGNETIC CIRCUIT

25. Pole Cores.—The useful flux per pole is 10,500,000 lines of force, and for preliminary calculations the leakage coefficient can be assumed to be 1.3, making the total flux $1.3 \times 10{,}500{,}000 = 13{,}650{,}000$ lines. The pole will be laminated, and the flux density may be assumed at 95,000 lines per square inch, requiring a pole section of $13{,}650{,}000 \div 95{,}000 = 144$ square inches.

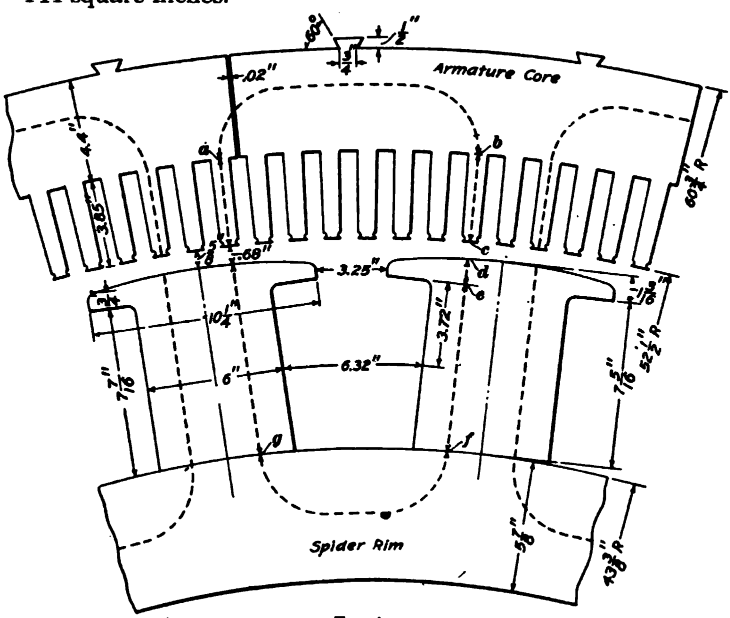


FIG. 4

The length of the pole parallel to the shaft has been taken at 25.5 inches (Art. 11). Each lamination will be $\frac{1}{16}$ inch thick, and the net steel may be taken as 93 per cent. of the pole length, or $.93 \times 25.5 = 23.7$ inches. The width of the pole, or the *pole waist*, must, therefore, be $144 \div 23.7 = 6.08$, or, say, 6 inches.

For this type of machine the radial length of the pole cores is usually from 1 to 1.5 times the waist. It will here be taken at $7\frac{5}{16}$ inches, or approximately 1.22 times the waist.

26. Spider Rim.—The spider rim must be at least 5.875 inches thick in order to be strong enough mechanically. Its width parallel to the shaft will be 25.5 inches, and the area across which the flux must pass will be $5.875 \times 25.5 = 150$ square inches. This section carries one-half the flux in the pole core, and the density is, therefore, $\frac{13,650,000}{2 \times 150} = 45,500$ lines per square inch, which is lower than is necessary in steel for purely magnetic reasons.

27. Leakage Flux.—The leakage flux is calculated by the formula $\Phi_l = 3.19\ X\left[\left(\frac{4\,h'}{a'}+\frac{2\,h}{a}\right)L + 3.2\ h\,k + 6.42\ h'\,k'\right]$, in which the quantities have the following values, dimensions being in inches, as indicated in Fig. 4. The distance across the pole tip $h' = .75$; the distance between pole tips $a' = 3.25$; the length of space available for the insulated field coil is $h = 7\frac{5}{16}$; the distance between poles at their centers is $a = 6.32$; the axial length of the pole $L = 25.5$; the thickness of the pole waist is $w = 6$ inches; the chord of the pole arc is $w' = 10.25$; the ratio $\frac{w}{a} = \frac{6}{6.32} = .95$, the ratio $\frac{w'}{a'} = \frac{10.25}{3.25} = 3.15$, and by the curve in *Design of Alternating-Current Machines*, Part 1, under the heading Field Leakage, the values of k and k' for these ratios are, respectively, .4 and .76. Then,

$$\Phi_l = 3.19\ X\left[\left(\frac{4 \times .75}{3.25}+\frac{2 \times 7.3125}{6.32}\right) \times 25.5 + 3.2 \times 7.3125 \times .4 + 6.42 \times .75 \times .76\right] = 305\ X$$

When the number of ampere-turns X required to set up the useful flux through the air gap and stator teeth is known, the leakage flux can be readily calculated by this expression.

28. Density in Air Gap.—The **effective polar arc** equals the actual arc plus $k\,l_g$, in which expression the value of k depends on the ratio $\frac{\text{pole pitch} - \text{pole arc}}{l_g}$, or $\frac{13.74-10.25}{.68}$

$=5.13$. By the curve for determining the effective polar arc, given in Fig. 25, *Design of Alternating-Current Machines*, Part 1, the factor k for the ratio 5.13 is 1.6. Then, $k\,l_g=.68\times1.6=1.09$. The effective polar arc is $10.25+1.09=11.34$ inches.

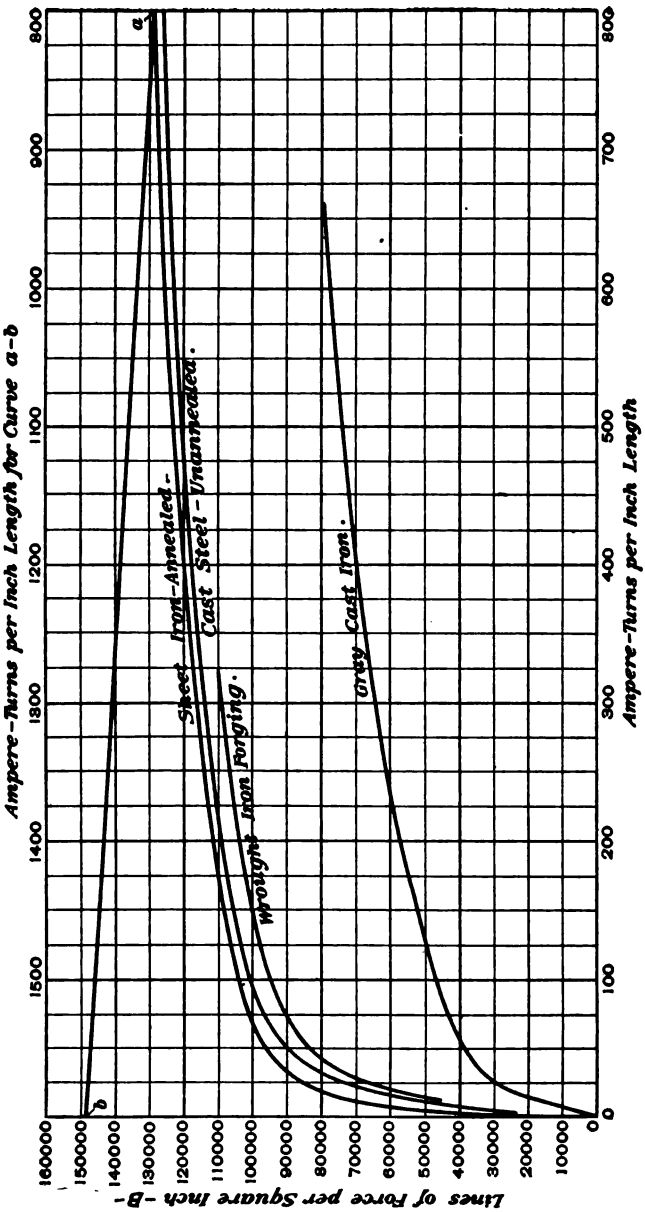
The effective length of the pole face will be greater than the actual length, because of fringing of the lines of force, and may be taken as the actual length plus the actual air gap, or $25.5+.68=26.18$ inches.

The effective area of the air gap will then be $11.34\times26.18=296.9$ square inches, and the effective gap density will be $\frac{10{,}500{,}000}{296.9}=35{,}400$, nearly.

29. Density in Armature Teeth.—As shown in Fig. 3, the width of each tooth at the root is .89 inch and at the tip .78 inch. The average width is $\frac{.89+.78}{2}=.835$. The net steel in each tooth parallel to the shaft has been found to be 20.2 inches, and the number of teeth opposite each pole 6.7. The combined sectional area of all the teeth opposite a pole is $6.7\times.835\times20.2=113$ square inches, nearly. The density in the teeth is $\frac{10{,}500{,}000}{113}=93{,}000$ lines per square inch.

30. Density in the Core.—The cross-section of the armature core is the product of the depth, 4.4 inches, and the net length, 20.2 inches, or 89 square inches. As this section must carry half the useful flux, the density is $\frac{10{,}500{,}000}{2\times89}=59{,}000$ lines per square inch.

31. Leakage Coefficient.—The leakage flux has been found to be $305\,X$, in which expression X is the ampere-turns required for the air gap, which is 8,361 (Art. **23**), plus the number required for the stator teeth. The average flux density in the stator teeth is 93,000, and the length of the teeth is 3.85 inches. The laminations are annealed sheet steel, and Fig. 5 shows that for 93,000 lines per square inch, 40 ampere-turns are required per inch length of tooth. The number of



Ampere-Turns per Inch Length for Curve a-b
800
900
1000
1100
1200
1300
1400
1500
a
b
Sheet Iron-Annealed-
Cast Steel-Unannealed.
Wrought Iron Forging.
Grey Cast Iron.
160000
150000
140000
130000
120000
110000
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
Lines of Force per Square Inch -B-
0
100
200
300
400
500
600
700
800
Ampere-Turns per Inch Length

FIG. 5

ampere-turns for the stator teeth is, therefore, $40\times3.85=154$, and the value of X is $8{,}361+154=8{,}515$. The leakage flux $\Phi_l=305\times8{,}515=2{,}600{,}000$ lines of force. The useful flux is 10,500,000, and the leakage coefficient is $\frac{10{,}500{,}000+2{,}600{,}000}{10{,}500{,}000}$ $=1.248$, instead of the value 1.3 assumed in the preliminary calculations.

32. Densities in Pole Core, Shoe, and Spider Rim. The total flux per pole is $10{,}500{,}000+2{,}600{,}000=13{,}100{,}000$. The dotted lines in Fig. 4 indicate the path of this flux. The spider rim carries all of it in two equal parts; the pole carries practically all of it, although the leakage increases along the path from the spider rim toward the armature; the pole shoe, the air gap, and the armature teeth and core carry only the useful flux.

The sectional area of a pole core is $6\times23.7=142.2$ square inches, and the density in this section is $\frac{13{,}100{,}000}{142.2}=92{,}000$ lines per square inch.

The sectional area of the pole shoe perpendicular to the direction of the flux is $10.25\times23.7=243$ square inches, and the density in this section is $\frac{10{,}500{,}000}{243}=43{,}200$ lines per square inch, approximately.

The section of the spider rim carrying half of the total flux is 5.875×25.5 inches, or 150 square inches, and the density in the rim is $\frac{13{,}100{,}000}{2\times150}=43{,}700$ lines per square inch.

33. Length of Magnetic Circuit.—A drawing of the proposed design is now made to scale, as in Fig. 4, and the lengths of the parts of a magnetic circuit either calculated or measured on the drawing. In some cases it is much more convenient to use the scale measurements, since the calculations would be of a complicated nature. This complete circuit consists of portions of the spider rim and armature core, two pole cores, two air gaps, and twice the length of the teeth.

Part	Length Inches
Armature core $a\,b$	15.000
Teeth $2\,b\,c$, or 2×3.85	7.700
Effective air gaps (Art. 24) are $2\times\frac{1.96}{1.77}\times.68$ $(c\,d)$	1.506
Pole shoes $2\,d\,e$, or 2×1.125	2.250
Pole cores $2\,e\,f$, or 2×7.375	14.750
Spider rim $f\,g$	11.500

SATURATION CURVE

34. A saturation curve showing the relation between excitation and volts in such a machine can be determined by calculating the ampere-turns needed for four voltages distributed over the useful part of the curve. The calculations for the machine under consideration are recorded in Table II. The terminal voltages for which calculations are made are 6,000, 6,600, 8,250, and 8,910. The volts per phase and the flux per pole are calculated as indicated, the formula for Φ being obtained by substituting values already found in the formula

$$\Phi=\frac{10^8\,E_p}{4.44\,T_p\,f\,k_w\,k_e}=\frac{10^8\,E_p}{4.44\times144\times60\times.96\times.982}=2{,}760\,E_p$$

The ampere-turns per inch are read on the curves, Fig. 5. Annealed sheet iron or steel is used for all the metallic parts of the circuit except the spider rim, which is unannealed steel.

35. The calculations thus made determine four points on the saturation curve A, Fig. 6, and these are plotted at *1*, *2*, *3*, and *4*; ampere-turns per pole are indicated by the abscissas and terminal volts by the ordinates. The air-gap line can be determined by plotting the gap ampere-turns per pole for any voltage. For example, for 6,000 volts, the gap ampere-turns per pole are $15{,}200\div2=7{,}600$; an abscissa of 7,600 and an ordinate of 6,000 determines the point *5* through which a straight line is drawn from the origin *O*. The departure of curve A from this line increases as the saturation of the steel increases.

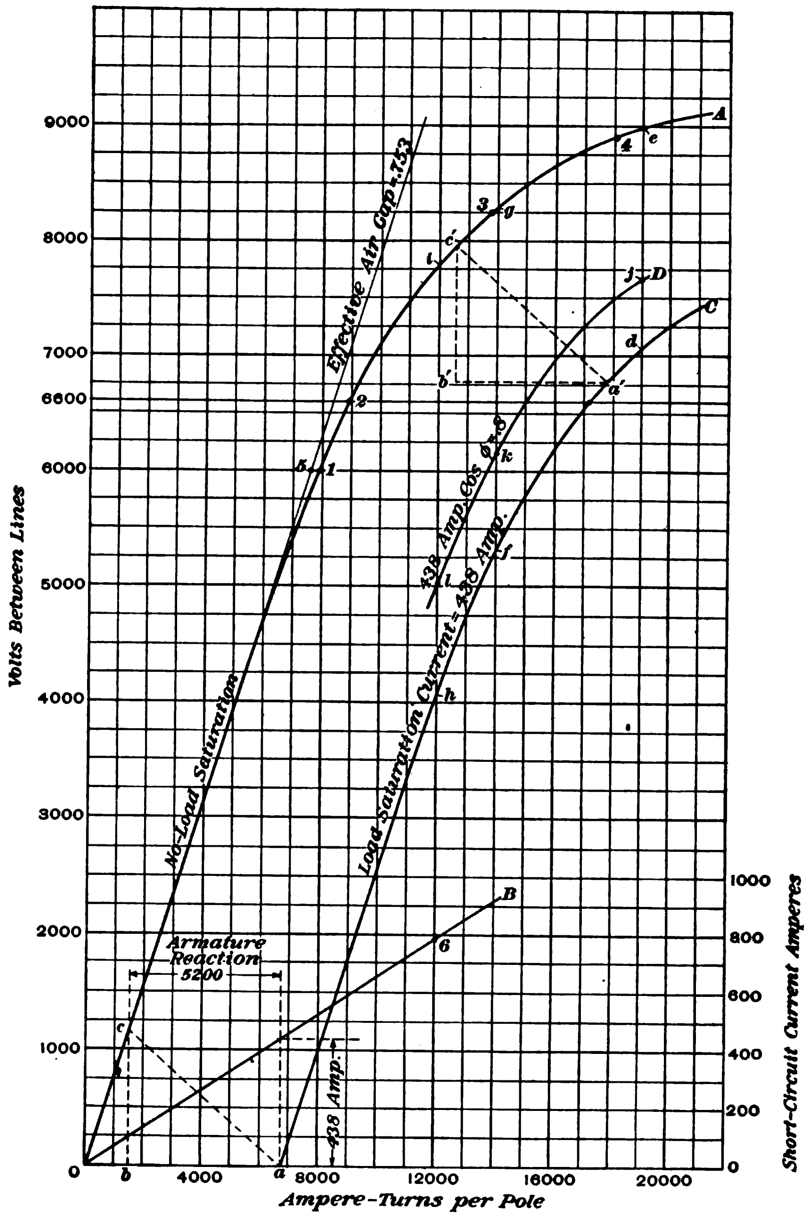

Volts Between Lines
9000
8000
7000
6600
6000
5000
4000
3000
2000
1000
0
b
4000
a
8000
12000
16000
20000
Ampere-Turns per Pole
Short-Circuit Current Amperes
1000
800
600
400
200
0
Effective Air Gap = .753
No-Load Saturation
Load Saturation Current = 438 Amp.
438 Amp. Cos φ = .8
Armature Reaction
5200
438 Amp.
A
B
C
D

FIG. 6

TABLE II
MAGNETIC CIRCUIT DATA

	Terminal Volts E			
	6,000	6,600	8,250	8,910
General				
Volts per phase $E_p = E \div \sqrt{3}$	3,470	3,815	4,770	5,150
Useful flux per pole ($\Phi = 2,760\ E_p$)	9,580,000	10,500,000	13,170,000	14,200,000
Total flux per pole $= 1.248\ \Phi$	11,960,000	13,100,000	16,400,000	17,700,000
Densities				
Air gap, 296.9 sq. in.	32,300	35,400	44,400	47,800
Teeth, 113 sq. in.	84,800	93,000	116,500	125,700
Armature core, 89 sq. in.	53,800	59,000	74,000	79,800
Pole shoes, 243 sq. in.	39,400	43,200	54,000	58,400
Pole core, 142.2 sq. in.	84,000	92,000	115,000	124,000
Spider rim, 150 sq. in.	39,900	43,700	54,700	59,000
Ampere-Turns				
Air gaps, $2 \times .313 \times .753\ B_g$	15,200	16,700	20,900	22,500
Teeth, per in.	25	41	300	600
Teeth, 7.7 in.	193	316	2,310	4,620
Armature core, per in.	6	8	15	20
Armature core, 15 in.	90	120	225	300
Pole shoes, per in.	4	5	6	8
Pole shoes, 2.25 in.	9	11	14	18
Pole cores, per in.	25	40	270	575
Pole cores, 14.75 in.	369	590	3,980	8,500
Spider rim, per in.	8	9	14	17
Spider rim, 11.5 in.	92	104	161	196
Total per pair of poles	15,953	17,841	27,590	36,134
Ampere-turns per pole	7,977	8,921	13,795	18,067

REGULATION

36. To ascertain whether the regulation will be within the specified limit, 30 per cent., with a power factor of 80 per cent., the *short-circuit characteristic* is first drawn. In the formula $I_p = \frac{I_f T_f}{k_c m T_{pp}}$, the field ampere-turns per pole $I_f T_f$ may be taken at any convenient number, say 12,000, the factor k_c may be taken at .85, the number of phases m is 3, and the armature-turns per pole per phase is 6. Then, with 12,000 field ampere-turns per pole, the current with the armature short-circuited would be

$$I_p = \frac{12{,}000}{.85 \times 3 \times 6} = 784 \text{ amperes}$$

In Fig. 6, a scale of amperes is indicated on the right-hand margin. Point *6* is located by the abscissa 12,000 and the ordinate 784, and a straight line B, representing the short-circuit characteristic, is drawn from the origin through this point. This line shows that for the full-load current, 438 amperes, about 6,700 ampere-turns will be required when the armature is short-circuited. The saturation curve for any power factor and full-load current, therefore, starts at the point a representing 6,700 ampere-turns.

37. Of the 6,700 ampere-turns per pole, the larger part, required to compensate for the armature demagnetizing ampere-turns per pole, is determined by the formula

$$D\,I\,T_p = .9\, m\, T_{pp}\, I_p\, k_w\, k_o\, k_p \sin a$$

in which $m=3$, $T_{pp}=6$, $I_p=438$, $k_w=.96$, $k_o=.982$, the ratio pole arc to pole pitch $=.746$, $k_p=.775$, and $\sin a$ may be taken as 1, because the short-circuit current will lag approximately 90° behind the electromotive force. Then,

$$D\,I\,T_p = .9 \times 3 \times 6 \times 438 \times .96 \times .982 \times .775 \times 1$$
$$= 5{,}200, \text{ approximately}$$

38. The ampere-turns effective in setting up the full-load current on short circuit are $6{,}700 - 5{,}200 = 1{,}500$. According

to the no-load saturation curve, 1,500 ampere-turns will set up 1,150 terminal volts, and the point *c* of the triangle *a b c* is thus located. By constructing triangles similar to *a b c* as at *a′ b′ c′*, the point *c′* being on the curve *A*, points on the **full-load saturation curve at zero power factor** are found, and the curve *C* is drawn through these points. This curve shows that 17,200 ampere-turns will be required to maintain 6,600 volts at full load with *0* power factor, or 17,200−8,921 =8,300 ampere-turns, approximately, more than will be required at no load.

39. The **full-load saturation curve at .8 power factor** can be drawn as explained in connection with saturation curves in *Design of Alternating-Current Machines*, Part 1, after the resistance and the resulting voltage drop have been calculated as explained later. The construction for obtaining points on this curve is indicated in Fig. 7, which is similar to Fig. 20, *Design of Alternating-Current Machines*, Part 1. In the actual work of designing, such a figure should be laid out to a scale larger than here shown, and all measurements should be made accurately. Along a horizontal line *a b* a distance *a c* is laid off proportional to the resistance drop, which will be a small percentage of the total voltage.

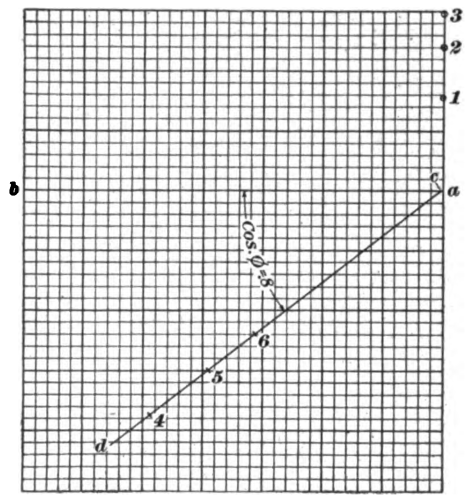
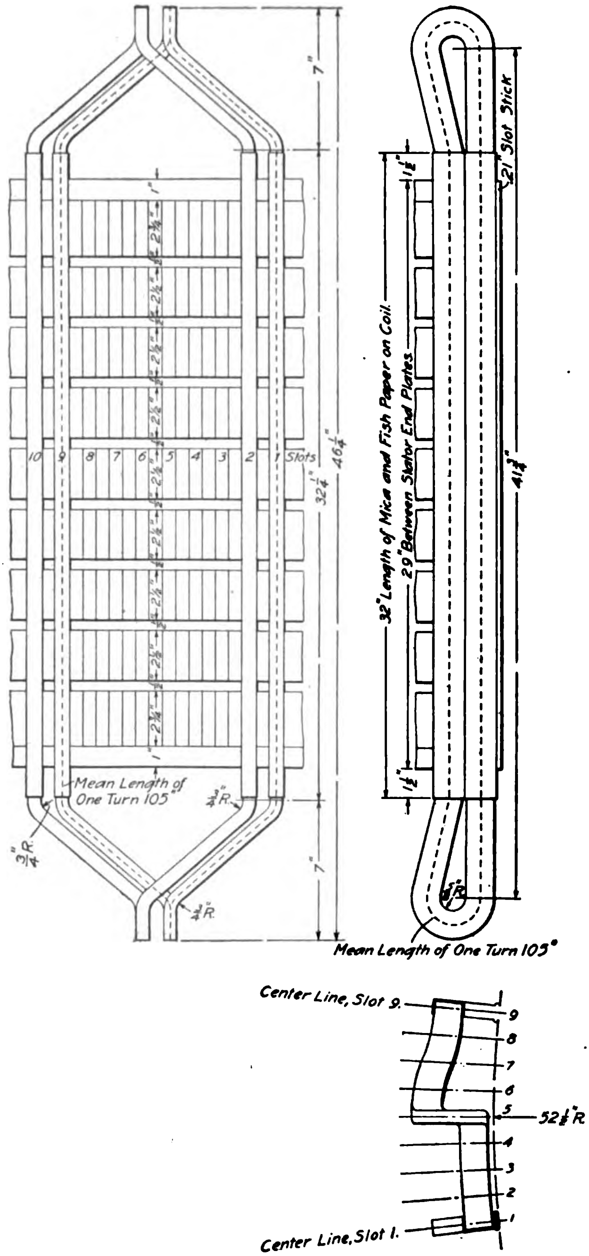


Fig. 7

A line *c d* is drawn at an angle ϕ with line *a b* so that cos ϕ =.8. With point *c* as a center and with radii proportionally equal to *d e*, *f g*, and *h i*, Fig. 6, points *1*, *2*, and *3*, Fig. 7, are located on a vertical line from *a*. With these three points as centers and with radii respectively and proportionally equal



7"
46¼"
32¼"
1"
2⅛"
2½"
10 9 8 7 6 5 4 3 2 1 Slots
Mean Length of One Turn 105"
¾" R.
32" Length of Mica and Fish Paper on Coil.
29" Between Stator End Plates
1½"
21" Slot Stick
41¾"
Mean Length of One Turn 105"
Center Line, Slot 9.
Center Line, Slot 1.
52½" R.

FIG. 8

to the ordinates of the points *e*, *g*, and *i*, Fig. 6, the points *4*, *5*, and *6*, Fig. 7, are located. The distances *c 4*, *c 5*, and *c 6* are proportional to the ordinates of points *j*, *k*, and *l* on the desired curve *D*, Fig. 6.

According to this curve, 15,000 ampere-turns will maintain 6,600 volts at full load; and if the load is thrown off with this excitation the voltage will rise to 8,475, or 28 per cent., which is 2 per cent. less than the specified limit.

RESISTANCE OF ARMATURE

40. In order to calculate the resistance of the armature, a coil is laid out to scale, as in Fig. 8; the mean length of a turn, indicated by the dotted line, is then measured and found to be about 105 inches. When the machine is operating and the coils are warm, the resistance will be approximately 1 ohm per circular-mil inch.

Each conductor consists of 4 strands of .18 in. $\times$.35 in. copper strip, and the sectional area of a conductor is $4\times.18\times.35\times 1{,}270{,}000=320{,}000$ circular mils, nearly. There are 72 coils of 2 turns each per phase, and the length of the conductors in series in each phase is $72\times 2\times 105=15{,}120$ inches. At 1 ohm per circular-mil inch, the resistance of each phase will be $\frac{15{,}120}{320{,}000}$ $=.0473$ ohm. To this number should be added, say, 3 per cent. to cover the resistance of end connections and joints, making the resistance per phase approximately .049 ohm.

41. At full load, 438 amperes, the voltage drop per phase due to resistance is $438\times.049=21.5$, and the drop between terminals is $\sqrt{3}\times 21.5=37.2$ volts. The full-load loss per phase due to this resistance is $438\times 21.5=9{,}400$ watts, and in the three phases $3\times 9{,}400=28{,}200$ watts.

DESIGN OF FIELD WINDING

42. According to the saturation curve, 17,200 ampere-turns are required to maintain 6,600 volts at full load with zero power factor. The field coils of an alternator should be designed to carry without overheating the maximum exciting current required when the machine is operating at very low power factor and possibly with an overload at the same time. In this case the design will be made for approximately 18,000 ampere-turns and for 60° C. maximum temperature rise.

43. The rate at which electric power can be dissipated as heat by the rotor with a temperature rise not exceeding 60° C. must be determined and the field winding calculated accordingly. The rotor diameter will be $105-(2\times .68)=103.64$ inches; the peripheral velocity will be $\dfrac{3.1416\times 103.64\times 300}{12}$ $=8,140$ feet per minute, and at this velocity 5 watts per square inch of coil surface can be dissipated.

44. The field coils will be made of copper strip wound edgewise, and this strip may be assumed as $1\frac{1}{2}$ inches wide. The pole core section has been found to be 6×25.5 inches; if the corners are rounded, the section of the core and field coil will appear as in Fig. 9. The mean length of the field turns, as indicated by the broken line, is $2\times 23+2\times 3\frac{1}{4}+2\times 2\frac{1}{4}\times 3.1416=66.63$ inches.

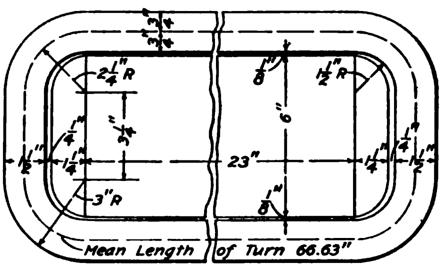


FIG. 9

The field resistance at maximum temperature may be taken at 1.1 ohms per circular-mil inch. In order to determine the dimensions of the field conductors,

let a = sectional area, in circular mils;

e = voltage at field terminals;

I_f = field current, in amperes;
L_f = mean length of field turn, in inches;
T_c = the number of turns per field coil;
p = number of field coils;
r = resistance of field circuit, in ohms.

Then, $$r = \frac{1.1\, L_f\, T_c\, p}{a} \quad (1)$$

$$I_f = \frac{e}{r} = \frac{e\,a}{1.1\, L_f\, T_c\, p}$$

The ampere-turns $I_f\, T_c = \dfrac{e\,a\,T_c}{1.1\, L_f\, T_c\, p} = \dfrac{e\,a}{1.1\, L_f p}$

and $$a = \frac{1.1\, I_f T_c\, L_f\, p}{e} \quad (2)$$

In this design, $I_f\, T_c$ is to be 18,000, $L_f = 66.63$, $p = 24$, and $e = 120$; then,

$$a = \frac{1.1 \times 18{,}000 \times 66.63 \times 24}{120} = 264{,}000 \text{ circular mils}$$

45. The number of square mils is $264{,}000 \times .7854 = 207{,}000$, and if the strip is $1\frac{1}{2}$ inches wide it must be $\dfrac{207{,}000}{1.5 \times 1{,}000{,}000} = .138$ inch thick. A copper strip $.14 \times 1.5$ inches has a sectional area of $\dfrac{.14 \times 1.5 \times 1{,}000{,}000}{.7854} = 267{,}400$ circular mils, nearly, and it may answer.

The outer periphery of the coil is $2 \times 23 + 2 \times 3\frac{1}{4} + 2 \times 3 \times 3.1416 = 71.35$ inches. The pole cores are $7\frac{5}{16}$ inches in radial length; $\frac{3}{4}$ inch must be allowed for two insulating collars, one at each end of the coil, leaving $6\frac{9}{16}$ inches winding space. The total radiating surface on 24 coils will be $24 \times 71.35 \times 6.5625 = 11{,}240$ square inches.

46. The insulation between field turns will be .013-inch paper, making the space occupied by an insulated turn $.14 + .013 = .153$ inch. The number of turns possible in the available

space is $\frac{6.5625}{.153} = 42.9$. In order to avoid any possibility of crowding, 42 turns should be allowed; as the coil terminals must come out on opposite sides of the machine in order to facilitate connections, there will be 41½ active turns per coil. The resistance of the field circuit, calculated by formula **1** of Art. **44**, is $r = \frac{1.1 \times 66.63 \times 41.5 \times 24}{267{,}400} = .273$ ohm.

The field current at full voltage will be $\frac{120}{.273} = 440$ amperes. The maximum watts to be dissipated by the field coils will be $440 \times 120 = 52{,}800$, and the watts per square inch will be $\frac{52{,}800}{11{,}240} = 4.7$, nearly, which is within the allowable limit, 5, for 60° C. temperature rise. The maximum excitation will be $440 \times 41.5 = 18{,}260$.

LOSSES AND EFFICIENCY

STEEL LOSSES

47. The losses in any generator may be classed as steel or iron losses, copper losses, and friction and windage losses. Steel, or eddy-current, losses occur in each steel part of the magnetic circuit in which the flux varies. These losses are estimated by calculating the weight in pounds of each part having a distinctive flux density and multiplying this weight by the number representing the loss per pound at the given density. Steel losses remain practically constant at all loads.

The volume of all the teeth is $.835 \times 3.85 \times 20.2 \times 216 = 14{,}000$ cubic inches, and as steel weighs .28 pound per cubic inch, the weight of the teeth is $14{,}000 \times .28 = 3{,}920$ pounds. The density in the teeth is 93,000 lines per square inch at 6,600 volts, and by the curve of losses given in connection with the discussion of efficiency in *Design of Alternating-Current Machines*, Part 1, the loss per pound will be 6 watts. The **loss in the teeth** will therefore be approximately $6 \times 3{,}920 = 23{,}520$ watts.

48. The diameter of the armature core at the tooth roots is $105+2\times3.85=112.7$, and the area of a circle with this diameter is 9,976 square inches. The outer diameter of the core is 121.5 inches, and the area of a circle of this diameter is 11,594 square inches. The volume of the core is $(11,594-9,976)\times20.2=32,684$ cubic inches. The weight of the core is $32,684\times.28=9,152$ pounds. The density in the core at normal voltage is 59,000, the loss per pound at this density is 2 watts, and the total loss in the armature core is $2\times9,152=18,304$ watts. The total steel losses are the sum of the losses in the teeth and core, or $23,520+18,304=41,824$ watts, or practically 41.8 kilowatts.

COPPER LOSS

49. The **I^2R loss in the armature conductors** varies with the load and should be calculated for three different load conditions. The loss at full load has been determined as 28,200 watts (Art. **41**), and this loss varies as the square of the load in amperes; that is, at three-quarters load it is $\frac{9}{16}\times28,200=15,860$ watts, and at one-half load it is $\frac{1}{4}\times28,200=7,050$ watts. To these values must be added approximately 10 per cent. to cover the eddy-current loss in the armature conductors, making 31, 17.45, and 7.76 kilowatts.

50. The **field copper loss** is calculated by squaring the number of amperes of field current and then multiplying by the hot field resistance. As shown by the saturation curve, Fig. 6, the excitation required to maintain 6,600 volts at full load and 80 per cent. power factor is 15,000 ampere-turns. If similar curves were plotted for three-quarter and one-half load the excitation would be found to be, respectively, 13,200 and 11,800 ampere-turns, approximately. The field current in each case is $I_f T_c \div 41.5$, and the field currents for the three load conditions are, respectively, 361, 318, and 284 amperes. The field resistance $r=.273$ ohm, and the three field losses $I_f^2 r=35,600$, 27,600, and 22,100 watts, or 35.6, 27.6, and 22.1 kilowatts. Field-rheostat losses are not usually charged to a generator unless the exciter is direct-connected.

WINDAGE AND FRICTION LOSSES

51. The losses due to the movement of air by the rotor and to friction in the bearings can be assumed constant at 40 kilowatts, this assumption being based on experience with other similar machines. This loss can be added to the iron losses, making the constant losses 81.8 kilowatts.

EFFICIENCY CALCULATIONS

52. The efficiency calculations for three points on the curve shown in Fig. 10 are recorded in tabular form, as follows,

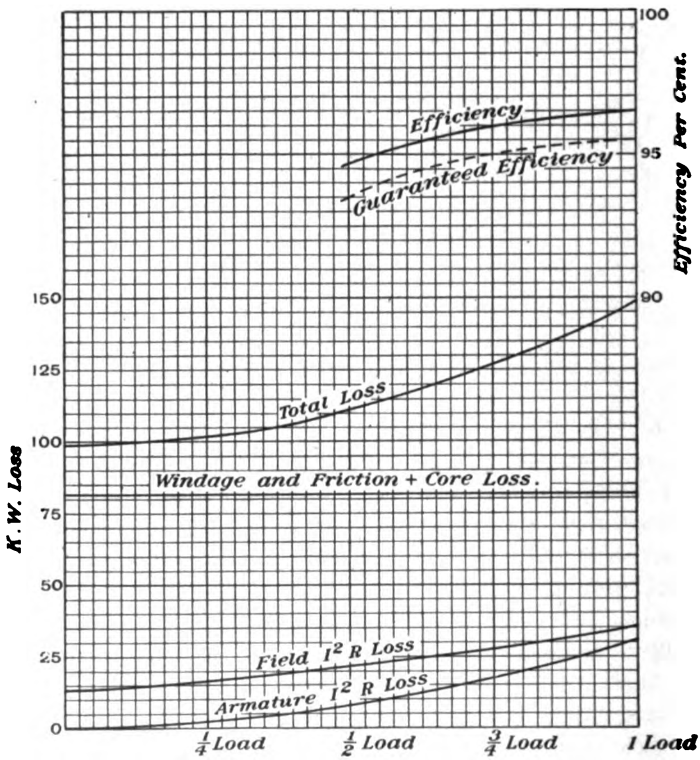
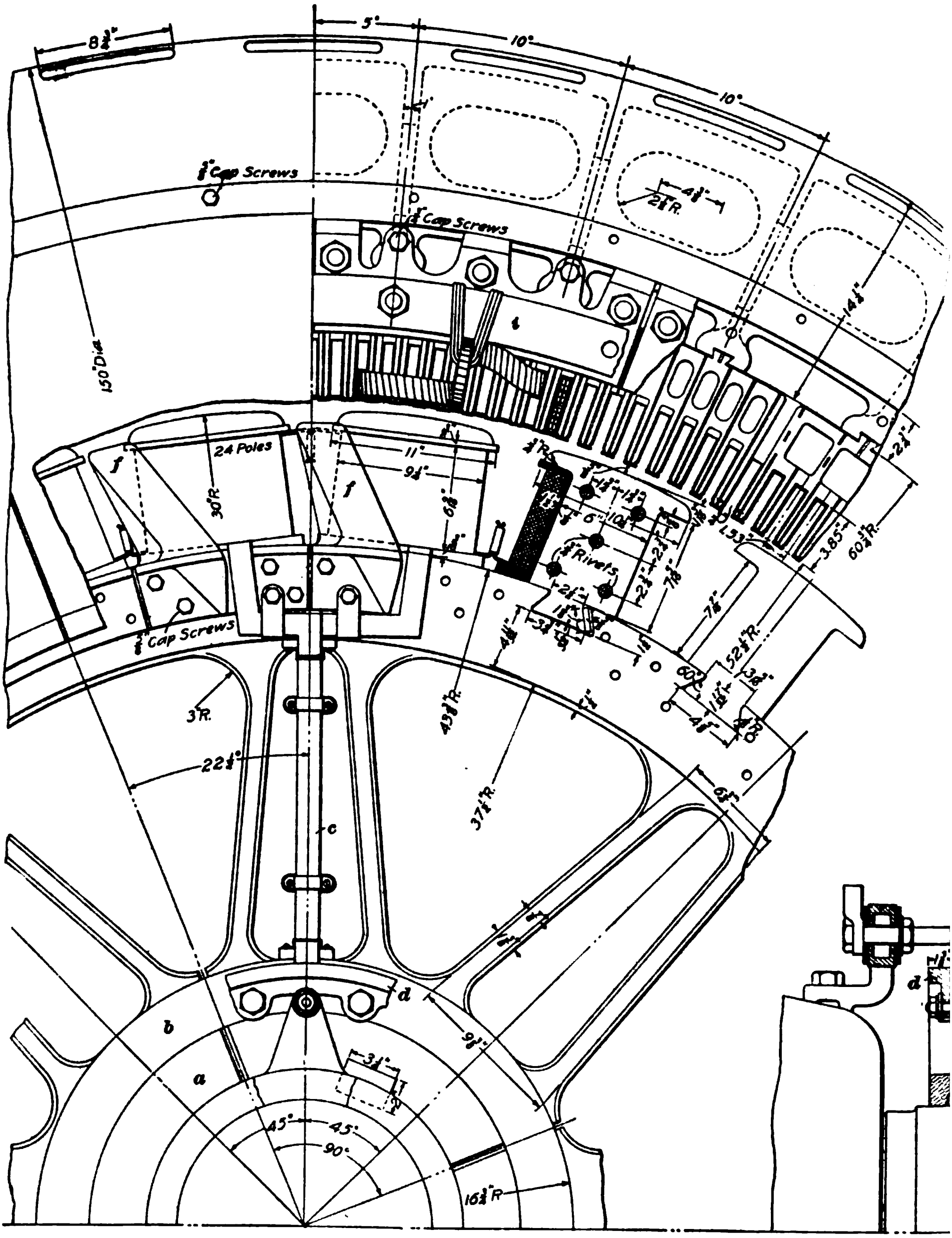
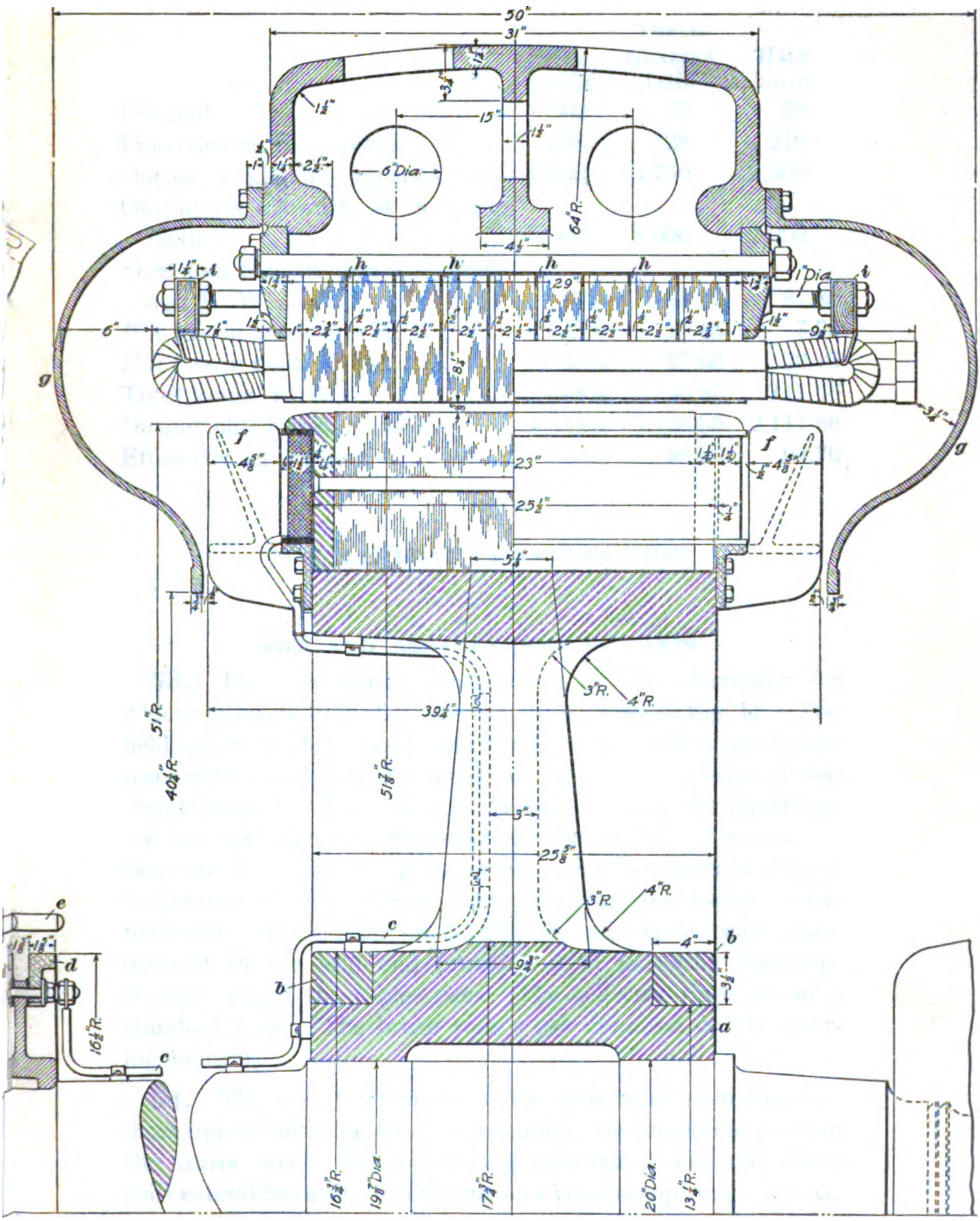


FIG. 10

and the curve is plotted. The variation of individual losses is also shown in the illustration.


5°
10°
10°
Cap Screws
Cap Screws
24 Poles
150" Dia
Rivets
Cap Screws
45°
45°
90°
a
b
c
d
d
f
f



50"
31"
15"
6"Dia
64"R.
4½"
29"
1"Dia
23"
25½"
5¼"
51"R.
40¼"R.
51⅞"R.
39¼"
3"R.
4"R.
25⅝"
3"
9¾"
4"
3½"
16½"R.
16¾"R.
19⅞"Dia.
17¾"R.
20"Dia.
13¼"R.
a
b
c
d
e
f
g
h
i

FIG. 11

	Full Load	Three-Quarter Load	Half Load
Per cent	100	75	50
Line current, in amperes	438	328	219
Output, in kilovolt-amperes	5,000	3,750	2,500
Output, in kilowatts at .8 power factor	4,000	3,000	2,000
Steel loss plus friction and windage, K. W.	81.8	81.80	81.80
I^2R loss in armature, K. W.	31.0	17.45	7.76
I^2R loss in field, K. W.	35.6	27.60	22.10
Total losses, K. W.	148.4	126.85	111.66
Output plus losses, K. W.	4,148.4	3,126.85	2,111.66
Efficiency, in per cent	96.4	96.00	94.70

MECHANICAL CONSTRUCTION

SIZE AND ARRANGEMENT OF PARTS

53. The mechanical construction of the alternator for which the calculations have been made is shown in Fig. 11. The field spider, made of cast steel, has eight arms and is cast in one piece with a split hub *a* clamped to the shaft by means of two shrink rings *b*. The poles are laminated, and the punchings are held between the end heads by five rivets. The laminations are dovetailed to the spider rim. Rubber-covered cables *c* lead from the collector rings *d* along the shaft and a spoke to the field coils. These leads are held firmly in place by brass cleats screwed to the arm and provided with insulating bushings through which the cables pass. The collector rings are of a standard type. The brush rods *e* are mounted on the bearing housing.

54. The *fans f*, which are made from aluminum castings, discharge air into the space surrounding the projecting parts of the stator coils. The castings *g* projecting over the stator coils extend inwardly to the fans and have no openings, so that

the air discharged by the fans must pass out through the openings in the yoke. When air is discharged into the space surrounding the ends of the stator coils, part of it passes around and through the coils, hence through openings *h* in the stator end plates and out through the yoke openings. The greater part, however, is caught up by the poles and thrown from the interpolar space through the stator-core ducts and thence out through the yoke openings. The yoke openings are so arranged that there can be no pocketing of the heated air. This type of ventilation insures a uniform temperature throughout the generator and prevents excessive heating in any part.

55. The armature laminations are clamped between segmental end plates. At frequent intervals in the core are ½-inch ventilating ducts *h*, and at each end is a 1-inch duct, all made by brass spacing segments. The end segments are made strong enough to support the core punchings. To guard the stator coils against being deformed by the stress during short circuits, the free ends are roped to wooden rings *i* held by studs screwed to the stator end plates.

MECHANICAL STRESSES

56. The stresses in the rotor structure must now be calculated in order to make sure that the sections chosen are strong enough. If any of them are found too small, the whole electrical design may have to be changed. All the calculations are made according to the explanations under the heading of Mechanical Design in the Section entitled *Design of Alternating-Current Machines*, Part 1. The centrifugal forces are first determined by the general formula $F = .00034\, W R n^2$. The weights W are the product of the volumes, in cubic inches, and the weights, in pounds, of the materials per cubic inch, which is taken at .28 for iron and steel and .32 for copper. The dimensions are shown in Figs. 4 and 11. Approximate results are satisfactory for such calculations.

57. The net length of steel in the pole parallel to the shaft is 23.7 inches. The pole arc is 10.25 inches, and the average thickness of the pole shoes is 1.125 inches. The pole-shoe

weight is $10.25\times1.125\times23.7\times.28=76.5$, or, say, 85 pounds, including the flanges that extend over the coil ends. The pole core, exclusive of the shoe and the dovetail, weighs $6\times7.375\times23.7\times.28=294$, making the weight of the pole core and shoe 379 pounds.

The width of the pole dovetail is $2\frac{1}{2}$ inches minimum and $3\frac{3}{4}$ inches maximum. The radial depth is 1.5 inches, and the weight is $\frac{3\frac{3}{4}+2\frac{1}{2}}{2}\times1.5\times23.7\times.28=31.1$ pounds. The pole wedges weigh $\frac{5}{8}\times1\frac{3}{8}\times25.5\times.28=6.15$, making the combined weight of the dovetail and wedges 37.25, or approximately 37 pounds.

The field conductor section is $.14\times1.5$ inches, the mean length of the turns is 66.63 inches, and the number of turns 41.5. The weight of copper in a field coil is $.14\times1.5\times66.63\times41.5\times.32=186$ pounds, and the complete insulated coil with end washers will weigh approximately 200 pounds.

The outer, or greater, arc of the spider dovetail is $\frac{2\times3.1416\times43\frac{3}{8}}{24}-3\frac{5}{16}=8$ inches. The radius of the inner arc is $43\frac{3}{8}-1\frac{17}{32}=41.845$ inches, and of the smaller arc is $\frac{2\times3.1416\times41.845}{24}-4\frac{5}{8}=6.3$ inches. The average arc is $\frac{8+6.3}{2}=7.2$ inches, approximately. The depth of this dovetail is $1\frac{17}{32}$ inches, the length parallel to the shaft is $25\frac{5}{8}$ inches, and the weight is $7.2\times1.53\times25.625\times.28=79.1$, or, say, 80 pounds.

The inner radius of the spider rim is $37\frac{1}{2}$ inches, the radial depth below the dovetail is $4\frac{11}{32}$ inches, and the mean radius is $37\frac{1}{2}+2\frac{11}{64}=39\frac{43}{64}$, or 39.6, inches. The axial width is $25\frac{5}{8}$ inches, and the weight is $2\times3.1416\times39.6\times4\frac{11}{32}\times25\frac{5}{8}\times.28=7{,}800$ pounds. To this must be added the weight of the plates and fan blades bolted to the rim, which will make the total weight to be considered in connection with the rim approximately 8,200 pounds.

58. The radii of revolution obtained from Fig. 11 are as follows:

For the pole core and shoe, $R_p=43\frac{3}{8}+\frac{7\frac{5}{16}+1\frac{3}{16}}{2}=47\frac{5}{8}$ inches $=3.97$ feet.

For the pole dovetail and wedges, $R_{pd}=43\frac{3}{8}-\frac{1\frac{1}{2}}{2}=42\frac{5}{8}$ inches $=3.55$ feet.

For the field coil and washers, $R_c=43\frac{3}{8}+\frac{7\frac{7}{16}}{2}=47\frac{3}{32}$ inches $=3.92$ feet.

For the spider dovetail, $R_{sd}=43\frac{3}{8}-\frac{1\frac{17}{32}}{2}=42\frac{39}{64}$ inches $=3.55$ feet.

For the spider rim, $R_r=37\frac{1}{2}+\frac{4\frac{11}{32}}{2}=39\frac{43}{64}$ inches $=3.3$ feet.

59. The centrifugal forces caused by these rotating parts are calculated by the formula $F=.00034\ W R n^2$, in which $n=500$, the specified runaway speed, and W and R are weight and radius, with the foregoing values for weights and radii. When the value of n is substituted, the formula reduces to $F=.00034\times 250{,}000\ W R=85\ W R$, from which the following forces, in pounds, are calculated:

Pole core and shoe,	$F_p=85\times\ 379\times 3\ 97=$	128,000
Pole dovetail and wedges,	$F_{pd}=85\times\ 37\times 3.55=$	11,160
Field coil and washers,	$F_c=85\times\ 200\times 3.92=$	66,600
One inch mean length of field turns,	$F_1=\frac{66{,}600}{66.63}$ $=$	1,000
Spider dovetail,	$F_{sd}=85\times\ 80\times 3.55=$	24,100
Spider rim,	$F_r=85\times 8{,}200\times 3.3$ $=$	2,300,000
$F_p+F_{pd}+F_c=F_t=128{,}000+11{,}160+66{,}600$	$=$	205,760
$(F_t+F_{sd})\times 24=F_s=(205{,}760+24{,}100)\times 24$	$=$	5,500,000

60. The stresses acting in different parts of the rotor can now be calculated according to formulas given in Art. **93**, *Design of Alternating-Current Machines*, Part 1. In using these formulas, the forces calculated in Art. **59** herein are substituted for F. The dimensions are taken from Fig. 11 or computed from dimensions there given and are in inches. The calculated stresses, as determined by these formulas, are in pounds per square inch.

61. The stress tending to bend or break the corner of the spider dovetail is found by the formula $S_{cc_1}=\dfrac{\dfrac{3F_t}{2}b}{c_1^2c}$, in which b is the overhang, or leverage of the dovetail corner, or $\dfrac{4\frac{5}{8}-3\frac{5}{16}}{2}=.66$; c_1 is the dimension along which the break would probably occur and is given in Fig. 11 as $1\frac{1}{2}$ inches; and c is the width of the spider, given as $25\frac{5}{8}$ inches.

Then,
$$S_{cc_1}=\frac{3\times\dfrac{205{,}760}{2}\times .66}{1.5^2\times 25.625}=3{,}530$$

The stress tending to separate the whole dovetail from the spider is $S_{cd}=\dfrac{F_{sd}+\dfrac{F_t\cos(A-B)}{\cos A}}{c\,d}$, in which B is half a polar angle, or, $\dfrac{360°}{2\times 24}=7\frac{1}{2}°$; cos. $(A-B)=\cos(60°-7\frac{1}{2}°)=\cos 52\frac{1}{2}°=.609$; $\cos A=.5$; and $c=25\frac{5}{8}$, as before. The dimension along which the dovetail would break, if it should give way, may be taken $\frac{1}{4}$ inch longer than the smaller arc found in Art. **57,** on account of the fillets in the dovetail corners; that is, $d=6.3+.25=6.55$ inches. Then,
$$S_{cd}=\frac{24{,}100+\dfrac{205{,}760\times .609}{.5}}{25.625\times 6.55}=1{,}633$$

The stress tending to separate the dovetail from the pole is $S_{ce}=\dfrac{F_p+F_c}{n\,c\,e}$. Here n is the number by which to multiply the pole width (parallel to shaft) in order to obtain the net steel, in this case .93; c is the pole width, 25.5 inches, and e is the short dimension of the neck of the pole dovetail, 2.5 inches. Then,
$$S_{ce}=\frac{128{,}000+66{,}600}{.93\times 25.5\times 2.5}=3{,}281$$

The stress tending to break the tip from the pole shoe is $S_f = \frac{6\,F_1\,t\cos C}{f^2}$. In this formula t is the perpendicular distance of the center of the coil from the side of the pole, or $\frac{1\frac{1}{2}}{2}+\frac{1}{8}=\frac{7}{8}$. C is the angle between radial lines through the center of the pole and the center of one side of the coil. The tangent of this angle is $\dfrac{\frac{6+1\frac{1}{2}}{2}+\frac{1}{8}}{43\frac{3}{8}+\frac{7\frac{5}{16}}{2}}=.0823$; $C=4°\ 42'$, and $\cos C=.997$.

In practice, the dimension f, Fig. 42, *Design of Alternating-Current Machines*, Part 1, along which the tip would probably break, can be scaled from the drawing. It can also be calculated with sufficient accuracy by considering the pole chamfer a straight line such that the maximum air gap is $\frac{3}{4}$ inch and the minimum $\frac{5}{8}$ inch, as previously designed. At the pole center the shoe is $1\frac{3}{16}$ inches thick, as indicated, and at the tip it is $\frac{1}{8}$ inch less. From the pole center to the tip is $5\frac{1}{8}$ inches, and to the edge, above which is the required dimension f, 3 inches.

Then, $3 : 5\frac{1}{8} = x : \frac{1}{8}$, or $x=\frac{3}{5\frac{1}{8}}\times\frac{1}{8}=\frac{3}{41}$ inch, so that f must be approximately $\frac{3}{41}$ inch less than $1\frac{3}{16}$, or 1.1 inches.

Then, $$S_f=\frac{6\times 1{,}000\times .875\times .997}{1.1^2}=4{,}326$$

The bending stress tending to distort the field copper is $S_t=\frac{l\,F_1\sin C}{2\,t_1^2\,t_2}$. In this formula, l is the distance between centers of the coil ends, or $25.5+2\times\frac{1}{4}+1\frac{1}{2}=27\frac{1}{2}$ inches; C is $4°\ 42'$, as previously found, and $\sin C=.082$; t_1 and t_2 are coil dimensions given in Fig. 11 as $1\frac{1}{2}$ and $6\frac{9}{16}$ inches, respectively.

Then, $$S_t=\frac{27.5\times 1{,}000\times .082}{2\times 1.5^2\times 6.56}=76.4$$

The stress tending to burst the rotor spider in its smallest section is $S_{cg} = \frac{F_s + F_r}{2\pi c g}$. In this formula c and g are the dimensions of the section given in Fig. 11 as $25\frac{5}{8}$ and $4\frac{11}{32}$ inches, respectively. To the smaller dimension can be added, say, $\frac{5}{32}$ inch, because of the $\frac{1}{2}$-inch rib inside the spider, making $g = 4\frac{1}{2}$.

Then, $$S_{cg} = \frac{5{,}500{,}000 + 2{,}300{,}000}{2 \times 3.1416 \times 25.625 \times 4.5} = 10{,}759$$

62. All these stresses are within the permissible working limits given in Table III, *Design of Alternating-Current Machines*, Part 1, and are therefore satisfactory. The design is now complete, and a machine built accordingly would probably meet all the specifications. The instructions from the engineering office to the shop must include drawings giving the dimensions of the parts and specifications covering the kind of material for each part, the size of conductors for armature and field, the number of coils, turns per coil, insulation, etc.

TURBO-ALTERNATOR DESIGN

5,000-KILOVOLT-AMPERE ALTERNATOR

63. The rotor design is the chief problem when designing a turbo-generator, because high speed is essential, and the rotor dimensions must be as small as possible to avoid excessive peripheral velocity. The construction must be substantial, in order to withstand the stresses. Other features sometimes requiring special attention are the difficulty of finding room on the smaller rotors for field conductors large enough to carry the exciting current at low power factors without overheating, the problem of laminating the armature conductors of large alternators so as to prevent excessive loss from eddy currents, the proper securing of the ends of the armature coils on large machines, and the problem of ventilating such machines.

Separate blowers are sometimes essential to obtain proper ventilation.

Large flux per pole and a comparatively small number of armature turns per phase are common in turbo-generators. Low-voltage generators of this type, therefore, offer some difficulty to the designer, as the number of turns can be varied but little. Two-, four-, and eight-circuit windings with short-pitch coils are sometimes resorted to on four-pole generators, but even with this arrangement voltages below 2,300 are often difficult to obtain. Multi-circuit windings are also sometimes used so as to reduce eddy currents in the armature conductors.

The rotor length is limited by the critical speed, which should not be within 20 per cent. of the normal speed. Rotors with laminated cores can be run satisfactorily between their first and second critical speeds, the second being 2.8 times the first. Rotors for very large generators are made of solid steel forgings.

64. To explain the theory of designing turbo-generators, a design will be developed. The quantities frequently mentioned in this discussion will be referred to by symbols, and for convenience some of these symbols are listed alphabetically below:

A_f = sectional area of field conductor, in square inches;
B_{gm} = maximum flux density in air gap, in lines per square inch;
B_{tm} = maximum flux density in teeth, in lines per square inch;
d = shaft diameter, in inches;
d_r = rotor diameter, in inches;
d_s = stator diameter, in inches;
E_p = volts per phase;
I_f = field current, in amperes;
I_p = armature current in amperes per phase;
K_r = armature ampere-conductors per inch of rotor periphery;
l_g = radial length of single air gap, in inches;
l_r = total length of rotor core, in inches;
R_p = rotor pole pitch, in inches;
S_p = number of rotor slots per pole;
t_o = stator slot pitch, in inches;
T_f = number of field turns per pole;
w_r = weight of rotor, in pounds.

65. Let it be assumed that a turbo-alternator is to be designed for an output of 5,000 kilovolt-amperes, 80 per cent. power factor, 6,600 volts, 3 phase, 60 cyles, 1,800 revolutions per minute, with temperature rises not exceeding 40° C. The rotor will be laminated and the exciter voltage will be 120.

For 60 cycles and 1,800 revolutions per minute, the number of poles $p = \frac{120 \times 60}{1,800} = 4$. For the given output, the current in each terminal is $I_p = \frac{5,000,000}{\sqrt{3} \times 6,600} = 438$ amperes. The phases of the armature winding will be Y connected, and the voltage per phase will be $E_p = \frac{6,600}{\sqrt{3}} = 3,815$.

66. The first step in the design is to select a **rotor diameter** as large as may be without causing the peripheral velocity to exceed a safe limit. This limit may be taken as 20,000 feet per minute for laminated rotors and 24,000 feet per minute for a rotor made from a steel forging. For the machine to be designed, let $d_r = 40$ inches, giving a peripheral velocity of $\frac{40 \times 3.1416 \times 1,800}{12} = 18,850$ feet per minute. The rotor pole pitch $R_p = \frac{40 \times 3.1416}{4} = 31.4$ inches.

67. The **number of armature conductors** can now be determined by first assuming a number of armature ampere-conductors per inch of rotor circumference based on experience with similar machines. In general, such assumptions can be made as follows:

Kilovolt-ampere output	=1,000	1,000 to 5,000	5,000 and up
Amp.-cond. per inch K_r	= 350 to 600	500 to 700	700 and up

For the machine to be designed, let $K_r = 800$. The total number of ampere-conductors will then be $K_r R_p p$, and the total number of conductors will be $\frac{K_r R_p p}{I_p} = \frac{800 \times 31.4 \times 4}{438}$

=229. This number cannot be evenly distributed in the slots, and it must be changed. Each slot must contain an even number of conductors, in order to connect them into coils. The number of slots per pole per phase could be made 3, 4, or 5, and the last number will be taken; the number per pole will then be 15 and the total number 60. With 4 conductors per slot, the total number of armature conductors will be 240, instead of 229, and $K_r = 837$ instead of the assumed number 800.

68. The **air gap** must be such that the number of field ampere-turns $I\,T_g$ required to establish the flux in it will be approximately 1.5 times the number of armature ampere-turns per pole. The latter number will be $\frac{K_r R_p}{2} = \frac{837 \times 31.4}{2}$ $= 13{,}141$, and $I\,T_g = 1.5 \times 13{,}141 = 19{,}700$. The flux density B_g in the air gap will be approximately 48,000 lines per square inch and the length of the air gap should be $l_g = \frac{I\,T_g}{.313\,B_g}$ $= \frac{19{,}700}{.313 \times 48{,}000} = 1.3$, or, say, 1.25 inches.

69. The **length of the stator core** is made such that the maximum flux density in the stator teeth will be approximately 100,000 lines per square inch. The average density will then be approximately $100{,}000 \div 1.5 = 67{,}000$, or, say, 70,000 lines per square inch. The diameter of the stator-core face is $d_s = 40 + 2 \times 1.25 = 42.5$ inches, and the slot pitch is $t_o = \frac{\pi\,d_s}{60}$ $= \frac{3.1416 \times 42.5}{60} = 2.23$ inches. The slot width can be made nearly half of this pitch, or, say, 1 inch, leaving the tooth width at the stator surface 1.23 inches.

With five slots per pole per phase, the distribution factor $K_w = .957$. The coils will be made to span 11 slots instead of 15, the full pitch, making the winding pitch $\frac{11}{15} = 73.3$ per cent., for which the pitch factor $k_o = .91$, as shown by the curve of pitch factors in *Design of Alternating-Current Machines*, Part 1. The number of conductors in series per phase is

$5\times4\times4$, and the number of turns $T_p=\frac{5\times4\times4}{2}=40$. The useful flux per pole can now be calculated by the general formula

$$\Phi=\frac{10^8 E_p}{4.44\,T_p f k_u k_o}=\frac{10^8\times3{,}815}{4.44\times40\times60\times.957\times.91}=41{,}000{,}000$$

The tooth section required to carry this flux at 70,000 lines per square inch is $41{,}000{,}000\div70{,}000=586$ square inches. There are 15 teeth per pole each 1.23 inches wide at the end, and the net length of the core must be not less than $\frac{586}{15\times1.23}=31.8$ inches. Such a core should be made of .014-inch steel laminations japanned on both sides, and the net length of steel will be about 88 per cent. of the gross length, making the latter approximately $31.8\div.88=36.1$ inches. The punchings will be assembled in 23 sections, each $1\frac{5}{8}$ inches thick, separated by $\frac{5}{8}$-inch ventilation spaces. The gross steel will be $23\times1\frac{5}{8}=37\frac{3}{8}$ inches, the net length of steel $l=37\frac{3}{8}\times.88=32.9$ inches, the combined width of the air spaces $22\times\frac{5}{8}=13\frac{3}{4}$ inches, and the length of the assembled core $37\frac{3}{8}+13\frac{3}{4}=51\frac{1}{8}$ inches. A $1\frac{3}{16}$-inch ventilating space will be allowed for at each end, making the total length of the stator core $53\frac{1}{2}$ inches. The total length of the rotor core can be slightly less, or, say, $l_r=52$ inches.

70. The **rotor winding** should next be calculated, and it should be designed for safe operation at overloads with low power factors, say for 80° C. maximum temperature rise under the worst probable condition. As 12° C. rise corresponds to heat dissipation equivalent to about 1 watt per square inch of rotor surface, 80° C. rise indicates $80\div12=6.67$ watts per square inch. At this rate the maximum field current with full exciter voltage, 120, is

$$I_f=\frac{6.67\,\pi\,d_r\,l_r}{120}=\frac{6.67\times3.1416\times40\times52}{120}=363 \text{ amperes}$$

71. In a 60-cycle machine the total number of field ampere-turns per pole must be from 2.8 to 3.8 times the number of

armature ampere-turns per pole, depending on the regulation required. The smaller ratio will be used here. For 25 cycles, each limit can be from 10 to 15 per cent. lower than for 60 cycles. The minimum number of field ampere-turns on the machine here being designed must be $I_f\,T_f = 2.8 \times 13{,}141 = 36{,}800$, approximately. The minimum **number of field turns** per pole $T_f = 36{,}800 \div 363 = 101$; 105 will be used.

72. The **sectional area of the field conductor** must be such that the field resistance at maximum temperature will be $\frac{E_f}{I_f} = \frac{120}{363} = \frac{1}{3}$ ohm, nearly. At 80° C. temperature rise the resistance per circular-mil inch will be about 1.18 ohms and per square-mil inch will be $1.18 \times .7854 = .93$ ohm. The sectional area of the conductor in square mils is $10^6 A_f$, in which A_f is its area in square inches. The mean length of a field turn is approximately $1.5\,R_p + 2\,l_r = 1.5 \times 31.4 + 2 \times 52 = 151.1$, or, say, 152 inches, and the total length of conductor will be $152 \times 105 \times 4 = 63{,}840$ inches. The mean length per turn is calculated more accurately later. The sectional area of this conductor, in order to have a resistance of $\frac{1}{3}$ ohm at .93 ohm per square-mil inch, must be $A_f = \frac{.93 \times 63{,}840}{\frac{1}{3} \times 10^6} = .178$ square inch.

73. The **diameter of the shaft** should next be determined, in order to ascertain the depth of metal in the rotor available for slots. The weight of the rotor complete will be about 40 per cent. greater than the weight of a solid steel cylinder having the diameter and length of the rotor core, or

$$w_r = \frac{1.4 \times .28\,\pi\,d_r^2\,l_r}{4} = \frac{1.4 \times .28 \times 3.1416 \times 40^2 \times 52}{4}$$
$$= 25{,}600 \text{ pounds}$$

A preliminary drawing can be made and the distance l between bearing centers estimated. If it is taken at 150 inches and the maximum deflection y is taken at 5 per cent. of the air gap l_g, the approximate shaft diameter will be

$$d = \sqrt[4]{\frac{w_r\,l^3}{10^6 \times 22\,y\,l_g}} = \sqrt[4]{\frac{25{,}600 \times 150 \times 150 \times 150}{10^6 \times 22 \times .05 \times 1.25}} = 15.8 \text{ inches}$$

The critical speed of a 16-inch shaft will be $N_c = \dfrac{1{,}570{,}000\, d^2}{\sqrt{w_r\, l^3}}$

$= \dfrac{1{,}570{,}000 \times 16 \times 16}{\sqrt{25{,}600 \times 150 \times 150 \times 150}} = 1{,}370$ revolutions per minute, or nearly 25 per cent. below the normal speed, 1,800. A 16-inch shaft will, therefore, be satisfactory.

74. The **dimensions of the rotor slots and the rotor conductor** can now be decided. From experimental data on similar machines, 10 slots per pole will be chosen, and they can safely be made $5\frac{3}{4}$ inches deep. The slot wedge will occupy $\frac{3}{4}$ inch and the slot insulation, say, $\frac{1}{2}$ inch, leaving $4\frac{1}{2}$ inches for insulated conductor. The conductor will be a copper strip wound flat in the slot, and each slot must contain $\dfrac{105 \times 2}{10}$ $= 21$ conductors. The thickness of an insulated conductor will therefore be $4.5 \div 21 = .214$ inch, of which .014 inch will be allowed for insulation and .2 inch for copper. As the cross-

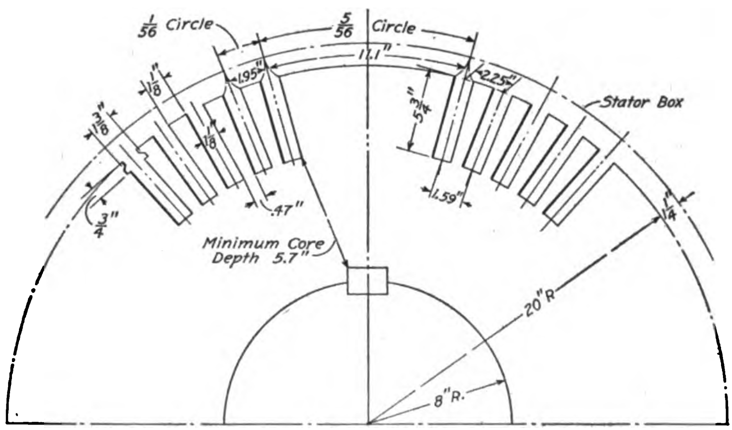


FIG. 12

section of the conductor was found to be .178 square inch, its width must be $.178 \div .2 = .89$ inch, or, say, .9 inch. About .1 inch should be allowed for insulation on each side of the slot, making the slot width $.9 + .2 = 1.1$, or, say, $1\frac{1}{8}$ inches.

75. At this point in the design outline drawings should be made, as in Figs. 12 and 13, and the centrifugal forces and stresses calculated. The drawings should not be completed until the calculations are made and the parts known to have sufficient mechanical strength. The calculations are made as explained in *Design of Alternating-Current Machines*, Part 1. The dimensions are shown on Figs. 12 and 13.

The weight of 1 inch of rotor tooth, of which 93 per cent. is net steel, is $\frac{1.125+.47}{2}\times 5.75\times .93\times .28=1.2$ pounds. Its radius of revolution may be taken at $18\frac{1}{2}$ inches, or $\frac{18.5}{12}$ feet, and the centrifugal force acting on it is

$$C_t=.00034\times 1.2\times\frac{18.5}{12}\times 1{,}800^2=2{,}040 \text{ pounds}$$

The weight of 1 inch of the slot contents, mostly copper, is $1.125\times 5.75\times .32=2.07$ pounds, and its radius of revolution may be taken as 1.5 feet. Then, the centrifugal force per inch of rotor length due to slot contents is

$$C_s=.00034\times 2.07\times 1.5\times 1{,}800^2=3{,}420 \text{ pounds}$$

The weight of 1 inch of total rotor core, including slot contents and teeth, is approximately $\left(\frac{3.1416\times 40^2}{4}-\frac{3.1416\times 16^2}{4}\right)\times .28=296$ pounds. The radius of revolution may be taken at .8 of the radius of the rotor, or $.8\times 20=16$ inches $=\frac{16}{12}$ feet. The centrifugal force per inch of rotor core is

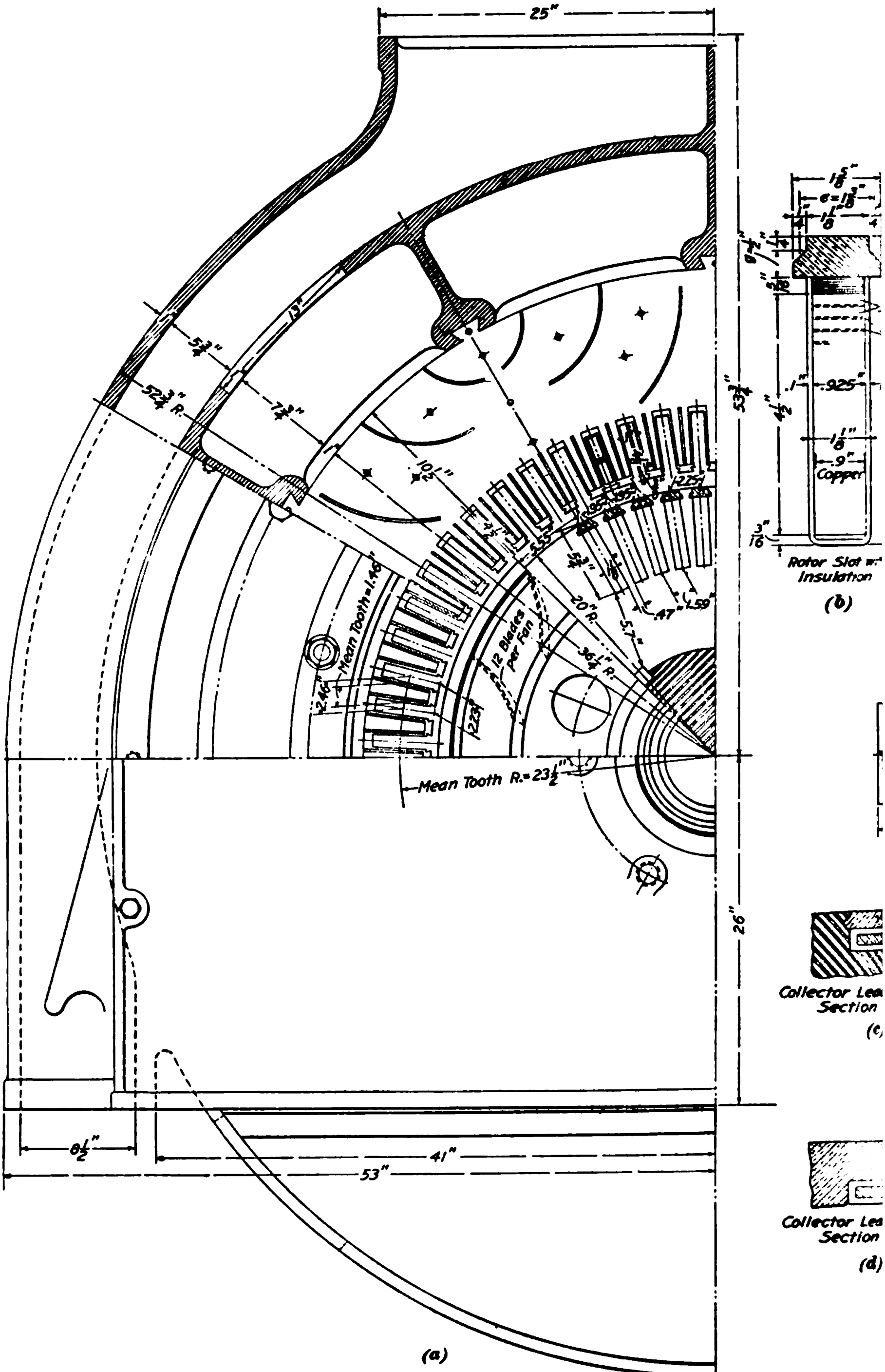
$$C_c=.00034\times 296\times\frac{16}{12}\times 1{,}800^2=435{,}000 \text{ pounds}$$

76. The stress tending to break the rotor teeth at their narrowest section, .47 inch, Fig. 12, is

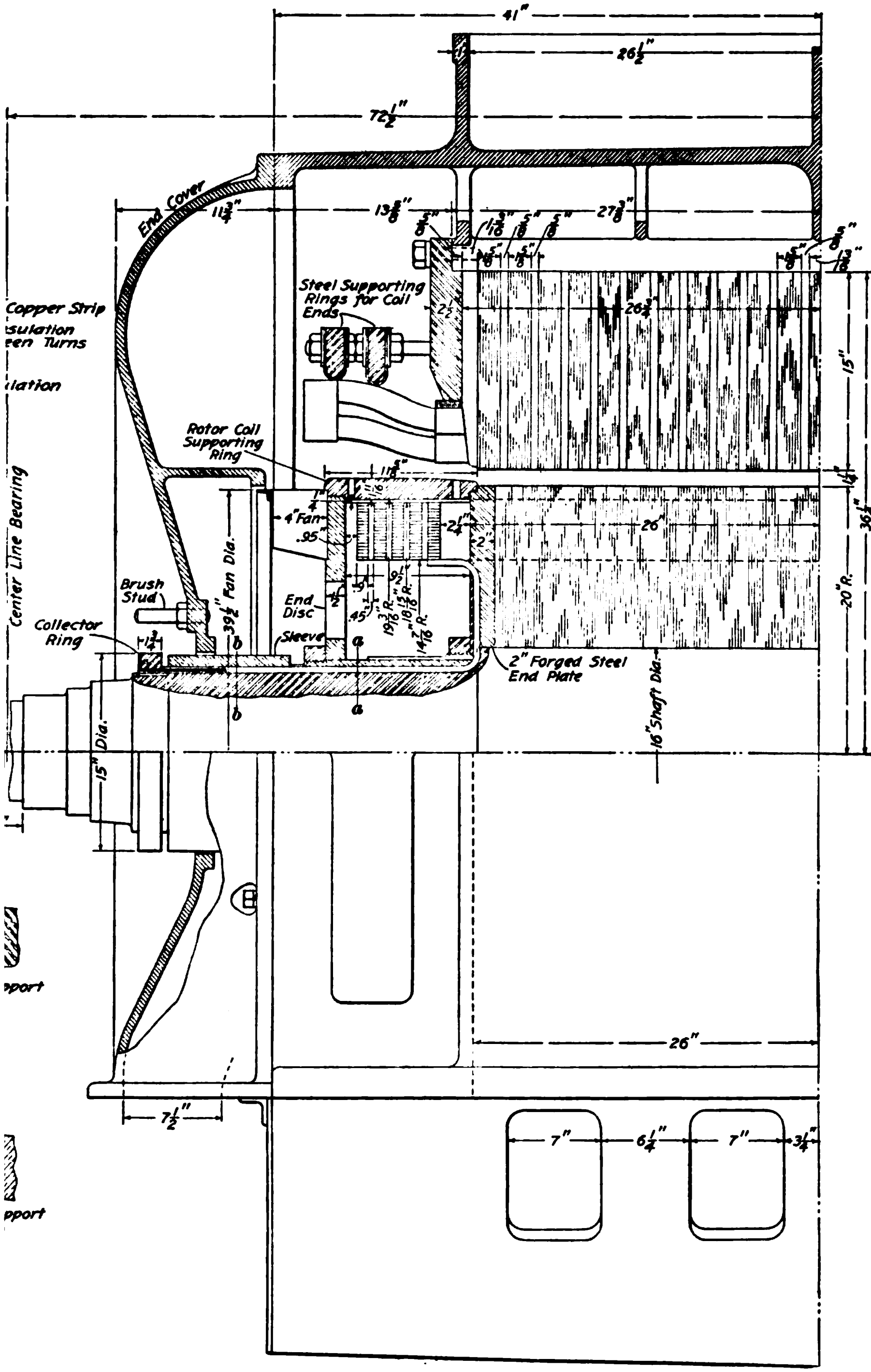
$$S_c=\frac{C_t+C_s}{x}=\frac{2{,}040+3{,}420}{.47}=11{,}600 \text{ pounds per square inch}$$

The stress tending to burst the rotor core in its smallest section, measuring 5.7 inches from a slot to the keyway, is

$$S_d=\frac{C_c}{2\pi y}=\frac{435{,}000}{2\times 3.1416\times 5.7}=12{,}150 \text{ pounds per square inch}$$


25"
$5\frac{3}{4}$"
$52\frac{3}{4}$" R.
$7\frac{3}{4}$"
13"
$10\frac{1}{2}$"
Mean Tooth = 1.46"
12 Blades per Fan
20" R.
$36\frac{1}{4}$" R.
5.7"
.47"
1.59"
.225"
.246"
Mean Tooth R. = $23\frac{1}{2}$"
$53\frac{3}{4}$"
26"
$8\frac{1}{2}$"
41"
53"
.925"
$1\frac{1}{8}$"
.9"
Copper
$\frac{3}{16}$"
Insulation
(b)
Section
Section
(d)

(a)


41"
26½"
72½"
End Cover
11¾"
13⅜"
27⅜"
Steel Supporting Rings for Coil Ends
Copper Strip
Insulation Between Turns
Rotor Coil Supporting Ring
4" Fan
39½" Fan Dia.
Center Line Bearing
Brush Stud
Collector Ring
End Disc
Sleeve
2" Forged Steel End Plate
16" Shaft Dia.
15" Dia.
15"
20" R.
36¼"
26"
26¾"
7½"
7"
6¼"
3¼"

(e)

The bending stress acting on the wedge center between supporting points $1\frac{3}{8}$ inches $(=e)$ apart is

$$S_o = \frac{.75\, C_s\, e}{f^2} = \frac{.75 \times 3{,}420 \times 1.375}{.75^2} = 6{,}270 \text{ pounds per square inch}$$

The shearing stress affecting the wedge corner, where $g = \frac{1}{2}$ inch, is

$$S_s = \frac{C_s}{2g} = \frac{3{,}420}{2 \times \frac{1}{2}} = 3{,}420 \text{ pounds per square inch}$$

The bending stress S_b affecting the wedge corner is

$$S_b = \frac{1.5\, C_s\, h}{g^2} = \frac{1.5 \times 3{,}420 \times \frac{1}{4}}{.5^2} = 5{,}130 \text{ pounds per square inch}$$

The combined bending and shearing stress acting on the wedge corner is

$$S_{os} = .35\, S_b + .65\sqrt{S_b^2 + 2.3\, S_s^2} = .35 \times 5{,}130$$
$$+ .65\sqrt{5{,}130^2 + 2.3 \times 3{,}420^2} = 6{,}530 \text{ pounds per square inch}$$

77. In order to calculate the stress tending to burst the ring over the rotor coil ends, the weights of these ends and the centrifugal force acting on them must be calculated. The coil ends have the general form indicated in Fig. 14. The mean length of the straight parts is 5.4 inches and of the arched parts 17 inches. The mean distance from the core around the coil end to the core again is $17 + 2 \times 5.4 = 27.8$ inches, from which should be deducted, say, 5 per cent., for rounded corners, leaving 26.5 inches, approximately. The mean radius of these connections from the center of revolution (see Fig. 13) is $20 - \frac{3}{4} - 2\frac{1}{2} = 16.75$ inches $= 1.4$ feet. There are 4 poles, 5 coils per pole, 21 turns per coil, the conductor section is .2 inch $\times$ .9 inch, and copper weighs .32 pound per cubic inch. The weight of copper in the connections at one end is, therefore, $4 \times 5 \times 21 \times .2 \times .9 \times 26.5 \times .32 = 640$ pounds, and the centrifugal force acting on it is $C_f = .00034\ W R n^2 = .00034 \times 640 \times 1.4 \times 1{,}800^2 = 987{,}000$ pounds.

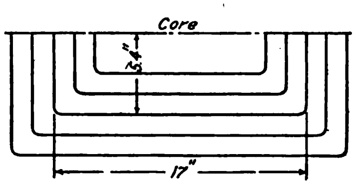


Fig. 14

The dimensions of a section of the support ring for the rotor coil ends, as given on Fig. 13, are $1\frac{11}{16}$ inches $\times 11\frac{5}{8}$ inches. The area of this section is approximately 19.6 square inches. The distance of the center of the section from the center of the shaft is $19\frac{3}{16}+\frac{1\frac{11}{16}}{2}=20$ inches. The weight of steel in the ring is approximately $2\times 3.1416\times 20\times 19.6\times .28=690$ pounds, and the centrifugal force acting on it is $C_r=.00034\times 690\times\frac{20}{12}\times 1,800^2$ $=1,267,000$ pounds. The bursting stress on the ring is

$$S_{ab}=\frac{C_f+C_r}{2\,\pi\,a\,b}=\frac{987,000+1,267,000}{2\times 3.1416\times 1\frac{11}{16}\times 11\frac{5}{8}}=18,300 \text{ pounds per square inch}$$

All the stresses on the rotor parts are thus found to be within the permissible working limits for ordinary steel as given in Table III, *Design of Alternating-Current Machines*, Part 1, except the stress on the rings over the ends of the coils, and chrome vanadium steel will be used for these rings.

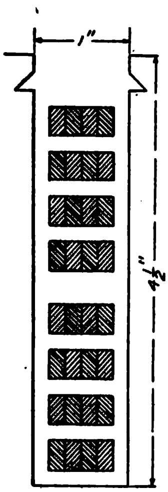


FIG. 15

78. The **stator slot dimensions** are calculated as for a salient-pole machine. The width has been assumed at 1 inch, the slot pitch being 2.23 inches, and the depth must be sufficient to carry conductors of the proper cross-section to prevent overheating. In order to minimize eddy currents in the conductors, a two-circuit winding will be used and the conductors will also be stranded. Each circuit will carry half the phase current, or $438\div 2=219$ amperes. A current density of 1,300 amperes per square inch of conductor cross-section can be assumed, necessitating a cross-section of $219\div 1,300=.168$ square inch. Of the 1-inch width of slot, about .65 inch will be available for copper, making the depth of copper in each conductor approximately $.168\div .65=.258$ inch.

79. Four conductors per slot have previously been chosen, and the two-circuit winding will require double this number.

The eight conductors will be arranged eight deep in the slot, each conductor consisting of four rectangular double-cotton-covered copper strips with cross-sectional dimensions .16 inch $\times$.28 inch, arranged as indicated in Fig. 15. The four conductors will require $4\times.16=.64$ inch of the slot width, leaving $1-.64=.36$ inch, for insulation. The slots will be made $4\frac{1}{2}$ inches deep, of which the conductors will occupy $8\times.28=2.24$ inches, the slot wedge $\frac{3}{8}$ inch, and the insulation $4.5-(2.24+.375)=1.89$ inches. This also allows for extra insulation spaces between the eight conductors.

80. In order to determine the **armature heating,** the watts dissipated per square inch are calculated by the formula

$$P_s=\frac{.395\ k_a\ t_o\ I_k}{W+D},$$

in which the kiloamperes per square inch $I_k=\frac{219}{4\times.16\times.28\times1{,}000}=1.22$; the slot pitch $t_o=2.23$ inches; the kiloampere-conductors per inch of stator circumference $k_a=\frac{40}{42.5}\times\frac{837}{1{,}000}=.788$; the slot width $W=1$ inch and the slot depth below the wedge $D=4.5-.375=4.125$ inches. Then, $P_s=\frac{.395\times.788\times2.23\times1.22}{1+4.125}=.165$ watts per square inch of stator coil surface.

In the formula, $T_s=\frac{P_s\,O}{.003}$ for temperature difference between the conductor and the outside of the insulation on it, and the thickness of insulation $O=.21$. Then, $T=\frac{.165\times.21}{.003}=11.55°$ C., which is a safe difference.

81. The maximum flux density in the teeth can be calculated by the formula $B_{tm}=\frac{\Phi\ (d-l_g)\ t_o}{\left[a'+\left(\frac{S_p}{2}-1\right)b'\right]d\ t_m\ l}$, in which values of, Φ d, l_g, t_o, S_p, and l are those found in the preceding discussion. From Fig. 12, a', the arc over the center pole

is 11.1 inches, and b', the arc over each rotor tooth is 1.95 inches. From Fig. 13, t_m, the mean width of each stator tooth is 1.46 inches. Then,

$$B_{tm} = \frac{41{,}000{,}000 \times (42.5 - 1.25) \times 2.23}{[11.1 + (\frac{10}{2} - 1) \times 1.95] \times 42.5 \times 1.46 \times 32.9}$$
$$= 97{,}700 \text{ lines per square inch}$$

The depth of the stator core back of the slots will be made 10.5 inches, and the density in it will be $\frac{41{,}000{,}000}{2 \times 32.9 \times 10.5} = 59{,}400$ lines per square inch. Both of these densities are within the limits of good practice.

82. The **field resistance** should be calculated for the maximum probable temperature, 105° C., at which the resistance per circular-mil inch will be approximately 1.18 ohms. There are 105 turns, or 210 conductors, per pole, making 21 conductors per slot. The connections between coils will render 1 conductor in every slot inactive, leaving $\frac{20 \times 10}{2} = 100$ active turns per pole.

The mean length of a field turn may be estimated from the data in Figs. 12 and 13. The rotor length is $2 \times 26 = 52$ inches, and the copper strips project straight out from the rotor a short distance at each end. The average length of these straight ends may be estimated as $\frac{9\frac{1}{2}}{2}$, Fig. 13, for each end, or $9\frac{1}{2}$ inches for both ends, making the two straight portions of the mean turn $2\ (52 + 9\frac{1}{2}) = 123$ inches. The radial distance of the outer turn of a coil will be $20 - \frac{3}{4}$ (wedge) $= 19\frac{1}{4}$ inches. The radial distance of the lower turn of a coil will be $20 - 5\frac{3}{4}$ (slot depth) $= 14\frac{1}{4}$ inches. The mean radial distance will be $\frac{19\frac{1}{4} + 14\frac{1}{4}}{2} = 16\frac{3}{4}$ inches, and the mean diameter $33\frac{1}{2}$ inches. The mean span of the coils will be $\frac{5}{56} + 4\ (\frac{1}{56}) = \frac{9}{56}$ of a circle, Fig. 12, or $\frac{9}{56} \times 33.5 \times 3.1416 = 17$ inches, nearly. The mean length of the field turn will be the sum of the straight and end portions, or $123 + 2 \times 17 = 157$ inches.

The field-conductor section is .2 inch$\times$.9 inch, and its area in circular mils is $\frac{.2\times.9}{.7854}\times 10^6 = 229{,}000$. The hot field resistance is $\frac{1.18\times157\times100\times4}{229{,}000} = .324$ ohm. The maximum field current when the machine is at maximum temperature will be $I_f = \frac{120}{.324} = 370$ amperes, and the ampere-turns with this current will be $100\times370 = 37{,}000$.

83. The **regulation** can be checked approximately by ascertaining that the ratio of the gap ampere-turns to the demagnetizing armature ampere-turns is between the limits 1.5 and 2.5. In the formula for the effective air gap, namely, $l'_g = \left(\frac{1+\frac{s}{t}}{1+k'\frac{s}{t}}\right) l_g$, the slot opening $s = 1$ inch, the tooth width $t = 1.23$ inches, the actual air gap $l_g = 1.25$ inches, and for the ratio $\frac{s}{l_g} = \frac{1}{1.25} = .8$, $k' = .82$, as shown by the curve in *Design of Alternating-Current Machines*, Part 1, for calculating the effective air gap. Then,

$$l'_g = \frac{1+\frac{1}{1.23}}{1+.82\times\frac{1}{1.23}}\times 1.25 = 1.36 \text{ inches}$$

The total number of gap ampere-turns can now be calculated by the formula $I\,T_g = \frac{.313\,l'_g\,\Phi}{l_r\left[a' + \left(\frac{S_p}{2} - 1\right)b'\right]}$, in which l'_g, Φ, l_r, and S_p have the values previously found, $a' = 11.1$, and $b' = 1.95$, as indicated in Fig. 16. Then,

$$I\,T_g = \frac{.313\times1.36\times41{,}000{,}000}{52\times(11.1+4\times1.95)} = 17{,}750$$

The number of demagnetizing armature ampere-turns per pole is calculated by the formula $D\,I\,T_p = .81\ m\ T_{pp}\ I_p\ k_w\ k_o$ sin a, in which all the values are as previously determined except sin a, which may be considered as 1. Then, $D\,I\,T_p = .81 \times 3 \times 10 \times 438 \times .957 \times .91 = 9{,}270$, and the ratio $\frac{I\,T_p}{D\,I\,T_p} = \frac{17{,}750}{9{,}270} = 1.91$, which is a safe value.

84. The foregoing calculation of demagnetizing armature ampere-turns per pole is based on the assumption that the ratio of the maximum number of field ampere-turns per pole to the average number is 1.5. This assumption can now be checked by drawing the flux distribution curve, as explained in *Design of Alternating-Current Machines*, Part 1. Fig. 12 shows that at the center of the air gap the flux from each tooth crosses an arc of 1.95 inches and the flux from the pole center crosses an arc of 11.1 inches. The entire base of the curve will, therefore, be $8 \times 1.95 + 11.1 = 26.7$ inches. Each distance a, Fig. 16, represents the flux set up by the ampere-turns in one field coil, and the maximum flux $B_m = 5\,a$. The total area between the curve and its base line is $5\,a \times 26.7 - 5\,a \times 8.775 = 89.625\,a$, and the average ordinate $B_a = \frac{89.625\,a}{26.7} = 3.36\,a$. The ratio $\frac{B_m}{B_a} = \frac{5\,a}{3.36\,a} = 1.49$, nearly, which is very close to the assumed ratio.

85. Forced ventilation must be used in such a machine, and the designer must provide air passages large enough to carry approximately 150 cubic feet of air per minute per kilovolt-ampere loss at air velocities not exceeding 6,000 feet per minute. The losses can be calculated, but the more practical way is to estimate them from an efficiency curve of such machines. Such a curve is given in *Design of Alternating-Current Machines*, Part 1, and this curve shows that a full-load efficiency of approximately 96.2 per cent. may be expected. The losses will be 3.8 per cent., or $.038 \times 5{,}000 = 190$ kilovolt-amperes. The volume of air forced through the machine must be approximately $190 \times 150 = 28{,}500$ cubic feet per minute.

There will be six air inlets and six outlets, one of each being outlined in Fig. 13. Their arrangement is shown in Fig. 17;

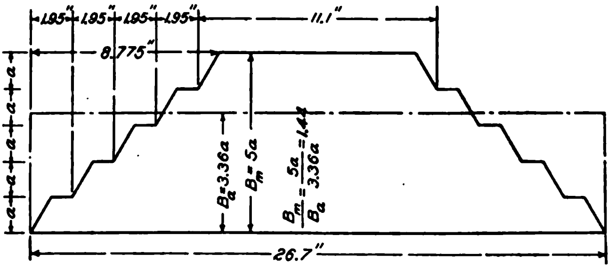


FIG. 16

air enters through alternate pairs of openings, passes in toward the shaft through ducts in the stator core and through openings in the rings covering the ends of the rotor coils, and then

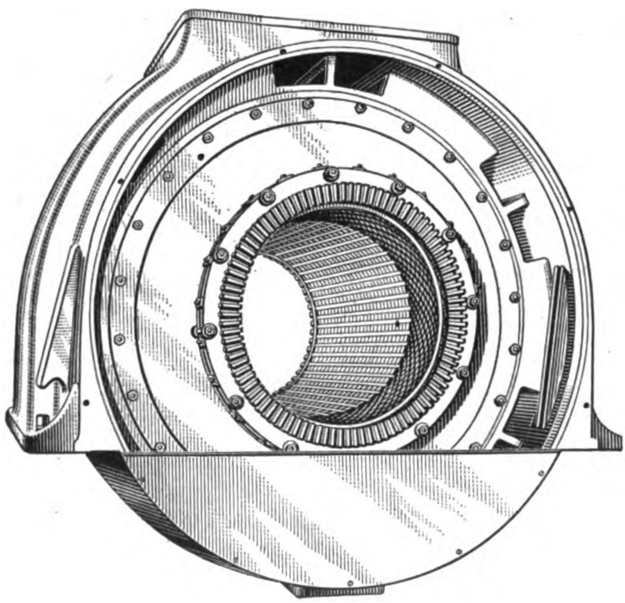


FIG. 17

through the outlets. Fans, one at each end, on the rotor keep up the air circulation, and vanes in the passages guide it over

and through the heated parts, thus keeping their temperature below danger limits.

Of the 22 ducts in the stator core, 11 are outlined in the half section view (*e*), Fig. 13. The radial depth of these ducts outside the teeth is $10\frac{1}{2}$ inches, and the combined sectional areas of the ducts through which air must pass is

$$\frac{6\times22\times.625\times10.5}{144}=6 \text{ square feet.}$$

The air velocity in the ducts will be $28{,}500\div6=4{,}750$ feet per minute.

The mean diameter of the fan is 35 inches, and the width of the blades is 4 inches. The combined areas of the spaces through which air passes in the two fans is $\frac{2\times4\times35\times3.1416}{144}$ $=6.11$ square feet, and the air velocity is $28{,}500\div6.11=4{,}660$ feet per minute. The opening in the fans, as well as the air ducts in the stator, are, therefore, of ample size.

86. Fig. 17 shows the appearance of a complete stator such as here designed, and Fig. 18 shows the outer casing in place.

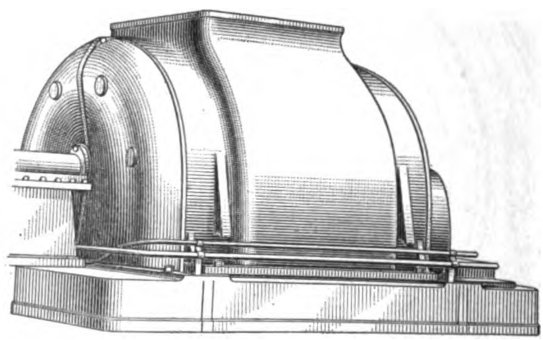


Fig. 18

The bearings, as determined from experience, are $7\frac{1}{2}\times26$ inches. The mechanical construction is indicated in Fig. 13, which is largely self-explanatory.

The rotor core consists of $\frac{1}{16}$-inch steel punchings keyed on the forged-steel shaft and clamped between 2-inch forged-steel end plates. The centrifugal force acts against the flat side of

the field conductors, thus eliminating tendency to cut their insulation or the slot insulation. The latter consists of three molded troughs with a total thickness of .1 inch. The rotor coils are held in the slots by bronze wedges.

Spacers are secured between the ends of adjacent coils and between the outer coil on each pole and the adjoining bronze disk. Over the coil ends are the insulated support rings containing ventilating holes.

87. The leads from the collector rings to the field coils are located in special grooves milled into the steel end plates, the shaft, and a holding sleeve. The leads are heavily insulated and held in the grooves in the end plate and shaft by wedges, as indicated in Fig. 13 (*c*); view (*d*) shows how the sleeve fits over the lead. One end of each lead is fastened to a collector ring by means of screws and the other end is joined to a field coil terminal by means of rivets and solder. A cast-steel collector ring is shrunk on the shaft at each end and insulated from it by a $\frac{1}{8}$-inch mica sleeve. The rotor fans are made of sheet steel and designed so as to supply an adequate amount of ventilating air, with a minimum expenditure of power.

88. The stator yoke is cast in one piece, thoroughly ribbed and bored to receive the armature punchings. It is shaped to guide the air delivered by the rotor fan through all parts of the machine by means of short, regular paths, with a minimum amount of friction. The air is finally discharged either out of the foot of the machine or out of the top, as circumstances may require.

The spacers in the ventilating ducts in the stator core are made from steel punchings. The spacing ribs are riveted to punchings $\frac{1}{16}$ inch thick and are curved so as to serve as guides for the air. At the ends of the core the stator teeth are supported by heavy ventilating segments, which clamp them solidly through the whole core length. The stator core is clamped between heavy cast-iron end plates bolted to the stator yoke.

89. The stator coils are supported where they leave the core by fiber rings mounted on the stator end plates; the ends

are roped to two insulated steel rings, mounted on steel studs screwed into the stator end plates and are braced to withstand lateral movement by means of spacers fastened between the coils. One of these rings with the coil ends inside appears in Fig. 17.

90. Cast-iron end covers, one of which is outlined in Fig. 13 (*e*), are bolted to the stator yoke in two equal parts, split vertically, as shown in Fig. 18. These covers enclose the end windings of the stator and rotor and form part of the walls of the ventilating chamber. A brush stud, carrying a holder with a carbon brush, is secured to the cover at each end of the machine, as indicated in Fig. 13 (*e*). The holder is separated from the stud by an insulating tube.

DESIGN OF ALTERNATING-CURRENT MACHINES

(PART 3)

INDUCTION MOTORS

LIMITING FEATURES OF DESIGN

1. The design of an induction motor has some features in common with the design of both alternators and transformers. The stator of an induction motor usually has more slots than that of an alternator of the same size and characteristics, but otherwise the two are alike. The rotor of an induction motor bears to its stator practically the same relation that a transformer secondary bears to its primary. For this reason, the name secondary is frequently used when referring to the rotating part of the motor, and the name primary is then given to the stator.

2. The chief considerations in the design of an induction motor are heating, overload capacity, starting torque, power factor, and efficiency. In importance, these considerations usually rank about in the order named, but the designer must aim to produce a machine with characteristics balanced to suit the application for which it is to be used. For long-continued service at steady load, the heating limit, efficiency, and power factor are especially important; in some cases, heavy overloads may be applied for short periods, and the motor must be able to carry them; again, the most important characteristic

may be high starting torque. The efficiency and power factor should always be as high as can be obtained with the proper balance of the other characteristics.

HEATING

3. The temperature rise in the stator during continuous operation will depend on the power loss in the stator windings and on the effectiveness of the ventilation. High peripheral velocity of the rotor usually causes good ventilation. In general, the temperature rise of the stator will remain below 40° C. if the power loss is not over .35 watt per square inch of coil surface. Phase-wound rotors will usually dissipate from .4 to .5 watt per square inch of cylindrical surface with the same temperature rise, and squirrel-cage rotors will do slightly better than indicated by these figures.

OVERLOAD CAPACITY

4. Induction motors for general service are usually designed to develop full load continuously with temperature rises not exceeding 40° C.; also, 25 per cent. overload for periods of 1 or 2 hours with temperature rises not exceeding 55° C., and from 50 to 100 per cent. overload momentarily. In some cases the full-load rating can be exceeded by 25 per cent. continuously without overheating. Motors designed especially for a particular application, such as crane and hoist motors, elevator motors, mill motors, etc., must be designed with suitable characteristics.

STARTING TORQUE

5. The full-load torque of a motor is the turning effort exerted by the rotor when the motor is developing full load at normal rated speed. With the full rated voltage applied to the stator terminals, the average squirrel-cage motor will develop starting torque from 1.25 to 1.5 times full-load torque. They can be designed, however, to develop two or more times full-load torque when starting with full voltage.

When starting with full voltage, however, the starting current is excessive, and the usual practice is to reduce the voltage for starting by means of autotransformer starters, variously called *autostarters* and *compensators*, or by resistance starters, called rheostats. The starting torque varies approximately as the square of the voltage; that is, at 60 per cent. of the full voltage the motor will start with $.6 \times .6 = .36$ of the torque that it would develop at full voltage.

6. In order to obtain uniform starting torque from all positions of a squirrel-cage rotor, the number of rotor slots and

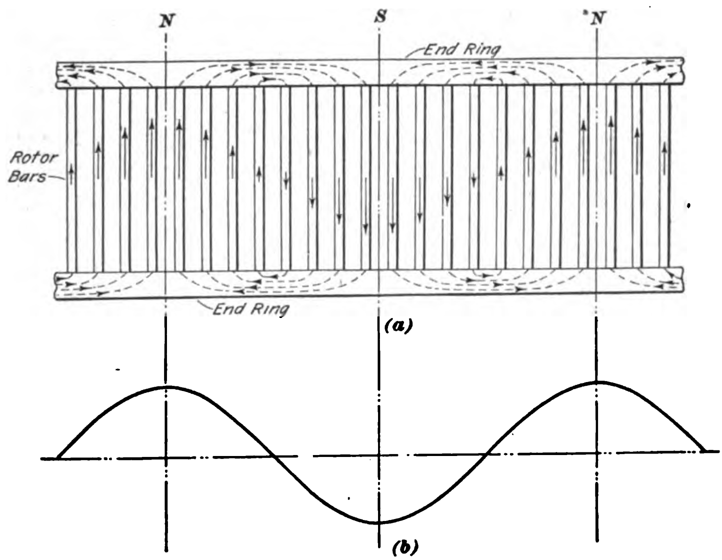


Fig. 1

the number of stator slots must be correctly proportioned to each other. Otherwise, the starting torque will depend on the position in which the rotor happens to stop; with very bad proportions the rotor might stop on *dead points*. The proportion of five rotor slots to six stator slots gives good results.

7. Starting Conditions With Squirrel-Cage Rotors. The conditions in the conductors of a squirrel-cage rotor may be explained by reference to Fig. 1, which represents an

instantaneous condition. The vertical lines N, S, N represent centers of three adjacent poles. The vertical arrows on the rotor bars, view (a), indicate the direction of current, and the lengths of the arrows are proportional to the strengths of the currents. The bars directly under the poles carry the most current and those between the poles the least. The variation in current strength in the bars located in successive positions is according to a sine curve, as indicated in view (b).

8. As the rotor turns, the strength of the current in any given section of the end rings also varies according to the sine law. This current is always maximum midway between the poles, where it is the sine of the average currents in the rotor bars at that instant between pole centers. The average current in any given section of the end ring is .636 times the maximum current, and the effective current is .707 times the maximum current. These statements can be represented mathematically as follows:

Let I_a = average current in each rotor bar;
I_e = effective current in each rotor bar;
I_m = maximum current in each rotor bar;
I = effective current in end ring;
I_r = maximum current in end ring;
N = total number of rotor bars;
N_p = number of rotor bars per pole $= \frac{N}{p}$;
R = resistance of end ring between adjacent rotor bars;
p = number of poles.

Then,

$$I_e = .707\, I_m, \text{ and } I_m = \frac{I_e}{.707}$$

$$I_a = .636\, I_m = \frac{.636}{.707}\, I_e = .9\, I_e$$

$$I_r = \tfrac{1}{2} \times N_p \times .9\, I_e = .45\, N_p\, I_e = \frac{.45\, N\, I_e}{p}$$

$$I = .707\, I_r = \frac{.318\, N\, I_e}{p}$$

The total resistance loss in both rings is

$$2\,N\,I^2R = 2\,N\left(\frac{.318\,N\,I_e}{p}\right)^2 R = \frac{.2\,N^3\,I^2_e R}{p^2}$$

and

$$R = \frac{5\,p^2 \times I^2 R \text{ loss in both rings}}{N^3\,I^2_e}$$

By this formula the resistance of the end ring can be calculated when the loss in both rings, the number of rotor bars, and the effective current in each bar are known. End rings are generally made of composition metal, and the resistance depends on the materials used and their relative proportions. The calculation of the exact cross-section to produce a given resistance at operating temperature is therefore difficult. Small errors can scarcely be avoided; these sometimes result in speeds slightly different from those found by calculations, but not enough to interfere with practical motor applications.

9. Starting Conditions in Phase-Wound Rotors. The starting conditions in phase-wound rotors require less attention than in squirrel-cage rotors, because the starting current can be restricted by the use of adjustable resistance in the secondary circuit without its being necessary to reduce the primary voltage. Dead points or weak points are not likely to occur unless the ratio of the number of rotor slots to the number of stator slots is made 1 to 1 or 2 to 1.

POWER FACTOR

10. The power factor of an induction motor depends inversely on the magnetizing current, and this current is affected by the flux density in the air gap and by the length of the air gap. In order to restrict the magnetizing current, so as to improve the power factor, the air gap is usually made as small as safe mechanical clearance will permit, the rotor slots are closed or nearly closed, and in some cases the stator slots are also closed. Closed slots cause more even distribution of the flux in the air gap and thus lower densities and lower magnetizing current. But the stator windings must be repaired occasionally, and repairs are difficult if the stator slots are

closed; therefore, closed stator slots are rarely used except in motors for extremely slow speeds and in small motors. The power factor is also affected by the leakage reactance, and this reactance is inherently lower in a squirrel-cage motor than in a phase-wound motor, causing a given motor to operate with higher power factor when the rotor is of the former type.

11. Rotating Magnetic Field.—When the stator winding of a polyphase induction motor is connected with the supply circuit, magnetic poles exist on the inner face of the stator. An instantaneous condition for a distributed winding is represented in Fig. 2. But this condition is continually changing; the magnetic poles advance around the inner periphery of the stator, forming a rotating magnetic field. The flux of each of the six poles is most dense at the pole center and the density varies according to the sine law. Then when the stator windings are distributed in a large number of slots per pole the following relations exist:

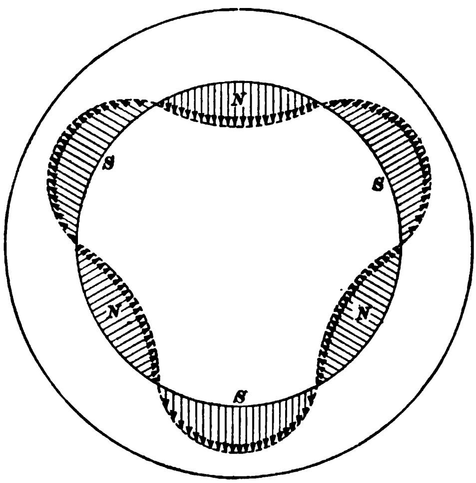


FIG. 2

$$B_a = \frac{2}{\pi} B_m = .636\, B_m$$

and $$\Phi = S_i\, l\, B_a = .636\, S_i\, l\, B_m$$

in which B_a = average flux density in the air gap, in lines per square inch;

B_m = maximum flux density in the air gap;

Φ = flux per pole;

S_i = pole pitch, in inches;

l = axial length of stator core, in inches.

12. Induction motors usually have from three to eight slots per pole per phase, and the flux density at any instant varies somewhat as indicated in Fig. 3, that is, in small steps. Moreover, changes in the current affect the form of this flux curve, but the form is nearly enough that of a sine wave to make the error small when the sine laws are assumed, as is generally done in practice.

13. Magnetizing Current.—When an induction motor is connected with an alternating-current circuit and allowed to run free of load, nearly all the current taken from the circuit is required to set up the magnetism in the motor, a very small part being required for the power necessary to overcome friction

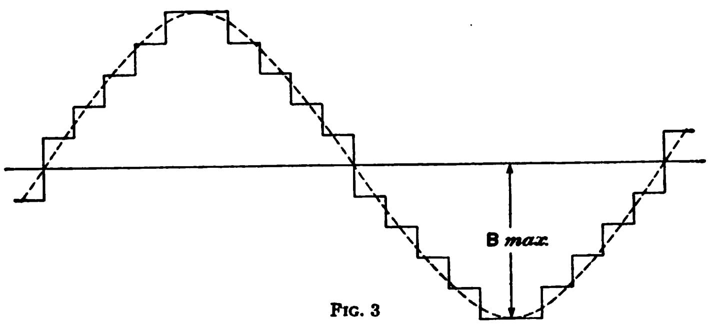


FIG. 3

in the moving parts. The magnetizing component of the no-load current is called wattless, because it lags 90° from the voltage of the circuit; the power component is in phase with the voltage. As nearly all the reluctance of the magnetic circuit is in the air gap, and the axial length of this air gap is small, variations in this gap, which are almost unavoidable, with ordinary shop methods of manufacture, often make the actual no-load current differ considerably from the calculated current. For example, if the magnetizing current is calculated for a motor having a .03-inch air gap and the motor is made with a .033-inch air gap, the increase of .003 inch will make a difference of 10 per cent. in the magnetizing current. With low flux densities, the magnetizing current, or the wattless component of the no-load current, can be calculated by the formula,

$$I_o = \frac{\Phi\, l_g\, k_1}{.85\, m\, b\, n S_i\, l}$$

in which I_o = magnetizing current, in amperes per phase;

Φ = flux per pole;

l_g = radial length of air gap, in inches;

k_1 = fringing coefficient, a factor to allow for fringing of lines of force at the ends of the teeth;

m = number of phases;

b = number of stator conductors per slot;

n = number of stator slots per pole per phase.

The fringing coefficient depends on the slot width at the air gap and on the ratios of the air gap to the slot width and the slot pitch to the tooth width. If the stator and rotor slots are closed, or nearly so, k_1 may be taken at 1.05; if the stator slots are open and the rotor slots closed, the value of k_1 will lie between the limits 1.35 and 1.55. With high flux densities, the magnetizing current may be from 10 to 20 per cent. larger than found by the formula, depending on the density in the teeth and their length.

EXAMPLE.—The inside diameter of the stator of a 10-pole 3-phase 25-cycle induction motor is 50 inches, and the axial length of the core is 9¾ inches. The flux per pole is 5×10^6 lines of force, and the radial length of the air gap is .055 inch. The stator has 18 open slots per pole with 8 conductors per slot, and the rotor has closed slots. Calculate the magnetizing current per phase, assuming that the teeth are long and the density in them rather high.

SOLUTION.—The fringing coefficient k_1 may be taken at 1.5; the number of slots per pole per phase $n=6$; the pole pitch $S_i = \frac{3.1416 \times 50}{10} = 15.7$. The values of the others quantities in the formula are clearly stated in the problem. The value given by the formula should be increased 20 per cent., because of the long teeth and high density, and the factor 1.2 is therefore placed in the numerator. Then

$$I_o = \frac{5 \times 10^6 \times .055 \times 1.5 \times 1.2}{.85 \times 3 \times 8 \times 6 \times 15.71 \times 9.75} = 26.4 \text{ amp.} \quad \text{Ans.}$$

EFFICIENCY

14. The usual aim of the designer of an induction motor is to obtain the best efficiency possible with the other four limiting features satisfactory. Improving the efficiency beyond

a certain point causes some sacrifice of overload capacity, starting torque, and possibly of power factor. The efficiencies of induction motors compare favorably with those of direct-current motors when the quotients obtained by dividing the outputs by the speeds are approximately the same.

FORMULAS FOR INDUCTION-MOTOR DESIGN

15. The basic formula for induction-motor design is the same as for alternator design, namely,

$$E_p = \frac{4.44\ \Phi\ T_p f}{10^8} \times k_w = \frac{k_2\ \Phi\ T_p f}{10^8}$$

in which E_p = counter electromotive force generated in each phase;

Φ = flux per pole;

T_p = turns in series per phase;

f = frequency, in cycles per second;

k_w = distribution factor;

k_2 = the product 4.44 k_w.

The value of the factor k_2 thus depends on the number of phases and the number of slots per pole per phase, but, in general, it can be taken at 4.25 for three-phase motors and at 4 for two-phase motors.

When an induction motor is to be designed, the horsepower, speed, power factor, efficiency, line voltage, and the frequency are usually specified; the flux per pole and the number of turns in series per phase must then be determined so that at normal speed and with the calculated flux densities the counter electromotive force will equal the line voltage less the voltage drop due to the resistance of the stator windings. The flux densities in induction motors will be satisfactory at or below the following values:

Part	60 Cycles	25 Cycles
Stator teeth (maximum)	95,000	115,000
Rotor teeth (maximum)	125,000	150,000
Stator core (average)	65,000	85,000
Rotor core (average)	95,000	105,000
Air gap (maximum, B_m)	35,000	35,000

16. A formula must now be developed for determining the cylindrical inches (diameter squared times length) of the stator core surface. In this development the symbols have the meanings given in the foregoing discussion, and additional symbols are used as follows:

I_p = number of amperes per phase;
C_p = number of conductors per phase $= 2\,T_p$;
P = apparent watts input;
d = diameter of stator-core face, in inches;
K = number of ampere-conductors per inch $= \frac{m\,I_p C_p}{\pi\,d}$.

The basic formula of Art. **15** can now be written $E_p = \frac{k_2\,\Phi C_p f}{2\times 10^8}$.

Also, $P = m\,E_p\,I_p = \frac{m\,I_p\,k_2\,\Phi\,C_p f}{2\times 10^8}$, in which $m\,I_p C_p$, the total ampere-conductors, can be represented by $K\,\pi\,d$. The frequency $f = \frac{p\,S}{120}$, and the flux per pole Φ = pole face area $\times B_a$ $= \frac{\pi\,d\,l}{p}\times B_a = \frac{\pi\,d\,l}{p}\times\frac{2}{\pi}\,B_m = \frac{2\,d\,l\,B_m}{p}$.

Then,

$$P = \frac{K\,\pi\,d\,k_2\times\frac{2\,d\,l\,B_m}{p}\times\frac{p\,S}{120}}{2\times 10^8} = \frac{\pi\,d^2\,l\,k_2 K S\,B_m}{120\times 10^8}$$

and

$$d^2\,l = \frac{120\times 10^8\,P}{\pi\,k_2\,K S B_m} = \frac{38.2\times 10^8\,P}{k_2\,K S\,B_m}$$

The cylindrical inches thus determined are accurate for small motors, but should be decreased from 7 to 10 per cent. for large 60-cycle motors and from 15 to 20 per cent. for large 25-cycle motors.

17. In the formula of Art. **16**, the factor k_2 has the values given in Art. **15**, the quantities K and B_m are assumed from experimental data, the speed S is always specified in the conditions given the designer, and the apparent input in watts is calculated from the specified horsepower, efficiency, and power

factor; thus, $P = \frac{\text{H. P.} \times 746}{\text{efficiency} \times \text{power factor}}$. For motors of moderate size the product of efficiency times power factor is approximately .746, making $P = 1{,}000$ H. P., or $P \div 1{,}000 =$ H. P. As $P \div 1{,}000 =$ kilovolt-amperes, it is not uncommon to assume that the input in kilovolt-amperes is equal to the output in horsepower.

DESIGN OF A 500-HORSEPOWER INDUCTION MOTOR

STATOR DESIGN

18. To illustrate the electrical calculations required in the design of induction motors, the design of a motor will be worked out for the following specifications: Output, 500 horsepower; synchronous, or no-load, speed, 300 revolutions per minute; supply circuit 2,200 volts, 25 cycles, three phase; maximum temperature rise 40° C. with full load continuously and 55° C. after carrying 25 per cent. overload 2 hours; maximum momentary overload 75 per cent.; maximum starting torque not less than two times full-load torque; efficiency not less than 92 per cent. at full load, and power factor not less than 91 per cent. The motor will have two bearings and be coupled to a line shaft.

The formulas for determining the number of poles, number of cylindrical inches, the peripheral velocity, etc. are applied to the stator as though it were to rotate.

19. The **number of poles** must be $p = \frac{120\,f}{S} = \frac{120 \times 25}{300} = 10$.

The apparent input at full load will be $P = \frac{500 \times 746}{.92 \times .91} = 446{,}000$ watts, and the **full-load current input** will be $\frac{446{,}000}{\sqrt{3} \times 2{,}200} = 117$ amperes.

20. The **number of cylindrical inches** will be approximately 80 per cent. of the number given by the formula $d^2\,l$

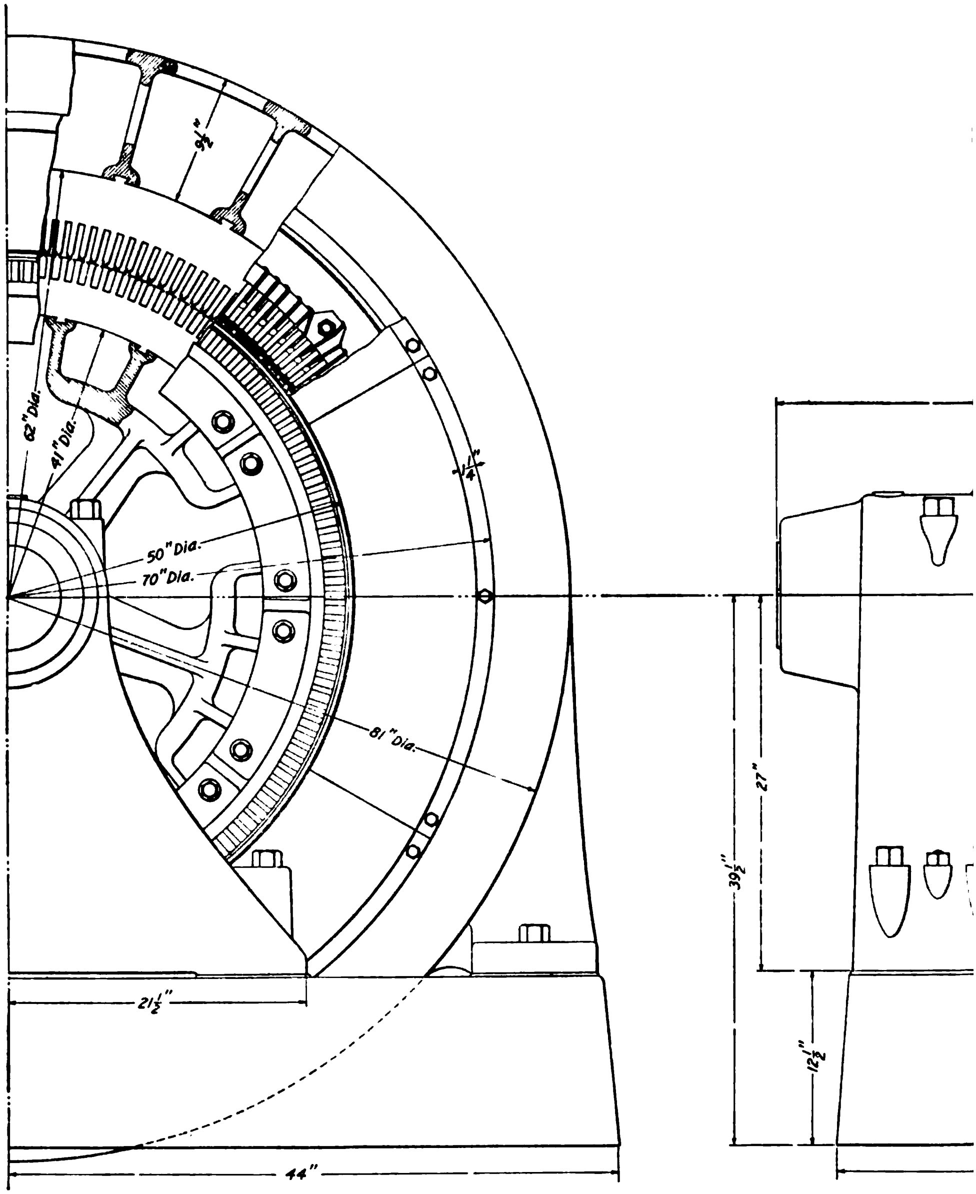
$=\frac{38.2\times10^8 P}{k_2 K S B_m}$ when average values $K=1{,}100$ and $B_m=32{,}500$, and the known values of P, k_2, and S are substituted. Thus,

$$d^2 l=\frac{38.2\times10^8\times446{,}000}{4.25\times1{,}100\times300\times32{,}500}\times.8=30{,}000\text{, approximately}$$

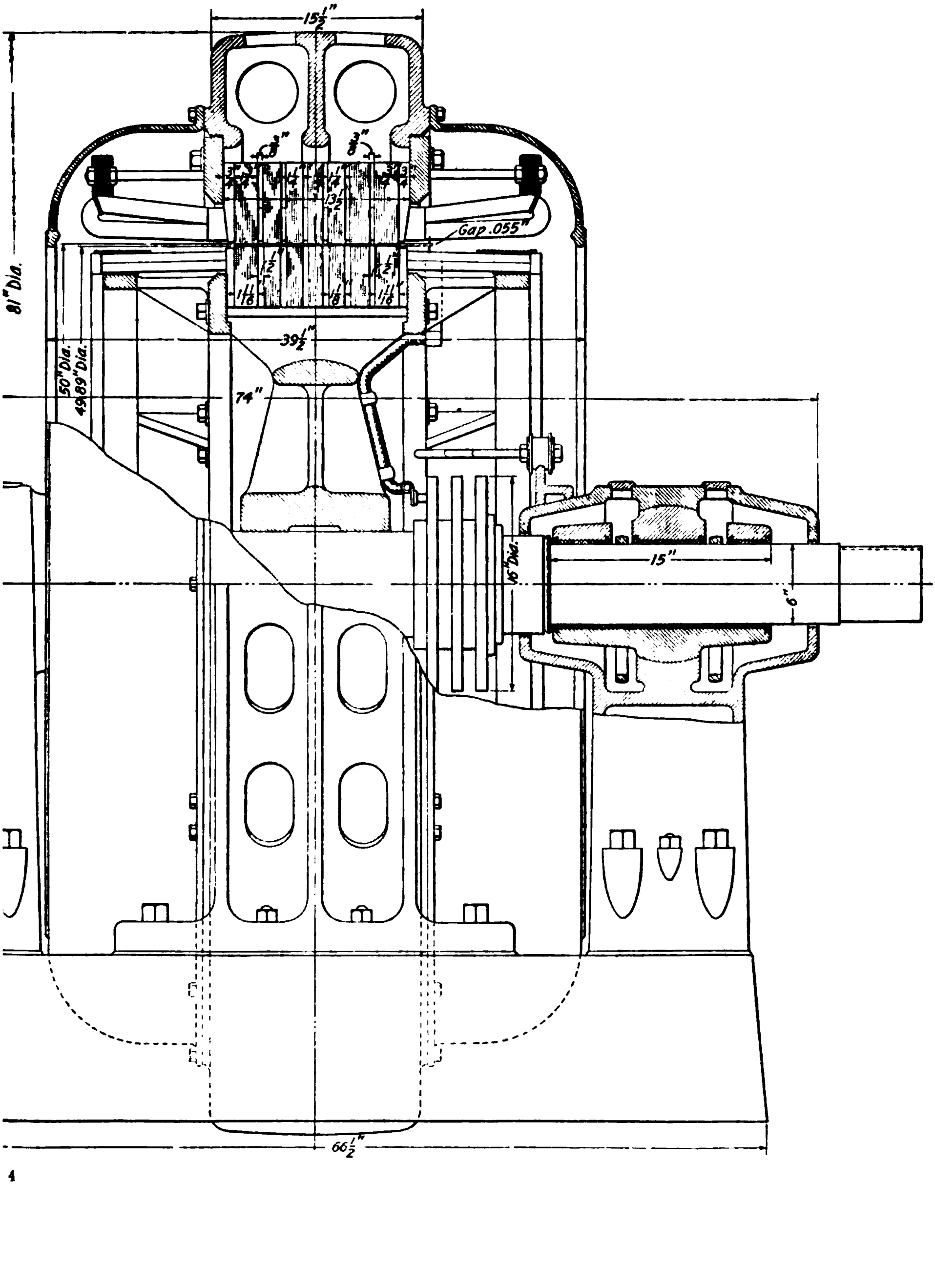
The diameter should be chosen first, with a view to using the same punchings for motors of different outputs by simply making the cores of different lengths. The pole pitch S_i and length l depend on the diameter chosen, and the diameter should be such that the lengths of the different cores using this diameter may be between the limits $.75\,S_i$ and $2\,S_i$. Large diameters are desirable, but care must be taken that the peripheral velocity is not excessive. In this case the value $d=50$ inches will be chosen, making the peripheral velocity $\frac{\pi d}{12}\times300=3{,}930$, which is not too high. Then, $d^2 l=50\times50\times l$ $=30{,}000$, and $l=\frac{30{,}000}{2{,}500}=12$ inches. $S_i=\frac{\pi d}{10}=15.7$ inches, and $l=.764\,S_i$.

21. The **number of stator slots** per pole in induction motors usually is made 12 or 18 so that the stator can be wound for either two-phase or three-phase circuits. The larger number will be chosen here, because with 50 inches diameter the teeth will still be large enough to carry the flux without excessive densities. The total number of slots will be 180, and the slot pitch will be $\frac{50\,\pi}{180}=.873$ inch.

22. The **number of stator conductors** will depend on the method of connecting them, that is, whether the connection is star or delta. Star connection will require fewer conductors of larger size and will be tried. The number of conductors per slot will then be $\frac{K}{I_p}\times$slot pitch, or approximately $\frac{1{,}100}{117}\times.873=8.2$. Eight conductors per slot in two layers will



FIG



15 1/2"
3/8"
3/8"
Gap .055"
81" Dia.
50" Dia.
49.89" Dia.
39 1/2"
74"
16" Dia.
15"
6"
66 1/2"

be used, each coil having four turns. The number of ampere-conductors per inch with eight conductors per slot will be $K=\frac{8\times117}{.873}=1{,}070$. The number of turns in series per phase will be $T_p=\frac{180}{3}\times4=240$.

23. The **flux per pole** can now be determined by the general formula $E_p=\frac{k_2\,\Phi\,T_p\,f}{10^8}$, or $\Phi=\frac{10^8\,E_p}{k_2\,T_p\,f}$. With star connections, the counter electromotive force generated in each phase will be $E_p=\frac{2{,}200}{\sqrt{3}}$, and $\Phi=\frac{10^8\times2{,}200}{4.25\times240\times25\times\sqrt{3}}=4{,}990{,}000$ lines of force, approximately.

24. The **construction of the stator core** is indicated in Fig. 4. The axial length of the core punchings must be 12 inches and outside the punchings at each end is a $\frac{3}{4}$-inch finger plate, making the over-all length of the core $13\frac{1}{2}$ inches. Six $\frac{3}{8}$-inch ducts are placed so that the core laminations will be in two $1\frac{3}{4}$-inch blocks and five $1\frac{1}{4}$-inch blocks.

25. The **width of the stator teeth** must be such that the maximum flux density in the teeth B_t will be approximately 115,000 lines per square inch, making the average density $.636\times115{,}000=73{,}140$ lines per square inch. There are 18 teeth per pole, and the combined areas of their ends must be $\frac{4{,}990{,}000}{73{,}140}=68$ square inches. The gross axial length of the laminations is $2\times1\frac{3}{4}+5\times1\frac{1}{4}=9\frac{3}{4}$, and the net length is about $.9\times9.75=8.78$ inches. The width of each tooth must then be approximately $\frac{68}{18\times8.78}=.43$ inch, leaving $.873-.43=.443$ inch for the slot. In this case the **slot width** will be made .445 inch and the tooth width .428 inch, making the maximum density in the teeth $B_t=\frac{4{,}990{,}000}{.636\times18\times8.78\times.428}=116{,}000$ lines per square inch.

26. The **insulation on the stator coils** and the space required in the slot are as follows:

MATERIAL	WIDTH INCHES	DEPTH INCHES
1 layer .005-inch staybinding, butt joint in the slot	.012	.012
1 layer .005-inch varnished cloth half lapped	.024	.024
1 turn .015-inch flexible mica lapped on top	.030	.045
$2\frac{3}{4}$ turns .007-inch fish paper	.042	.035
Total space required in slot	.108	.116

NOTE.—The staybinding and the varnished cloth swell to .006 inch when impregnated.

27. The **dimensions of the stator conductors** will be chosen with a view to arranging them one wide and eight deep in the slot, four conductors in each coil. The cotton covering on the conductor will occupy .016 inch, the coil insulation .108 inch; an allowance of .025 inch should be made for an oversized conductor, and an allowance of .035 inch clearance between the coil and the sides of the slot, making a total of approximately .184 inch difference between the width of bare copper and the width of the slot. The width of the conductor will, therefore, be about $.445-.184=.261$ inch.

The cross-section should be approximately 700 circular mils per ampere, or $117\times700=81{,}900$ circular mils. No. 2 square wire is .2576 inch on a side and with beveled corners has a sectional area of 81,500 circular mils. The current density with this wire will be $81{,}500\div117=696$ circular mils, and the kilo-amperes per square inch $I_k=\dfrac{1{,}000{,}000}{.7854\times696\times1{,}000}=1.83$.

28. The **slot depth** must be sufficient to contain two sides of coils, each side containing four double-cotton-covered conductors measuring $.2576+.016=.2736$ inch over the insulation. The coil insulation will be .116 inch, a clearance of .05 inch will be allowed for each coil, and the slot wedge will occupy .1 inch, making the total depth required for two sides of coils, clearance, and wedge $2\times(4\times.2736+.116+.05)+.1=2.62$ inches.

29. The **heating of the stator coils** can now be checked by the formula $P_s = \frac{.395\, k_a\, t_o\, I_k}{W+D}$, in which the kiloamperes per inch of inside armature circumference $k_a = \frac{1,070}{1,000} = 1.07$, the slot pitch $t_o = .873$, $I_k = 1.83$, the slot width $W = .445$, and the slot depth under the wedge $D = 2.52$. Then, the watts per square inch of coil surface dissipated as heat will be

$$P_s = \frac{.395 \times 1.07 \times .873 \times 1.83}{.445 + 2.52} = .227$$

This is a low rate of heat dissipation, and an experienced designer will know that the temperature of the conductor will not be excessive. However, the excess of this temperature above that measured by a thermometer can be checked by the formula $T = \frac{P_s O}{.003}$, in which the thickness of insulation may be taken as half the difference between the widths of the slot and the conductor, or $O = \frac{.445 - .2576}{2} = .0937$. Then, $T = \frac{.227 \times .0937}{.003} = 7.1°$ C., and when a thermometer applied to the stator core shows a temperature rise of 40° C. above that of the surrounding atmosphere at 25° C., the conductor temperature will still be considerably below the safe maximum limit.

30. The **depth of the stator core** back of the slots must be enough to carry $\frac{4,990,000}{2} = 2,495,000$ lines of force at a density not exceeding 85,000 lines per square inch. The cross-section must therefore be not less than $\frac{2,495,000}{85,000} = 29.4$ square inches. The net length of the core is 8.78 inches, and the minimum depth must be $\frac{29.4}{8.78} = 3.35$ inches. It will be made 3.38 inches, making the total thickness of the core and the teeth $3.38 + 2.62 = 6$ inches. The outside diameter of the core will be $50 + 2 \times 6 = 62$ inches.

PHASE-WOUND ROTOR DESIGN

31. The **outside diameter of the rotor** will be made as large as may be and leave safe mechanical clearance, or air gap, between the rotor and the stator. For a 50-inch stator the minimum safe clearance is .055 inch, and the rotor diameter will then be $50-2\times.055=49.89$ inches.

32. The **number of rotor slots** can be chosen between fairly wide limits, but best results will be obtained if this number is five-sixths of the number of stator slots, or, in this case, $\frac{5}{6}\times180=150$, or 15 per pole. The function of the rotor winding is to provide paths for the induced current, and the smaller the number of conductors the larger must be their size in order to carry this current without overheating. Too few rotor slots and conductors would necessitate the use of conductors so large as to localize the heating too much, while too many slots would make the construction unnecessarily expensive.

33. The **rotor core** can be made of .025-inch punchings, and only alternate punchings need be japanned, because the frequency with which the magnetic density changes in this core is very low, and the eddy-current loss as compared with that of the stator will be small. The net length of steel in the rotor core will therefore be about 95 per cent. of the gross length. The rotor slots should be shallow so that the conductors will be near the surface. The magnetic leakage will thus be minimized and the power factor improved.

The slot pitch at the rotor surface will be $\frac{49.89\times3.1416}{150}$ $=1.045$ inches, and the **slot width** may be approximately half of this, or, say, .53 inch. Of this width, .14 inch should be allowed for insulation and clearance, leaving .39 inch for conductor.

34. The **rotor winding** will be of the two-layer wave type with two conductors per slot, full-pitch coils, star connected. The secondary voltage with the rotor at rest and 2.200 volts applied to the stator terminals will be $E_s=\frac{150\times2}{180\times8}$

$\times 2{,}200 = 458.3$. The full-load output in watts is $500 \times 746 = 373{,}000$, and the rotor power factor will be about .93. The secondary current I_s in each phase can be calculated from the fact that $\sqrt{3}\, I_s E_s \times .93 = 373{,}000$, or $I_s = \frac{373{,}000}{\sqrt{3} \times 458.3 \times .93} = 506$ amperes.

The current density in the rotor conductors may be made approximately 2,200 amperes per square inch, for which a cross-section of $506 \div 2{,}200 = .23$ square inch. is required, and this cross-section divided by the assumed thickness, .39 inch, gives .59 inch as the depth of the conductor. Best results will be obtained if the conductor is made of thin strips, say three strips .1 in.$\times$.6 in. and one strip .09 in.$\times$.6 in. The cross-sectional area will be $.39 \times .6 = .234$ square inch, and the current density $506 \div .234 = 2{,}160$ amperes, or 2.16 kiloamperes, per square inch.

35. The **slot depth** must, in this case, be enough to contain two sides of coils made of .6-inch conductor with, say, .125-inch insulation, and about .1 inch must be allowed for the overhang of the teeth above the winding, making the total depth $2 \times (.6 + .125) + .1 = 1.55$ inch.

36. The **flux density** in the rotor-tooth roots should be checked. The rotor diameter at the tooth roots will be $49.89 - 2 \times 1.55 = 46.79$ inches, and the slot pitch with this diameter will be $\frac{46.79 \times 3.1416}{150} = .98$ inch, making the minimum width of a tooth $.98 - .53 = .45$ inch.

The axial length of the rotor core, excluding the finger plates, will be made the same as the stator core, 12 inches, as indicated in Fig. 4, and the number of ventilating ducts will be the same, namely, 6. But to allow for end play of the rotor, each of its ducts will be made $\frac{1}{2}$ inch wide, making the combined widths of the ducts 3 inches. The gross length of laminated core will be 9 inches, and the net length $9 \times .95 = 8.55$ inches. The combined sectional areas of the 15 tooth roots under each pole will be $15 \times .45 \times 8.55 = 57.7$ square inches. The maximum

flux density in these roots, neglecting leakage, will be $B_t = \frac{4,990,000}{57.7 \times .636} = 136,000$ lines per square inch, which is satisfactory.

37. The **internal diameter of the rotor core** should be selected so as to leave enough metal to make the flux density about 100,000 lines per square inch. The core section should, therefore, be $\frac{4,990,000}{2 \times 100,000} = 24.95$ square inches, and as the net length of iron in the core is 8.55 inches, the depth under the slots must be approximately $24.95 \div 8.55 = 2.92$ inches. The internal diameter would then be $46.79 - 2 \times 2.92 = 40.95$; it will be made 41 inches, making the core density 101,000 lines per square inch.

SQUIRREL-CAGE ROTOR

38. Either a phase-wound rotor or a squirrel-cage rotor can be used in an induction-motor stator. The outside and inside diameters of the rotor punchings and the number of rotor slots can be the same in both cases, but the size of the rotor slots must be suited to the winding.

Let it be assumed that a squirrel-cage rotor is to be designed for the stator already designed and that this rotor is to start with 20 per cent. of full-load torque when the starting voltage is 40 per cent. of full-rated voltage.

39. The **rotor bars** will be designed for the current that will be necessary to make the total number of ampere-conductors on the rotor about 4 per cent. less than the total number on the stator, because the rotor carries no current corresponding to the magnetizing current in the stator conductors. The effective current in each rotor bar, of which there is one per slot, will therefore average $\frac{.96 \times 180 \times 8 \times 117}{150} = 1,080$ amperes. These bars will need little insulation, and heat can escape from them readily; therefore, the current density in them can be made approximately 3,000 amperes per square inch, for which a sectional area of $1,080 \div 3,000 = .36$ square

inch is required. If the section is made .6 inch square, the teeth will be too thin, as the slot pitch at the rotor surface is only 1.045 inches. Bars .55 in. × .65 in. will therefore be used, and the slots will be made .6 inch wide and .8 inch deep, .1 inch of this depth being the allowance for overhang of the teeth to hold the bars in place.

The total length of each rotor bar will be 21 inches, and the effective length between centers of the end rings will be approximately 19.5 inches. The sectional area of each bar in circular mils is $\frac{.65\times.55\times10^6}{.7854}=455{,}000$, and its resistance at 1 ohm per circular-mil inch will be $\frac{19.5}{455{,}000}$ ohm.

40. The slot pitch at the tooth roots is $\frac{(49.89-2\times.8)\times3.1416}{150}$ $=1.01$ inches, and the minimum tooth width is $1.01-.6$ $=.41$ inch. The **maximum flux density in the tooth roots** is, therefore, $B_t=\frac{4{,}990{,}000}{.636\times15\times.41\times8.55}=149{,}000$ lines per square inch, which is within practicable limits.

The calculation of the flux density in the rotor core is unnecessary, because the slots are not as deep as for the phase winding and the core density will therefore be lower.

41. The **size of the end rings** will depend on the material of which they are made and on the loss in them. The slip of such a motor at full load should be between $3\frac{3}{4}$ and 4 per cent. and the energy loss in the rotor winding will be the same percentage of the full-load output, 373,000 watts, or, say, 14,500 watts. The rotor current per bar is 1,080 amperes, the resistance per bar $\frac{19.5}{455{,}000}$ ohm, and the number of bars 150. The copper loss in the bars will be $\frac{1{,}080^2\times19.5\times150}{455{,}000}=7{,}500$ watts, leaving $14{,}500-7{,}500=7{,}000$ watts for the end rings.

The end rings are a little less in diameter than the rotor core and the slot pitch for the end rings may be taken as .96 inch. The

length of the current path in the end ring between adjacent bars may be taken at $.9\times.96=.864$, or, say, .86 inch. Copper end rings with resistance at 1 ohm per circular-mil inch would have resistance between bars $R=\frac{.86}{\text{circular mils}}=\frac{.86\times.7854}{10^6 \text{ square inches}}$. By the formula of Art. 8, $R=\frac{5p^2\times I^2 R \text{ loss in both rings}}{N^3 I^2_e}$.

Then, $\frac{.86\times.7854}{10^6 \text{ square inches}}=\frac{5p^2\times I^2 R \text{ loss}}{N^3 I^2_e}$, and the approximate sectional area in square inches is

$$\frac{.86\times.7854 N^3 I^2_e}{10^6\times5p^2\times I^2 R\text{ loss}}=\frac{.86\times.7854\times150^3\times1{,}080^2}{10^6\times5\times10^2\times7{,}000}=.76$$

The rings will be made of hard-rolled copper $\frac{3}{8}$ inch by 2 inches in cross-section.

42. Power Factor.—The magnetizing current of an induction motor lags 90 electrical degrees behind the line voltage and remains practically constant in value at all loads. The power component of the no-load current is very small and is in phase with the line voltage; this component is required to make up the steel and friction losses, called *fixed losses*, because they remain practically constant at all loads. The copper losses, which are almost negligible at no load, vary with the square of the current.

The power factor at no load is therefore very low, as can be proved by measuring the watts, volts, and amperes input when the motor is running idle, and then dividing the watts by the volt-amperes. If these measurements are repeated with the motor loaded, it will be found that the power factor increases as the load is increased, this increase being due to the increased power component of the current.

CIRCLE DIAGRAM

43. The performance of an induction motor can be well represented by a **circle diagram,** also frequently called a *Heyland diagram* after the mathematician who first demonstrated its use By constructing such a diagram when designing a motor, the designer can predict very closely the current and

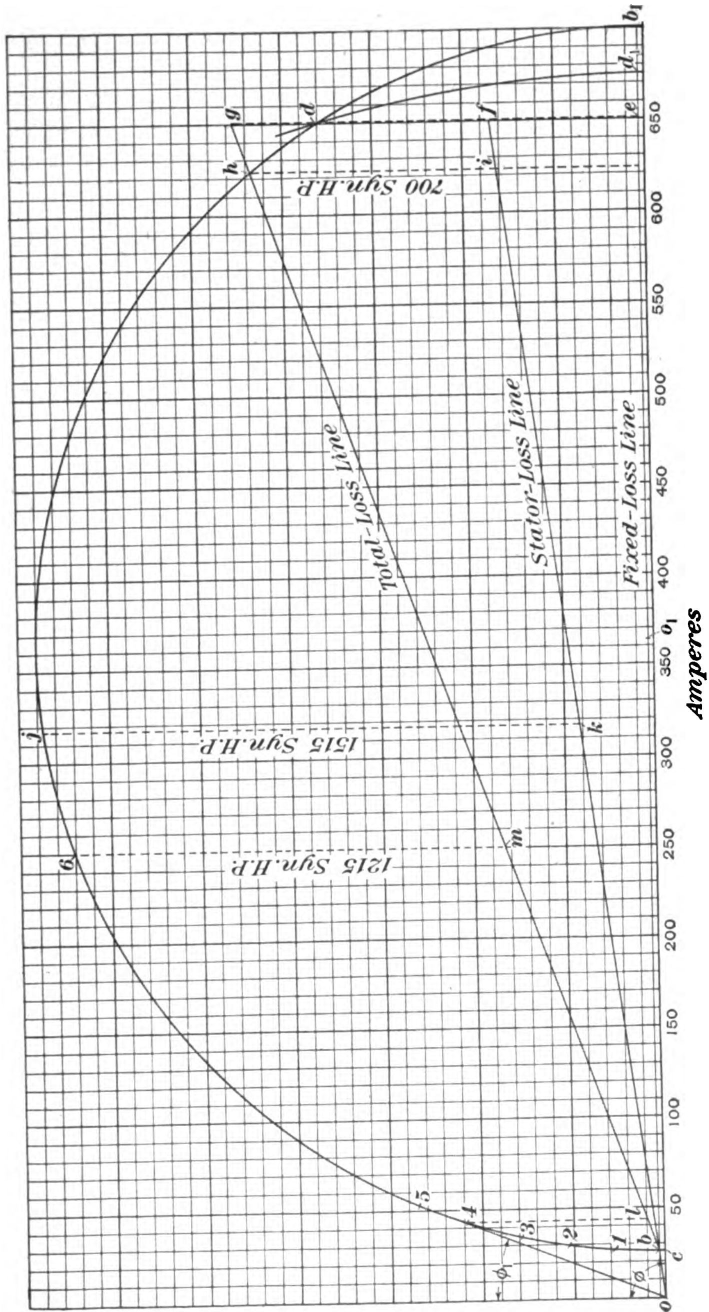

Total-Loss Line
Stator-Loss Line
Fixed-Loss Line
700 Syn.H.P
1515 Syn.H.P
1215 Syn.H.P
0
50
100
150
200
250
300
350
400
450
500
550
600
650
Amperes

FIG. 5

torque conditions, the efficiency, and the power factor. Such a diagram for the squirrel-cage motor here being designed is shown in Fig. 5 and the method of constructing it will be explained. The appearance of the diagram for the phase-wound motor as well as the method of construction is approximately the same.

44. Let the vertical line *o a* represent an instantaneous direction of the applied voltage. A scale is chosen such that a given distance in any direction on the diagram represents a certain value of primary, or stator, current. In this case, let one side of a small square represent 10 amperes. From the point *o* a horizontal *base line* of indefinite length is drawn. Parallel with this line and at a distance above it representing the component of the stator current required to meet the fixed losses is drawn the **fixed-loss line** $b\ b_1$.

In order to locate the fixed-loss line, the steel losses must be calculated as for an alternator. The weight of steel in the stator teeth will be about 550 pounds and in the core about 1,950 pounds. The loss in the teeth will be approximately 6 watts per pound and in the core 2.5 watts per pound. These values are a little higher than those indicated in Fig. 39, *Design of Alternating-Current Machines*, Part 1, owing to the small air gap and the high frequency of the tooth pulsations in an induction motor. The steel loss in the rotor at full speed will be small, because the frequency will then be very low. The friction and windage loss in a machine of this size and speed, together with the small steel loss in the rotor, may be assumed at 3,000 watts. The total fixed losses will be $550\times6+1{,}950\times2.5+3{,}000=11{,}175$, or, say, 12,000 watts. The primary current equivalent will be $\frac{12{,}000}{\sqrt{3}\times2{,}200}=3.15$ amperes, which is represented by the distance between the base line and the fixed-loss line.

45. In the example of Art. **13**, the conditions assumed are those of the phase-wound motor under consideration here, and the magnetizing current was found to be 26.4 amperes. For the squirrel-cage motor the magnetizing current will be about 26 amperes, and a distance *o c* is laid off on the base

line proportional to this current. From c a perpendicular is erected to the fixed-loss line at b and the line $o\,b$ is drawn. This line represents the no-load current, which lags by an angle ϕ behind the voltage.

46. The circle diagram of an induction motor can readily be drawn from experimental data by first observing the value and phase position of the no-load current and drawing its vector, as $o\,b$, Fig. 5. The circle and the fixed-loss line will intersect in this point. The current and its phase relation are then measured at one or more additional loads; the ends of the current vectors thus formed will be in points, as *1*, *2*, ... *6*, through which a semicircle can be drawn from a center on the fixed-loss line.

47. When designing a motor, however, the diameter of the circle may be taken as equal to the quotient $\dfrac{\text{magnetizing current}}{\text{leakage factor}}$. The leakage factor depends on the dispersed magnetic flux that surrounds either the primary or the secondary conductors alone. The larger this magnetic leakage, or dispersion, the larger will be the leakage factor and the greater will be the reactance of the windings. After a motor is completed and its circle diagram drawn from experimental data, its leakage factor is known, because it is equal to the quotient $\dfrac{\text{magnetizing current}}{\text{diameter of circle}}$.

In most cases the leakage factor will lie between the values .03 and .08. It is equal to the quotient $\dfrac{\text{air gap} \times C}{\text{pole pitch}}$ where C is a coefficient with limiting values 10 and 20. Experience with other motors will soon teach a designer to estimate the value of this coefficient for any given machine. For the squirrel-cage motor now being designed it may be taken at 11 and for the phase-wound motor at 15. The air gap is .055 inch and the pole pitch 15.7 inches; therefore, the leakage factor is $\dfrac{.055 \times 11}{15.7} = .0385$ for the squirrel-cage motor and $\dfrac{.055 \times 15}{15.7}$

$=.0525$ for the phase-wound motor. The circle diameter should therefore represent $\frac{26}{.0385}=675$ amperes for the squirrel-cage motor and $\frac{26.4}{.0525}=503$, or, say, 500 amperes, for the phase-wound motor. The semicircle, Fig. 5, is therefore drawn with a center o_1 and a radius $o_1\,b$ representing $675\div2$ amperes.

48. Copper Losses.—The stator winding is in 180 slots with eight conductors per slot, and the conductor section is 81,500 circular mils. The mean length of turn is 78 inches, and the resistance at 1 ohm per circular-mil inch will be $\frac{180\times8\times78}{2\times81{,}500}=.69$ ohm. The maximum current, 675 amperes, would therefore cause maximum stator loss of $675^2\times.69=314{,}000$ watts, and the equivalent primary current component per phase is $\frac{314{,}000}{\sqrt{3}\times2{,}200}=82.5$ amperes.

With a center at o and a radius equivalent to 675 amperes, draw an arc intersecting the semicircle at d and the fixed-loss line at d_1. Draw the line $d\,e$ perpendicular to the fixed-loss line, and from e lay off on this perpendicular a distance $e\,f$ equivalent to 82.5 amperes. Draw the **stator-loss line** $b\,f$.

49. The rotor winding is designed for a copper loss of 14,500 watts at full load, 117 amperes. At 675 amperes the corresponding loss would be $\frac{675^2}{117^2}\times14{,}500=483{,}000$ watts. With the rotor locked and full voltage applied to the stator, the rapid flux changes in the rotor would cause a *stray loss* in the rotor steel and in the rotor conductors amounting to probably 50 or 60 kilowatts. The total rotor loss at the lock point will therefore be approximately 540,000 watts, corresponding to $\frac{540{,}000}{\sqrt{3}\times2{,}200}=142$ amperes stator current. From the point f, Fig. 5, lay off a vertical distance $f\,g$ representing 142 amperes and draw the *total-loss line* $b\,g$.

50. Starting Torque.—The vertical distance between the stator-loss line and the total-loss line represents the input to the rotor, which is proportional to the starting torque. For example, the distance *h i* scales 137 amperes, indicating $\frac{137\times\sqrt{3}\times 2{,}200}{746}=700$ synchronous horsepower input to the rotor at the lock point with full voltage applied to the stator. If the motor is started with full voltage, the stator current will be as represented by the distance *o h*, which scales 660 amperes, or approximately 5.64 times the assumed full-load current of 117 amperes. The maximum input to the rotor is indicated by the distance *j k*, which scales 297 amperes, corresponding to 1,515 synchronous horsepower. The point *j* is located where a tangent to the curve is parallel to the stator-loss line.

The motor was specified to develop 20 per cent. of full-load torque with 40 per cent. of full voltage, or a torque equivalent to $.2\times 500=100$ horsepower at synchronous speed. Since the torque varies as the square of the applied voltage, the starting torque with full voltage, 100 per cent., must be equivalent to $\left(\frac{100}{40}\right)^2\times 100=625$ synchronous horsepower. The 700 horsepower indicated by the circle diagram will leave ample margin for stray losses when starting.

51. Pull-Out Torque.—The vertical distance between the total-loss line and the curve represents the rotor output, in terms of stator current. With the assumed full-load current, 117 amperes, represented by the distance *o 4*, Fig. 5, the rotor output is represented by the distance *l 4*, which scales about 100 amperes, indicating approximately 510 synchronous horsepower; this shows that the assumed full-load current is a little too large. If the load on the motor is increased, the full-load point rises higher on the curve, and the output increases. With an input *o 6*, the output becomes maximum and is represented by the distance *6 m*, which scales 238 amperes, indicating approximately 1,215 synchronous horsepower.

This is the *break-down horsepower* of the motor, and the torque corresponding to it is the *pull-out torque*. If the load is increased

beyond this point the motor will stop, the current input will increase to a value represented by the distance *o h*, and all the input will be converted into losses. Under such a condition the motor would probably burn out immediately if a circuit-opening device did not operate; in fact, the increase of load even to the pull-out point would injure the motor unless the increased load were of very brief duration.

52. Effect of Resistance in Rotor Circuit.—By increasing the resistance of the rotor winding, the lock point *h* can be carried back toward the point of maximum rotor input at *j*, thus enabling the motor to start a given load with less current input. Increasing the resistance of a squirrel-cage rotor, however, increases the losses and the heating while the motor is operating, and thus lowers the efficiency. The chief advantage of a phase-wound motor is the possibility of starting with high rotor resistance and cutting resistance out of the rotor circuit as the speed accelerates. Such motors can therefore be made to start heavy loads with less starting current than is required by squirrel-cage motors. The chief disadvantage of such motors is the higher cost of construction.

53. Power Factor and Efficiency.—The **power factor** with any current input can now be calculated from data given by the diagram. From the point *o* a distance proportional to the current input gives a point on the circle, and the distance from this point to the base line represents the power component with the given input. The power factor is equal to the power component divided by the total input. Points on the curve of power factors may be found for inputs of 38, 60, 86, 117, and 145 amperes, which represent approximately $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, full, and $1\frac{1}{4}$ load, respectively. With a center at *o* and with radii proportional to these currents, points *1*, *2*, *3*, *4*, and *5* are located on the curve. The amperes represented by the vertical distances of these points from the base line are 27, 52.5, 79, 111, and 137, and the corresponding power factors are $\frac{27}{38} = .71$, $\frac{52.5}{60} = .875$, $\frac{79}{86} = .918$, $\frac{111}{117} = .949$, and $\frac{137}{145} = .945$.

54. The **efficiency** can be obtained more accurately by calculating the losses rather than by scaling them from the diagram, Fig. 5, since the vertical distances from the base line to the total-loss line cannot be scaled accurately at working loads. The stator resistance is .69 ohm, and the copper loss in the rotor at full load, 117 amperes, is 14,500 watts. From these numbers can be calculated the copper losses at any load. For example, at $\frac{3}{4}$ load, the rotor $I^2 R$ loss will be $\frac{86^2}{117^2}=\frac{x}{14{,}500}$, or $x=\frac{86^2\times 14{,}500}{117^2}=7{,}830$, approximately. These losses plus the fixed loss, 12,000 watts, give the total loss, and this total loss taken from the input gives the output. The efficiencies can be calculated from the data on input and output, as shown in the following tabulation:

Stator current I...	38	60	86	117	145
Power factor, $\cos\phi$	.71	.875	.918	.949	.945
Input, $\sqrt{3}\,I\,E\cos\phi$	103,000	200,000	301,000	423,000	522,000
Stator I^2R loss ...	1,000	2,485	5,100	9,450	14,500
Rotor I^2R loss....	1,530	3,810	7,830	14,500	22,300
Fixed loss.........	12,000	12,000	12,000	12,000	12,000
Output, watts	88,470	181,705	276,070	387,050	473,200
Efficiency, per cent.	85.9	90.9	91.7	91.5	90.7

SYNCHRONOUS MOTORS

DESIGN OF AN 800-HORSEPOWER SYNCHRONOUS MOTOR

55. The design of a synchronous motor follows the same general lines as the design of an alternator except that the squirrel-cage winding on the rotor is designed like that of an induction motor. The specifications submitted to the designer usually cover output, speed, number of phases, frequency, voltage, power factor, efficiency, exciter voltage, temperature limits, overload capacity, starting torque, and starting current.

The designer first calculates the input in kilovolt-amperes corresponding to the specified horsepower, efficiency, and power factor, and then proceeds to design an alternator for this number of kilovolt-amperes output. If the overload capacity is to be large, the alternator is designed for good regulation; otherwise, the design will be for poorer regulation.

56. For example, suppose that a synchronous motor of the revolving-field, salient-pole type is to be designed for 800 horsepower output, 450 revolutions per minute on a 3-phase, 60-cycle, 2,150-volt circuit, with 90 per cent. power factor and leading current, 94 per cent. efficiency at full load, 120 volts excitation, 35° C. maximum temperature rise, overload capacity $2\frac{1}{4}$ times full load, and starting torque 45 per cent. of full-load torque at synchronous speed without taking more than 1.2 times full-load kilovolt-amperes from the line when starting with the aid of autotransformers.

57. The approximate input at full load will be $\frac{800 \times 746}{.9 \times .94 \times 1{,}000} = 705$ kilovolt-amperes and the overload capacity is large; hence, the design of an alternator with good regulation

will be worked out for this output. The general method will be precisely the same as followed when designing an alternator, and only a general outline of the calculations will accordingly be given. The following assumptions will be made:

Density in air gap, $B_g = 45{,}000$ lines per square inch.

Ratio $\frac{\text{pole arc}}{\text{pole pitch}} = 70$ per cent.

Number of ampere-conductors per inch of armature circumference $K = 465$.

Distribution factor $k_w = .96$.

Pitch factor $k_o = 1$.

Then,

$$d^2 l = \frac{5.48 \times 10^8 P}{\% B_g K S k_w} = \frac{5.48 \times 10^8 \times 705{,}000}{.7 \times 45{,}000 \times 465 \times 450 \times .96} = 61{,}000$$

58. The diameter d will be made 65 inches, and the length l will be $14\frac{1}{2}$ inches, making $d^2 l = 61{,}260$, and the peripheral velocity $\frac{\pi d}{12} \times 450 = 7{,}660$ feet per minute. The number of poles $p = \frac{120 f}{S} = \frac{120 \times 60}{450} = 16$, and the pole pitch is $\frac{65 \pi}{16} = 12.75$. The pole arc will be $12.75 \times .7 = 8.925$, or, say, 9 inches. The number of slots per pole per phase will be made 3, giving 9 slots per pole and 144 slots in all. The slot pitch $t_o = \frac{65 \pi}{144} = 1.42$ inches. The slot width will be made .6 inch and the minimum tooth width is $1.42 - .6 = .82$ inch.

59. The active flux per pole will be $\Phi = 14\frac{1}{2} \times 9 \times 45{,}000 = 5{,}870{,}000$ lines of force. The armature windings will be delta connected, and $E_p = 2{,}150 = \frac{4.44 \times 5{,}870{,}000 \times T_p \times 60 \times .96}{10^8}$, from which the number of turns per phase $T_p = 143$, or, say, 144, making nine turns per pole per phase and six conductors per slot.

The total kilovolt-amperes is $705 = 3\, E_p\, I_p$; and the current per phase $I_p = \frac{705{,}000}{3 \times 2{,}150} = 109.3$ amperes, and the current per

terminal $109.3\times\sqrt{3}=189$ amperes. The actual number of ampere-conductors per inch of armature circumference is

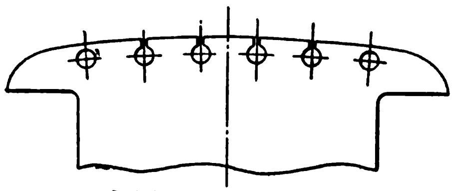
$$K=\frac{109.3\times 6\times 144}{65\,\pi}=463$$

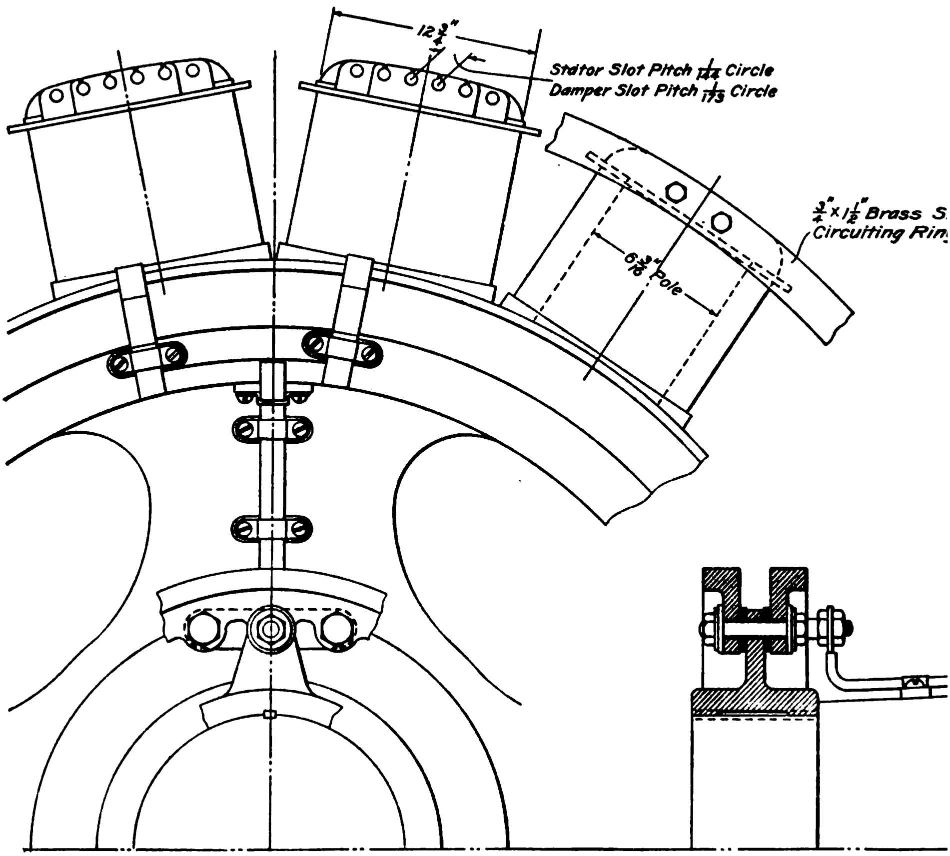
60. The conductors will be arranged six deep in the slots and .2 inch of the slot width will be allowed for insulation, leaving .4 inch for the width of conductor. For 1,800 amperes per square inch, $109.3\div 1{,}800=.061$ square inch of conductor section will be needed, making the required depth of conductor $.061\div .4=.1525$ inch. The conductor will be made of two strands of rectangular copper rod .16 inch by .2 inch. The number of kiloamperes per square inch will be

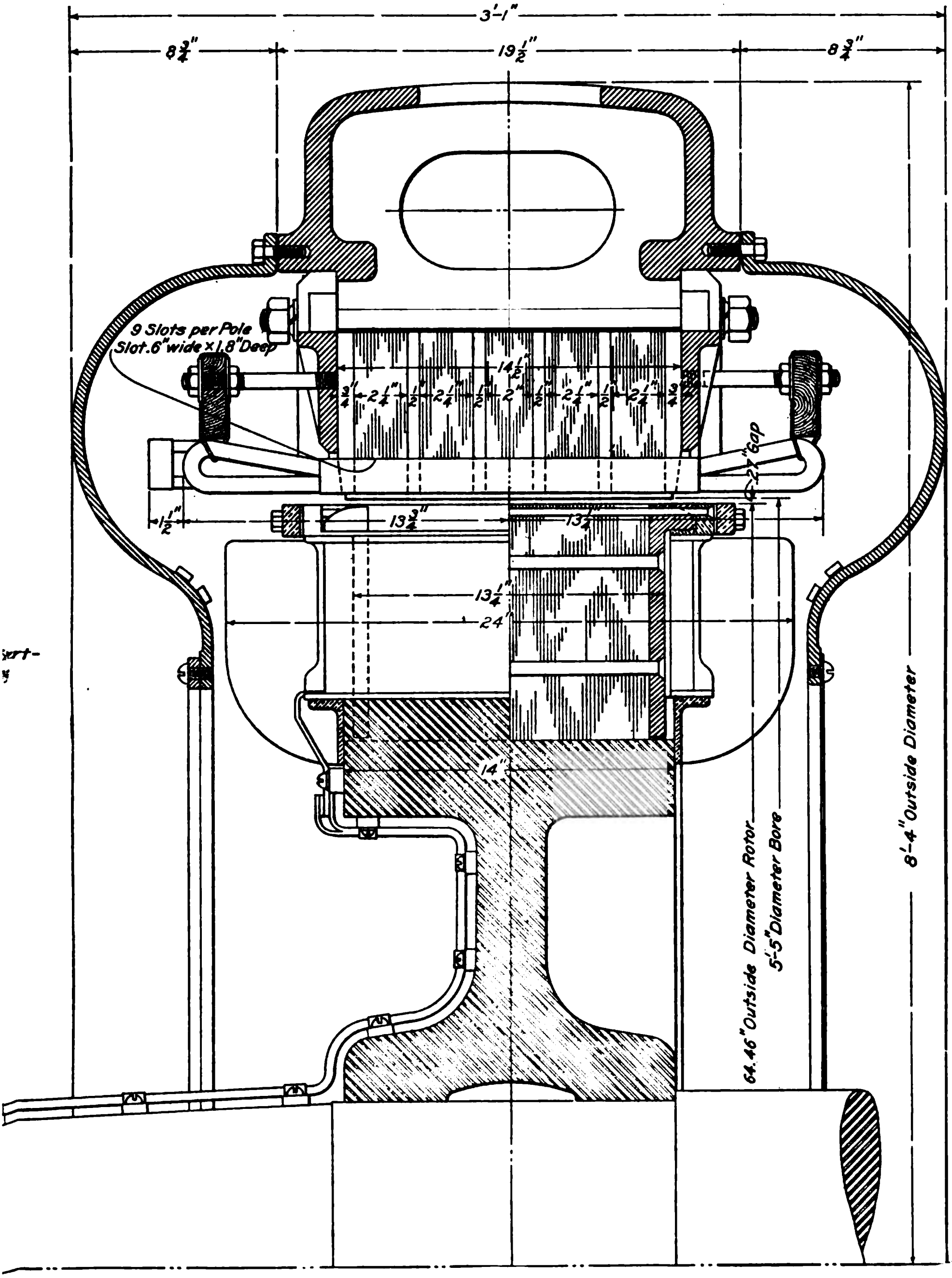
$$I_k=\frac{109.3}{2\times .16\times .2\times 1{,}000}=1.71$$

61. The conductors will occupy $6\times .16=.96$ inch in slot depth; .64 inch will be allowed for insulation, and .2 inch for a slot stick, making the depth D under the stick 1.6 inches and the total depth 1.8 inches. The number of kiloampere-conductors per inch of armature circumference is $k_a=.463$, and the number of watts per square inch of coil surface will be $P_s=\frac{.395\,k_a\,t_o\,I_k}{W+D}=\frac{.395\times .463\times 1.42\times 1.71}{.6+1.6}=.202$, indicating the excess temperature of the conductor above the temperature measured on the core surface $T=\frac{P_s O}{.003}=\frac{.202\times .2}{.003}=13.5°$ C., approximately.

62. The armature core will contain four $\frac{1}{2}$-inch air ducts, as indicated in Fig. 6, and at each end will be a $\frac{3}{4}$-inch finger plate, leaving $14.5-4\times\frac{1}{2}-2\times\frac{3}{4}=11$ inches for punchings, of which 90 per cent., or 9.9 inches, will be net steel. The slot pitch at the tooth roots is $\frac{(65+2\times 1.8)\,\pi}{144}=1.5$ inches, and the average pitch is $\frac{1.42+1.5}{2}=1.46$ inches. The average tooth width is $1.46-.6=.86$ inch, and the number of teeth opposite


Detail of Slots in Pole Face


12 3/4"
Stator Slot Pitch 1/144 Circle
Damper Slot Pitch 1/173 Circle
3/4" x 1 1/2" Brass S
Circuiting Rin
6 3/16" Pole



3'-1"
8 3/4"
19 1/2"
8 3/4"
9 Slots per Pole
Slot .6" wide × 1.8" Deep
14 1/2"
3/4"
2 1/4"
1/2"
2 1/4"
1/2"
2"
1/2"
2 1/4"
1/2"
2 1/4"
3/4"
.27" Gap
1 1/2"
13 3/4"
13 1/2"
13 1/4"
24"
14"
64.46" Outside Diameter Rotor
5'-5" Diameter Bore
8'-4" Outside Diameter

FIG. 6

each pole is $\frac{9}{12.75}\times 9=6.35$. The average flux density in the teeth is $\frac{5,870,000}{6.35\times .86\times 9.9}=109,000$ lines per square inch, approximately.

The outer diameter of the stator core will be made 79 inches, making the core depth back of the slots $\frac{79-(65+2\times 1.8)}{2}$ $=5.2$ inches. The core density will be $\frac{5,870,000}{2\times 5.2\times 9.9}=57,000$ lines per square inch.

63. The mean length of a turn of the armature winding is approximately 66 inches, and the sectional area of the conductor is $\frac{2\times .16\times .2\times 10^6}{.7854}=81,500$ circular mils. At 1 ohm per circular-mil inch the hot resistance of the armature will be approximately $\frac{144\times 66}{82,500}=.115$ ohm. The voltage drop in each phase, due to resistance, will be $109.3\times .115=12.6$, and the $I^2 R$ loss in the three phases at full load will be $3\times 109.3\times 12.6$ $=4,130$ watts.

64. The number of gap ampere-turns should be about 2.5 times the number of armature ampere-turns per pole, or $2.5 \times .707\ m\ T_{pp}\ I_p = 2.5 \times .707 \times 3 \times 9 \times 109.3 = 5,200$. The length of the effective air gap must be $\frac{5,200}{.313\times 45,000}=.37$. The minimum air gap at the pole center will be made .27, and the pole will be chamfered so that the average actual gap will be .325 inch. When the correction is made for the bunching of the lines at the teeth the effective air gap will be .37 inch.

65. The dimensions of the field poles can be closely approximated, the leakage calculated, and the dimensions then corrected if necessary, as is done in the design of an alternator. The leakage will be found to be about 17 per cent. with the pole

dimensions shown in Fig. 6, and the total flux per pole will be $5,870,000 \times 1.17 = 6,870,000$ lines of force. The axial length of the entire pole core is $13\frac{1}{4}$ inches, and the laminated part is 12 inches. The net steel can be taken at 93 per cent., or

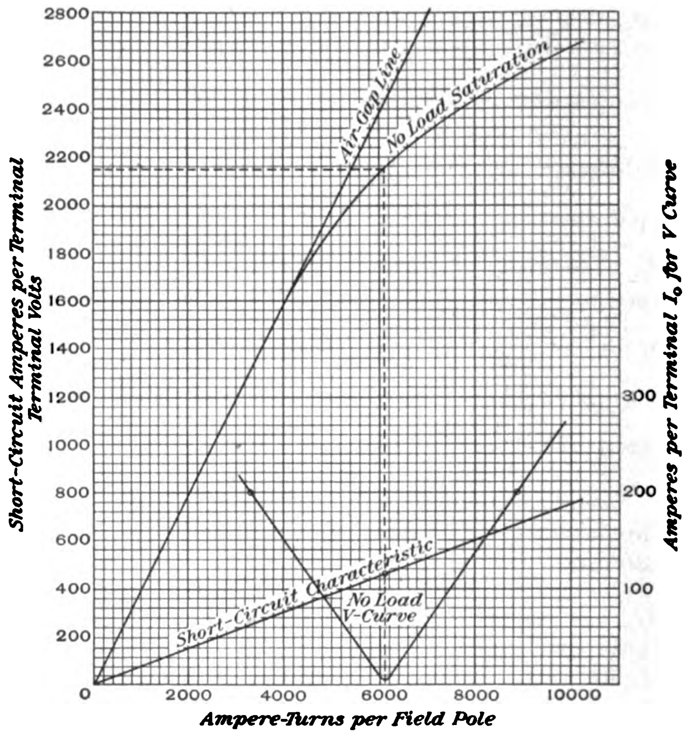


FIG. 7

$.93 \times 12 = 11.16$ inches. The pole waist is $6\frac{3}{16}$ inches, making the density $\dfrac{6,870,000}{11.16 \times 6.1875} = 99,500$ lines per square inch.

The radial length of the pole core will be made $6\frac{3}{4}$ inches, of which approximately $\frac{3}{4}$ inch will be taken by the insulating collars at the coil ends, leaving about 6 inches for the coil.

66. Outline drawings are now laid out as in Fig. 6 and the no-load saturation curve calculated and drawn as in Fig. 7.

This curve shows that 6,100 ampere-turns are required to set up 2,150 volts at the normal speed. With this excitation the current in each phase on short circuit would be $I_p = \frac{I_f T_f}{k_c m T_{pp}}$ $= \frac{6{,}100}{.85 \times 3 \times 9} = 266$ amperes, and the short-circuit current in each terminal of delta-connected phases would be $\sqrt{3} \times 266 = 460$ amperes. This current represents the maximum input to the motor when it pulls out of step. The full-load input is 109.3 amperes per phase, or 189 amperes per terminal. The overload capacity is therefore $\frac{460}{189} = 2.43$ times full load, which is slightly more than specified.

67. The short-circuit characteristic, Fig. 7, is drawn from the origin through the point determined by the coordinates 6,100 ampere-turns and 460 amperes. If the armature terminals are short-circuited when the machine is driven as an alternator, the current between them will be practically all wattless and may be treated as the wattless component of a full-load current. By assuming different wattless currents of this nature, both lagging and leading, and calculating the field excitations necessary to produce them, the relation between field excitation and wattless armature current may be determined, and from the data thus found a curve may be plotted. This curve will assume a form that gives it the name **V** *curve*, as shown for no load in Fig. 7.

Let I_o = wattless current, in amperes between terminals;
X = armature reactance, in ohms;
E_m = counter electromotive force per phase, in volts;
e = line voltage;
$I\,T_m$ = ampere-turns corresponding to E_m on no-load saturation curve;
$I\,T_r$ = total ampere-turns to set up flux corresponding to E_m.

The demagnetizing armature ampere-turns per pole are calculated by the formula $D\,I\,T_p = .9 \; m\,T_{pp}\,I_p\,k_w\,k_p$, in

which $m=3$, $T_{pp}=9$, $I_p=\frac{I_o}{\sqrt{3}}$ the armature current per phase, $k_w=.96$ and the ratio $\frac{\text{pole arc}}{\text{pole pitch}}=.7$, and $k_p=.81$. Then $D\,I\,T_p$ $=.9\times3\times9\times\frac{I_o}{\sqrt{3}}\times.96\times.81=10.9\,I_o$.

68. The armature reactance for wattless current can be calculated as follows:

Assume any short-circuit current, say 400 amperes, and find on the short-circuit characteristic the corresponding excitation, 5,300 ampere-turns. The demagnetizing ampere-turns are $D\,I\,T_p=10.9\times400=4{,}360$, leaving 940 effective ampere-turns. On open circuit 940 ampere-turns would set up 380 volts according to the no-load saturation curve, and $X=\frac{380}{400}=.95$ ohm.

For a power component the reactance is approximately 50 per cent. greater than thus calculated, and for current with a power factor of .35 to .45 the reactance X_o is about 20 per cent. greater than calculated for wattless current; that is, $X_o=1.2\,X$. These statements are based on experimental data.

69. The counter electromotive force of the motor will be greater or less than the applied electromotive force by the quantity $\frac{I_o X}{\sqrt{3}}$, depending on whether motor current leads or lags the applied electromotive force. That is, $E_m=e+\frac{I_o X}{\sqrt{3}}$ if I_o leads, and $E_m=e-\frac{I_o X}{\sqrt{3}}$ if I_o lags. Likewise, the total field excitation $I\,T_r$ required to establish a flux corresponding to E_m will be $I\,T_r=I\,T_m+10.9\,I_o$ for leading current, and $I\,T_r=I\,T_m-10.9\,I_o$ for lagging current. Different wattless currents assumed and the total field excitation for each can now be calculated as follows:

70. The no-load **V** curve, Fig. 7, is plotted with the coordinates in the columns headed I_o and $I\,T_r$ in Table I. The

abscissa of the lowest point on this curve is the number of ampere-turns corresponding to $E_m = 2{,}150$ volts, because at this point there is no phase displacement and the armature current has practically no effect on the field magnetization. If the short-circuit current were all wattless, the lowest point on the

TABLE I

CALCULATIONS FOR NO-LOAD V CURVE

I_o	$\frac{I_o X}{\sqrt{3}}$	$10.9\ I_o$	Leading Current			Lagging Current		
			E_m	$I\ T_m$	$I\ T_r$	E_m	$I\ T_m$	$I\ T_r$
50	27	545	2,177	6,200	6,745	2,123	5,950	5,405
100	55	1,090	2,205	6,400	7,490	2.095	5,800	4,710
150	82	1,635	2,232	6,550	8,185	2,068	5,700	4,065
200	110	2,180	2,260	6,700	8,880	2,040	5,500	3,320

no-load V curve would occur where $I_o = 0$; but the core loss, friction, and windage of the machine, amounting to about 30 kilowatts, causes a power component of $\frac{30{,}000}{3 \times 2{,}150} = 4.65$, or approximately 5 amperes at no load, which prevents the no-load V curve from touching the axis of abscissas.

71. The curve indicates that for full excitation at low power factors from 9,000 to 10,000 ampere-turns may be required. The mean length of a field turn will be about 42.5 inches, and at 1 ohm per circular-mil inch the approximate sectional area of the field conductor may be found by applying formula **2**, Art. **44**, *Design of Alternating-Current Machines*, Part 2, $a = \frac{I_f T_c L_f p}{e} = \frac{10{,}000 \times 42.5 \times 16}{120} = 56{,}700$ circular mils, or .0445 square inch. A copper strip .04 inch thick and $1\frac{1}{8}$ inches wide will be wound on edge with 12 mils insulation between turns, and there will be room in the 6 inches available space for $115\frac{1}{2}$ turns per coil.

The field resistance hot will be $r = \dfrac{16 \times 115.5 \times 42.5 \times .7854}{.045 \times 10^6}$ $= 1.37$ ohms. The field current at 120 volts will be $\dfrac{120}{1.37}$ $= 87.6$ amperes, the maximum excitation $115.5 \times 87.6 = 10{,}100$ ampere-turns, and the ratio of the maximum field excitation to the gap ampere-turns per pole $\dfrac{10{,}100}{5{,}200} = 1.94$.

72. A squirrel-cage winding will be embedded in slots in the pole faces and short-circuited by rings at the edges of the pole faces, as is shown in the Section *Alternating-Current Motors and Synchronous Converters*. This winding will increase the starting torque and the pull-out torque and will also decrease the inclination of the machine to ***hunt***, or periodically increase and decrease peripheral speed within narrow limits.

The squirrel-cage winding is designed like that of an induction motor. If this winding is to increase starting torque, the pitch of the rotor bars must differ from that of the stator slots and must be such that if the bars were evenly distributed over the complete cylindrical surface including the pole faces, the number of rotor bars should be prime to the number of stator slots, that is, one is the only common factor of the number of rotor bars and the number of stator slots. Moreover, the pole arc and the number of stator teeth must be so proportioned that the number of teeth opposite a pole is the same with the pole in any position.

If the squirrel-cage winding is to act only as a damper to prevent hunting, its resistance is made low and the rotor bars are placed with the same pitch as the stator slots. This equality of pitch minimizes the current in the squirrel-cage winding by equalizing the influence of the flux variations caused by the stator slots.

73. In the motor being designed, for example, the pitch of the stator slots is $\frac{1}{144}$ of a circumference. Six rotor bars will be placed in each pole face with a pitch of $\frac{1}{173}$ of a circumference. The pole faces will be so proportioned that nine stator

teeth will be opposite each pole face in every position, whether the pole center is opposite a tooth or a slot.

The rotor bars will be round copper rods $\frac{5}{8}$ inch in diameter; they will be riveted at each end to a brass collar, and a short-circuiting ring $\frac{3}{8}$ in.$\times 1\frac{1}{2}$ in. in section will be bolted to each collar. In order to reduce the reactance leakage of the squirrel-cage winding, openings will be made in the pole face over each rotor bar, as shown in the detail sketch, Fig. 6, except over the bars nearest the pole tips; here the metal is needed for mechanical strength.

74. Synchronous motors are usually started by applying reduced voltage E_o to the armature winding. The torque exerted by the rotor decreases as the speed increases, and frequently the starting voltage must be increased or the field must be excited in order that the motor may pull in to synchronism. The power factor, $\cos\phi$, of the starting current I_s is low, ranging from .34 to .45 in salient-pole machines and from .45 to .6 in round rotor machines.

The starting voltage E_o required for a given starting torque can be calculated. The product of the full load, in horsepower, and the fractional part of full-load torque that must be exerted in starting is the synchronous horsepower H_s equivalent to the starting torque. The watts equivalent to this synchronous horsepower is 746 H_s, and this number equals $m E_o I_s \cos\phi$; then, $I_s = \frac{746\,H_s}{m\,E_o \cos\phi}$. But $I_s = \frac{E_o}{X_o} = \frac{E_o}{1.2\,X}$; then, $\frac{E_o}{1.2\,X} = \frac{746\,H_s}{m\,E_o \cos\phi}$, and $E_o = \sqrt{\frac{1.2\times 746\,H_s\,X}{m \cos\phi}}$. The product 1.2 $\times 746 = 895.2$, or approximately 900, and $E_o = \sqrt{\frac{900\,H_s\,X}{m \cos\phi}}$.

75. The 800-horsepower motor being designed is specified to exert 45 per cent. of full-load torque in starting, or $H_s = .45 \times 800 = 360$ synchronous horsepower. The reactance $X = .95$ ohm, $m = 3$, and the power factor of the starting current will be approximately $\cos\phi = .35$. Then the starting voltage must

be $E_o=\sqrt{\frac{900\times360\times.95}{3\times.35}}=542$. The starting current will be $\frac{542}{1.2\times.95}=475$ amperes in each phase, or $\sqrt{3}\times475=822$ amperes per terminal. The input will be $\frac{3\times475\times542}{1{,}000}=772$ kilovolt-amperes, which is $\frac{772}{705}=1.09$ times the normal input.

76. During the starting period the armature current causes rapid flux changes in the magnetic circuit of a synchronous motor. If all the field coils are in series, a voltage E_f will be induced in the field circuit proportional to the product of the starting voltage and about half the ratio of the number of field turns to the number of armature turns. This fractional part, one-half, is due to leakage, and to the influence of the squirrel-cage winding. The total number of field turns is 115.5×16, and the number of armature turns is $.9\,m\,T_{pp}\,k_w\,k_p\,p=.9\times3\times9\times.96\times.81\times16$.

Then,

$$E_f=\frac{542\times115.5\times16}{2\times.9\times3\times9\times.96\times.81\times16}=1{,}660 \text{ volts}$$

It is the usual practice of American engineers to insulate the field windings of synchronous motors and synchronous converters according to the rules of the American Institute of Electrical Engineers. These rules are that the insulation must be capable of withstanding a test of twice the induced voltage plus 1,000 volts, and in no case can the test be less than 1,500 volts. For the motor here being designed, the test should be not less than $2\times1{,}660+1{,}000=4{,}320$ volts; probably 5,000 volts would be used. The induced voltage can be reduced by closing the field circuit through a resistance high enough to limit the field current to a safe value. In many cases, a field-break-up switch is installed; by means of this the field circuit can be opened in several places during the starting period.

SYNCHRONOUS CONVERTERS

77. Synchronous converters nearly always operate as synchronous motors and deliver direct current, these two functions being combined in one frame structure. In rare cases such a machine may operate inverted, receiving direct-current energy and delivering alternating-current energy.

The essential specifications to which the designer must work include the frequency of the alternating current, the voltage and rate of direct-current energy flow, and the heating limits. The design is closely allied to the design of a direct-current generator, with certain limitations for this special service. The number of poles must be selected to give a practicable speed with the specified frequency as well as with a view to keeping the angular space between pole centers large enough to obtain ample spacing between adjacent brush-holder studs.

78. The alternating current per terminal and the voltage per phase will bear to the direct current and voltage a fixed

TABLE II

CONVERSION FACTORS FOR SYNCHRONOUS CONVERTERS

System	Sine Flux Distribution		$P_a \div P_p = .75$		$P_a \div P_p = .7$		$P_a \div P_p = .6$	
	Volts	Amperes	Volts	Amperes	Volts	Amperes	Volts	Amperes
Three phase..	.600	.97	.620	.940	.660	.89	.612	.943
Four phase...	.490	.73	.500	.707	.530	.67	.500	.707
Six phase....	.347	.48	.354	.470	.377	.44	.354	.472

relation, depending on the number of phases, on the flux distribution, and to some extent on the ratio of the pole arc P_a to the pole pitch P_p. The factors by which to multiply the

direct-current volts and amperes in order to obtain the line volts and amperes alternating current are given in Table II.

The factors in the second and third column are for any ratio of pole arc to pole pitch, provided the pole faces are chamfered to give sine flux distribution. All the other factors are for machines in which the pole faces are concentric with the armature.

79. As explained in *Alternating-Current Motors and Synchronous Converters*, the heating effect of a given current in a polyphase synchronous converter is less than it would be in the same machine mechanically driven as a generator. This difference is due to the fact that the current in the armature winding of a synchronous converter at any instant is the difference between the instantaneous values of the alternating current and the direct current. Some of the alternating current is commutated and passes into the direct-current circuit without traversing any of the armature winding except the coils with which the collector rings are connected. The heating will therefore be greatest in these tap coils; and the larger the number of tap coils, the better will be the heat distribution, and the less will be the maximum heating.

The tap coils lie in slots with other coils in which the loss is less and to which some of the heat from the tap coils is conducted; the maximum heating is therefore less than the maximum loss in the tap coils may indicate. To obtain the maximum loss in the tap coils and the average loss in all the coils at any given current, the loss per coil at the same current when the machine is mechanically driven as a generator can be multiplied by a factor selected from the following:

Power Factor	Three Phase		Six Phase	
	Tap Coil	Average	Tap Coil	Average
100 per cent........	1.22	.57	.46	.27
95 per cent........	1.90	.70	.80	.37

80. When the poles of a synchronous converter are laminated they must be provided with damper windings to increase the operating stability and to prevent hunting. These windings are designed the same as for synchronous motors. Synchronous converters that are to be self-starting with alternating current are provided with squirrel-cage windings embedded in the pole faces the same as on self-starting synchronous motors.

81. At the given frequency f and speed S in revolutions per minute the number of poles $p=\frac{120\,f}{S}$. The speed and the commutator diameter must be chosen so that the peripheral velocity of the commutator surface will not be over 5,000 feet per minute. There must be as many brush-holder studs as there are poles, and if d is the distance in inches along the commutator surface between adjacent studs, $\frac{d\,p}{12}$ represents the commutator circumference in feet, and $\frac{d\,p\,S}{12}$ represents the surface velocity in feet per minute. When the value of p is substituted in this expression and it is placed equal to 5,000, the equation becomes $\frac{d\,\frac{120\,f}{S}\,S}{12}=5{,}000$, or $10\,d\,f=5{,}000$; then $d=\frac{500}{f}$. When the frequency is 60, the distance between adjacent brush holders cannot be over $8\frac{1}{3}$ inches, and it was this difficulty that formerly made impracticable the operation of 60-cycle converters to feed railway circuits. The maximum available spacing between brush holders was not sufficient to prevent flashing over at the voltages and with the heavy load fluctuations common to electric-railway circuits. The development of commutating poles has so improved commutation that such machines are now made to operate practically as well as those for 25 cycles.

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NOTE.—In this volume, each Section is complete in itself and has a number. This number is printed at the top of every page of the Section in the headline opposite the page number, and to distinguish the Section number from the page number, the Section number is preceded by a section mark (§). In order to find a reference, glance along the inside edges of the headlines until the desired Section number is found, then along the page numbers of that Section until the desired page is found. Thus, to find the reference "Alternator losses, §63, p28," turn to the Section marked §63, then to page 28 of that Section.